Annals of Pure and Applied Mathematics
Vol. 19, No. 2, 2019, 55-56
ISSN: 2279-087X (P), 2279-0888(online)
Published on 15 October 2019
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.632v20n2a1

Annals of **Pure and Applied Mathematics**

Brief Note

A Short Note on Solutions of the Diophantine Equations $6^x + 11^y = z^2$ and $6^x - 11^y = z^2$ in Positive Integers x, y, z

Nechemia Burshtein

117 Arlozorov Street, Tel – Aviv 6209814, Israel
Email: anb17@netvision.net.il

Received 14 September 2019; accepted 12 October 2019

Abstract. In this short note we investigate the solutions of the equations $6^x + 11^y = z^2$ and $6^x - 11^y = z^2$ when x, y, z are positive integers. It is shown that the first equation has no solutions, whereas the second equation has exactly one solution when $x = 2$, and no solutions for each and every value $2 < x \le 16$.

Keywords: Diophantine equations

AMS Mathematics Subject Classification (2010): 11D61

This note is concerned with the two equations $6^x + 11^y = z^2$ and $6^x - 11^y = z^2$. In Lemma 1, we show that the equation $6^x + 11^y = z^2$ has no solutions.

Lemma 1. The equation $6^x + 11^y = z^2$ has no solutions in positive integers x, y, z.

Proof: For each and every value x, 6^x has a last digit which is equal to 6. For each and every value y, 11^y has a last digit which is equal to 1. The sum $6^x + 11^y$ is odd, and therefore has a last digit which is equal to 7. Since no odd square z^2 ends in the digit 7, it follows that the equation $6^x + 11^y = z^2$ has no solutions as asserted. □

Corollary 1. In the equation $6^x + 11^y = z^2$, suppose that the value 11^y is replaced by the value $(10N + 1)^y$ where $N \ge 2$ and $y \ge 1$. For each and every of the values N and y, the value $(10N + 1)^y$ ends in the digit 1. By Lemma 1 it then follows that the equation $6^x + (10N + 1)^y = z^2$ has no solutions.

Consider the equation $6^x - 11^y = z^2$, where $x \ge 2$ and $y \ge 1$. When $x = 2$ and $y = 1$, we have

Solution 1. $$6^2 - 11^1 = 5^2.$$

Each of the fourteen values x where $3 \le x \le 16$ (6^{16} = 2821109907456) has been examined. For each such fixed value x together with all its possible values y, no solutions have been found. Hence, if the equation has a solution, then $x > 16$, in which case a computer must be involved since 6^{16} is a 13 digits number.

We presume that for all values $x > 16$, the equation $6^x - 11^y = z^2$ has no solutions. If our presumption is indeed true, then **Solution 1** is the unique solution of the equation $6^x - 11^y = z^2$.

REFERENCES

1. N. Burshtein, On solutions to the diophantine equations $5^x + 103^y = z^2$ and $5^x + 11^y = z^2$ with positive integers x, y, z, *Annals of Pure and Applied Mathematics*, 19 (1) (2019) 75 – 77.
2. N. Burshtein, On solutions of the diophantine equations $p^3 + q^3 = z^2$ and $p^3 - q^3 = z^2$ when p, q are primes, *Annals of Pure and Applied Mathematics*, 18 (1) (2018) 51 – 57.
3. B. Poonen, Some diophantine equations of the form $x^n + y^n = z^m$, *Acta Arith*., 86 (1998) 193– 205.
4. B. Sroysang, On the diophantine equation $5^x + 7^y = z^2$, *Int. J. Pure Appl. Math*., 89 (2013) 115 – 118.