Annals of Pure and Applied Mathematics
Vol. 19, No. 2, 2019, 63-68
ISSN: 2279-087X (P), 2279-0888(online)
Published on 8 November 2019
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.636v20n2a3

Annals of
Pure and Applied
Mathematics

Correction on "Some Relations Related to Centralizers on Semiprime Semiring, Vol. 13, Issue 1, 2017"

***D. Mary Florence*[1], *R. Murugesan*[2] and *P. Namasivayam*[3]**

[1]Department of Mathematics, Kanyakumari Community College
Mariagiri – 629153, Tamil Nadu, India. E-mail: dmaryflorence@gmail.com
[2]Department of Mathematics, Thiruvalluvar College
Papanasam – 627425, Tamil Nadu, India. E-mail: rmurugesa2020@yahoo.com
[3]Department of Mathematics, The M.D.T Hindu College
Tirunelveli – 627010, Tamil Nadu, India. E-mail: vasuhe2010@gmail.com

Received 15 September 2019; accepted 24 October 2019

Abstract. In this paper, we generalize the following result. If S is a 2-torsion free semiprime semiring and $T: S \to S$ be an additive mapping such that $2T(xyx) = T(x)yx + xyT(x)$ holds for all $x, y \in S$, then T is a centralizer.

Keywords: Semiring, semiprime semiring, centralizer, jordan centralizer, left (right) centralizer.

AMS Mathematics Subject Classification (2010): 16Y60, 16N60

1. Introduction

Semirings has been formally introduced by Vandiver in 1934. Golan [4] discussed the notion of semirings and their applications. In [3], Chandramouleeswaran and Tiruveni worked on the derivations on semirings. Zalar [17] studied centralizers on semiprime rings and proved that Jordan centralizer and centralizers of this rings coincide. In [14], Vukman and Irena proved that if R is a 2-torsion free semiprime ring and $T: R \to R$ is an additive mapping such that $2T(xyx) = T(x)yx + xyT(x)$ holds for all $x, y \in R$, then T is a centralizer. In papers [6, 7, 8] the authors Hoque and Paul worked on centralizers on semiprime Gamma rings and developed the results of [14] in Gamma rings. Motivated by this Florence and Murugesan [10] studied the notion of semirings and proved that Jordan centralizer of a 2-torsion free semiprime semiring is a centralizer. Here we develop the results of [7,14] in semirings by assuming that S be a 2-torsion free semiprime semiring and $T : S \to S$ be an additive mapping such that $2T(xyx) = T(x)yx + xyT(x)$ holds for all $x, y \in S$. Then T is a centralizer. In [11], we use the commutator of x and y in $[x, y] = xy - yx$. Now, we change the commutator as $[x, y] = xy + y'x$.

Now we recall the following definitions and results:
Let S be a non empty set followed with two binary operation '+' and '.' such that
i) $(S, +)$ is a commutative monoid with identity element 0.

ii) $(S,.)$ is a monoid with identity element 1.

iii) Multiplication distributes over addition from either side.
That is, $a.(b+c) = a.b + a.c$,
$(b+c).a = b.a + c.a$. Then S is called a semiring.

A Semiring S is prime if $xSy = 0$ implies $x = 0$ or $y = 0 \forall x, y \in S$, and semiprime if $xSx = 0$ implies $x = 0 \forall x \in S$. A semiring S is 2-torsion free if $2x = 0$, $x \in S \Rightarrow x = 0$. The commutator $xy + y'x$ will be denoted by $[x,y]$. More over the set $Z(S) = \{x \in S: xy = yx \ \forall y \in S\}$. we shall use basic commutator identities $[x, yz] = [x,y]z + y[x,z]$ and $[xz, y] = [x,y]z + x[z,y]$. An additive mapping $T: S \to S$ is called a Left (Right) Centralizer if $T(xy) = T(x)y$ $((T(xy) = xT(y))$ holds for all $x, y \in S$. We call T is a centralizer which is both left and right centralizer. For a fixed $a \in S$ then $T(x) = ax$ is a left centralizer and $T(x) = xa$ is a right centralizer. An additive mapping $T: S \to S$ is called a left (right) Jordan centralizer if $T(xx) = T(x)x(T(xx) = xT(x))$ holds for all $x \in S$. Every left centralizer is a Jordan left centralizer but the converse is not in general true. An additive mapping $T: S \to S$ is a Jordan centralizer if $T(xy + yx) = T(x)y + yT(x)$ for all, $y \in S$. Every centralizer is a Jordan centralizer but Jordan centralizer is not in general a centralizer.

According to [9] $for\ all\ a, b \in S$ we have

$(a+b)' = a' + b'$
$(ab)' = a'b = ab'$
$a'' = a$
$a'b = (a'b)' = (ab)'' = ab$

Also the following implication is valid.
$a + b = 0\ implies\ a = b'$ and $a + a' = 0$

2. The centralizers of semiprime semiring

Lemma 2.1. Let S be a semiprime semiring. Suppose that the relation $axb + bxc = 0 \ \forall x \in S$ and some $a, b \in S$. In this case $(a+c)xb = 0, \forall x \in S$

Proof: By hypothesis we have $axb + bxc = 0$ (1)

Putting x by xby yield $axbyb + bxbyc = 0$ (2)

On the other hand right multiplying (1) by yb we get
$axbyb + bxcyb = 0 \ \forall x, y \in S.$

Replacing y by y' in the above, we get
$axby'b + bxcy'b = 0 \ \forall x, y \in S.$ (3)

Adding (2) and (3), we get $axbyb + bxbyc + axby'b + bxcy'b = 0$

This implies, $bxbyc + bxcy'b = 0$
$bx(byc + cy'b) = 0$ (4)

Putting x by ycx in (4) we get
$bycx(byc + cy'b) = 0$ (5)

Left multiplying (4) by cy we obtain
$cybx(byc + cy'b) = 0$

Replacing x by x' in the above, we get
$cybx'(byc + cy'b) = 0$ (6)

Adding (5) and (6) , we get
$(byc + cy'b)x(byc + cy'b) = 0$

By the semiprimeness of S implies, $byc + cy'b = 0$

This implies $byc = cyb, y \in S$

Replace y by x, in the above relation we get, $bxc = cxb$

So (1) becomes $axb + cxb = 0$

$(a + c)xb = 0, \forall x \in S$

Hence the proof is complete.

Lemma 2.2. Let S be a 2-torsion free semiprime semiring and Let $T: S \to S$ be an additive mapping such that $2T(xyx) = T(x)yx + xyT(x)$ holds for all $x, y \in S$. Then $2T(xx) = T(x)x + xT(x)$.

Proof:

By the assumption we have $2T(xyx) = T(x)yx + xyT(x)$ (7)

Linearizing the above by putting $x + z$ for x we obtain

$2T((x + z)y(x + z)) = T(x + z)y(x + z) + (x + z)yT(x + z)$

$2T(xyz + zyx) = T(x)yz + T(z)yx + xyT(z) + zyT(x)$ (8)

Substituting $z = x^2$ the relation (8) yields

$2T(xyx^2 + x^2yx) = T(x)yx^2 + T(x^2)yx + xyT(x^2) + x^2yT(x)$ (9)

Substitution for y by $xy + yx$ in (7) we arrive at

$2T(x(xy + yx)x) = T(x)(xy + yx)x + x(xy + yx)T(x)$

$2T(x^2yx + xyx^2) = T(x)xyx + T(x)yx^2 + x^2yT(x) + xyxT(x)$ (10)

Comparing (9) and (10), we get

$T(x^2)yx + xyT(x^2) = T(x)xyx + xyxT(x)$

Adding $T(x)xyx' + x'yxT(x)$ on both sides, we get

$T(x^2)yx + xyT(x^2) + T(x)xyx' + x'yxT(x) = 0$

$(T(x^2) + T(x)x')yx + xy(T(x^2) + x'T(x)) = 0$

From the above relation taking

$a = T(x^2) + T(x)x', x = y, b = x, c = T(x^2) + x'T(x)$

Now applying lemma 2.1 follows that

$(T(x^2) + T(x)x' + T(x^2) + x'T(x))yx = 0$

$(2T(x^2) + T(x)x' + x'T(x))yx = 0$

Taking $A(x) = 2T(x^2) + T(x)x' + x'T(x)$, then the above relation becomes,

$A(x)yx = 0$ (11)

Applying y by $xyA(x)$ in (11) gives $A(x)\, xyA(x)x = 0$

By the semiprimeness of S, $A(x)x = 0$ (12)

On the other hand left multiplying (11) by x and right multiplying by $A(x)$ we obtain $xA(x)yxA(x) = 0$

Since S is semiprime, $xA(x) = 0$ (13)

Putting $x + y$ for x in (12) we get

$A(x + y)(x + y) = 0$

$A(x)y + A(y)x + B(x, y)x + B(x, y)y + A(x)x + A(y)y = 0$ where

$B(x, y) = 2T(xy + yx) + T(x)y' + T(y)x' + y'T(y) + y'(T(x)$

Using (12) the above relation reduces to

$A(x)y + A(y)x + B(x, y)x + B(x, y)y$

Replacing x by x' and using the result $a+b=0$ then $a=b'$, we get
$A(x)y+B(x,y)x=0$
Right multiplication of the above relation by $A(x)$ gives because of (13)
$A(x)yA(x)=0\ \forall\, x,y\in S$
By the semiprimeness of S, we get $A(x)=0$.
Thus $2T(x^2)+T(x)x'+x'T(x)=0$
$$2T(x^2)=T(x)x+xT(x) \tag{14}$$
This completes the proof.

Lemma 2.3. Let S be a 2-torsion free semiprime semiring and let $T:S\to S$ be an additive mapping, suppose that $2T(xyx)=T(x)yx+xyT(x)$ holds for all pairs $x,y\in S$. Then $[T(x),x]=0$

Proof: We have $2T(xx)=T(x)x+xT(x)$
Linearizing the above by replacing x by $x+y$ we obtain
$$2T(xy+yx)=T(x)y+T(y)x+xT(y)+yT(x) \tag{15}$$
Replacing y by $2xyx$ in (15) and using the assumption of the theorem yields
$$\begin{aligned}4T(x^2yx+xyx^2)&=2T(x)xyx+2T(xyx)x+x2T(xyx)+2xyxT(x)\\&=2T(x)xyx+(T(x)yx+xyT(x))x+x(T(x)yx\\&\quad+xyT(x))+2xyxT(x)\end{aligned}$$
$$\begin{aligned}2\big(2T(x^2yx+xyx^2)\big)&=2T(x)xyx+T(x)yx^2+xyT(x)x+xT(x)yx\\&\quad+x^2yT(x)+2xyxT(x)\end{aligned} \tag{16}$$
Applying (10) in (16) gives
$$\begin{aligned}2(T(x)xyx+T(x)yx^2+x^2yT(x)+xyxT(x))&=2T(x)xyx+T(x)yx^2\\&\quad+xyT(x)x+xT(x)yx+x^2yT(x)+2xyxT(x)\end{aligned}$$
$$T(x)yx^2+x^2yT(x)=xyT(x)x+xT(x)yx \tag{17}$$
Replacing y by yx in (17) we arrive at
$$T(x)yx^3+x^2yxT(x)=xyxT(x)x+xT(x)yx^2\ \forall x,y\in S$$
$$T(x)yx^3=xyxT(x)x+xT(x)yx^2+x^2y'xT(x)\ \forall x,y\in S \tag{18}$$
Right multiplication of (17) by x yields,
$$T(x)yx^3+x^2yT(x)x=xyT(x)x^2+xT(x)yx^2 \tag{19}$$
Substituting (18) with (19), we get
$$xyxT(x)x+xT(x)yx^2+x^2y'xT(x)+x^2yT(x)x=xyT(x)x^2+xT(x)yx^2$$
$$xyxT(x)x+x^2y'xT(x)+x^2yT(x)x=xyT(x)x^2+xT(x)yx^2+xT(x)y'x^2$$
$$xyxT(x)x+x^2y(T(x)x+x'T(x))=xyT(x)x^2$$

$$x^2y[T(x),x]=xy[T(x),x]x \tag{20}$$
Applying y by $T(x)y$ in (20) leads to
$$x^2T(x)y[T(x),x]=xT(x)y[T(x),x]x$$
Replacing y by y' in the above, we get
$$x^2T(x)y'[T(x),x]=xT(x)y'[T(x),x]x \tag{21}$$
Left multiplication of (20) by $T(x)$ gives
$$T(x)x^2y[T(x),x]=T(x)xy[T(x),x]x \tag{22}$$
Adding (22) with (21), we get
$$T(x)x^2y[T(x),x]+x^2T(x)y'[T(x),x]=T(x)xy[T(x),x]x+xT(x)y'[T(x),x]x$$
$$\left(T(x)x^2+x'^2T(x)\right)y[T(x),x]=(\,T(x)x+x'T(x))y[T(x),x]x$$

$[T(x),x^2]y[T(x),x]=[T(x),x]y[T(x),x]x$
$([T(x),x]x+x[T(x),x])y[T(x),x]=[T(x),x]y[T(x),x]x$
$[T(x),x]x\,y[T(x),x]+\ x[T(x),x]\,y[T(x),x]=[T(x),x]y[T(x),x]x$
Replace xy by y in (20), we get $xy[T(x),x]=y[T(x),x]x$.
The above result becomes
$[T(x),x]\,y[T(x),x]x+x[T(x),x]y[T(x),x]=[T(x),x]\,y[T(x),x]x$
$x[T(x),x]y[T(x),x]=[T(x),x]\,y[T(x),x]x+[T(x),x]\,y[T(x),x]x'$
$x[T(x),x]y[T(x),x]=0$
Substituting $y=yx$ in the above relation
$x[T(x),x]yx[T(x),x]=0\ \forall\ x,y\in S$
By the semiprimeness of S, $\quad x[T(x),x]=0$ (23)
Replacing y *by* xy in (17) gives

$$T(x)xyx^2+x^2xyT(x)=xxyT(x)x+xT(x)xyx$$
$$T(x)xyx^2+x^3yT(x)=x^2yT(x)x+xT(x)xyx \quad (24)$$

Left multiplication of (17) by x we get

$$xT(x)yx^2+x^3yT(x)=x^2yT(x)x+x^2T(x)yx$$

Replacing y by y' in the above relation, we get
$xT(x)y'x^2+x^3y'T(x)=x^2y'T(x)x+x^2T(x)y'x$ (25)
Adding (25) and (24) we obtain

$$[T(x)x+x'T(x)]yx^2=x[T(x)x+x'T(x)]yx=0$$
$$[T(x),x]yx^2=x[T(x),x]yx$$

Using (23) in the above relation yields $\quad [T(x),x]yx^2=0$ (26)
Applying $yT(x)$ for y in (26) we obtain $[T(x),x]\,yT(x)x^2=0$ (27)
Right multiplication of (26) by $T(x)$ gives$[T(x),x]\,yx^2\,T(x)=0$
Replacing y by y' in the above relation, we get
$[T(x),x]\,y'x^2\,T(x)=0$ (28)
Adding (28) and (27) we get

$$[T(x),x]y(T(x)x^2+x'^2T(x))=0$$
$$[T(x),x]y[T(x),x^2]=0$$
$$[T(x),x]y([T(x),x]x+x[T(x),x])=0$$

Using (23) in the above relation reduces to

$$[T(x),x]\,y\,[T(x),x]x=0$$

Putting y *by* xy in the above implies $[T(x),x]xy[T(x),x]x=0$
Since S is semiprime, $\quad [T(x),x]x=0$ (29)
Putting x *by* $x+y$ in (23) yields
$(x+y)[T(x+y),x+y]=0$

$$x[T(x),x]+x[T(x),y]+x[T(y),x]+x\,[T(y),y]+y[T(x),x]+y[T(x),y]$$
$$+\,y[T(y),x]+y[T(y),y]=0$$

Using (23), the above relation reduces to
$x[T(x),y]+[T(y),x]+x[T(y),y]+y[T(x),x]+y[T(x),y]+y[T(y),x]=0$ (30)
Replacing x by x', in the above relation implies
$x[T(x),y]+[T(y),x]'+x'[T(y),y]+y[T(x),x]+y'[T(x),y]+y'[T(y),x]=0$ (31)
Adding (30) and (31), we get $x[T(x),y]+\ y[T(x),x]=0$
Left multiplying by $[T(x),x]$ and using (29), we get

$$[T(x), x]\ y\ [T(x), x] = 0$$

By the semiprimeness of S implies, $[T(x), x] = 0$.

Theorem 2.1. Let S be a 2-torsion free semiprime semiring. Let $T: S \to S$ be an additive mapping, suppose that $2T(xyx) = T(x)yx + xyT(x)$ holds for all $x, y \in S$. Then T is a centralizer.

Proof: We have by Lemma 2.3, $[T(x), x] = 0$

$$T(x)x + x'T(x) = 0$$

$$T(x)x = xT(x)$$

Applying the above results in (14) we obtain $2T(x^2) = 2T(x)x$

Adding $2T(x)x'$ on both sides, we get $T(x^2) + T(x)x' = 0$

which implies $T(x^2) = T(x)x$.

Similarly $T(x^2) = xT(x)$. This means that T is a Jordan Centralizer. By theorem 4.1 in [10] yields that T is a left and right centralizer. Thus the proof is completed.

REFERENCES

1. M.Bresar and J.Vukman, On some additive mapping in rings with involution, *Aequationes Math.*, 38 (1989) 178-185.
2. M.Bresar, Jordan derivations on semiprime rings, *Proc. Amer. Math. Soc.*, 104 (1988) 1003-1006
3. T.Chandramouleeswaran, On derivations of semirings, *Advances in Algebra*, 3(1) (2010) 123-131.
4. J.S.Golan, Semirings and their Applications, *Kluwer Academic Press*, (1969).
5. I.N.Herstein, Topics in ring theory, *University of Chicago Press*, (1969).
6. M.F.Hoque and A.C.Paul, An equation related to centralizers in semiprime gamma rings, *Annals of Pure and Applied Mathematics*,1(1) (2012) 84-90.
7. M.F.Hoque and A.C.Paul, Centralizers on semiprime gamma rings, *Italian Journal of Pure and Applied Mathematics*, 30 (2013) 289-302.
8. M.F.Hoque and A.C.Paul, On centralizers of semiprime gamma rings, *International Mathematical Forum*, 6(13) (2011) 627-638.
9. P.H.Karvellas, Inversivesemirings, *J. Aust. Math. Soc.*, 18 (1974) 277 – 288.
10. D.Mary Florence, R.Murugesan and P.Namasivayam, Centralizers on semiprime semiring, *IOSR Journal of Mathematics*, 12 (2016) 86-93.
11. D.Mary Florence, R.Murugesan and P.Namasivayam, Some Relations Related to Centralizers on Semiprime Semiring, *Annals of Pure and Applied Mathematics*, 13(1) (2017) 119-124.
12. K.Ray Chowdhury, A.Sultana, N.K.Mitra and A.F.M.K.Khan, Some structural properties of semirings, *Annals of Pure and Appl. Mathematics*, 5(2) (2014) 158-167.
13. T.Vasanthi and N.Sulochana, On the additive and multiplicative structure of semirings, *Annals of Pure and Applied Mathematics*, 3(1) (2013) 78-84.
14. J.Vukman and I.Kosi-ulbl, On centralizers of semiprime rings, *Aequationes Math.*, 66 (3) (2003) 277-283.
15. J.Vukman, An identity related to centralizers in semiprime rings, *Comment. Math. Univ. Carolinae*, 40 (3) (1999) 447-456.
16. J.Vukman, Centralizers on semiprime rings, *Comment Math. Univ. Carolinae*, 42(2) (2001) 237-245.

17. B.Zalar, On centralizers of semiprime rings, *Commentationes Mathematicae Universitatis Carolinae,* 32(4) (1991) 609-614