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Computing Forgotten Eccentric Topological Index of Complement of Line Graphs

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Abstract. The forgotten topological index of graph G is defined as the sum of the cube of the degrees of its vertices. In this paper, we establish the forgotten eccentric topological index of the complement of the line graph, denoted by $\mathrm{F}\xi\left(\overline{L(G)}\right)$. These are defined as $\mathrm{F}\xi\left(\overline{\mathrm{L(G)}}\right) = \Sigma_{\mathrm{e}\in\mathrm{V(G)}}\mathrm{d}^3(v).e(v)$, where $e = v = V\ (G)$ (edge vertex degree) and establish some upper and lower bounds for them.

Keywords: Forgotten Topological Index, Eccentricity, Complement of Line of Graph.

AMS Mathematics Subject Classification (2010): 05C12, 05C07, 05C35

1. Introduction

Topological index of a graph is a numerical quantity which is invariant under the automorphisms of the graph. Topological indices are important and useful tools in mathematical chemistry, non-materials, pharmaceutical engineering, etc. Used for quantifying information on molecules. Molecules and molecular compounds are modelled as molecular graphs, in which vertices correspond to the atoms and edges to the chemical bonds between them. Hundreds of various topological indices have been introduced in mathematical chemistry literature in order to describe the physical and chemical properties of molecules, especially for studying quantitative structure-activity relationships (QSAR) and quantitative structure-property relationships (QSPR) for predicting different properties of chemical compounds(see for example [10, 13, 15]). In mathematical medicine, the structure of the drug is represented as an undirected graph, where each vertex indicates an atom and each edge represents a chemical bond between the atoms [8, 9, 13]. In 1972, Gutman and Trinajsti'c [1, 6, 7], made the study on total π-electron energy of the molecular structure and introduced two vertex degree-based graph variants. These variants are defined as

$$M_1(G) = \sum_{\mathrm{v}\in\mathrm{V(G)}} \mathrm{d}^2\,(v)\ and\ M_2(G) = \sum_{u,v\in V(G)} d(u)d(v).$$

In Chemical literature M_1 and M_2 are named as first and second Zagreb indices [6]. The Zagreb indices are used by various researchers in the studies of quantitative structure property relationships (QSPR). In 2015, Furtula and Gutman [3] reinvestigated this index and they showed that the predictive ability of this index is almost similar to that of first Zagreb index and for the entropy and acentric factor, both of them yield correlation coefficients greater than 0.95. They named this index as forgotten topological index or F-index, denoted by F(G).

$$F(G) = \sum_{v \in V(G)} d^3(v).$$

The total eccentricity of a graph, denoted by $\theta(G)$ is the sum of all the eccentricities of G, [2], i.e.

$$\theta(G) = \sum_{v \in V(G)} e(v).$$

Recently several graph invariants are defined based on vertex eccentricities in a large number of studies. Analogously to Zagreb indices, Ghorbani et. al., [5], and Vukiˇcevi´c et. al., [14], defined the Zagreb eccentricity indices by replacing degrees by the eccentricity of the vertices. Thus the first and second Zagreb eccentricity indices of a graph G are defined by

$$E_1(G) = \sum_{v \in V(G)} e(v)^2$$

And

$$E_2(G) = \sum_{uv \in V(G)} e(u)e(v).$$

Lemma 1.1. [16] Let *G* be a connected graph with *n* vertices and *m* edges. Then

$$d_2(v) = \left(\sum_{u \in N_{1(v)}} d_1(u)\right) - d_1(v).$$

Equality holds if and only if *G* is a $\{C_3, C_4\}$ −free graph. By Lemma 1.1 if *G* is a (C_3, C_4)-free graph, we obtain

$$\sum_{v \in V(G)} d_2(v) = M_1 - 2m.$$

Corollary 1.2. [16] Let *G* be a (C_3, C_4)-free *k*-regular graph with *n* vertices. Then

$$d_2(v) = k(k-1).$$

Let $G = (V, E)$ be such a graph with vertex set *V* (*G*) and edge set *E*(*G*). As usual, we denote the number of vertices and edges in a graph *G* by *n* and *m*, respectively. The distance $d_G(u,v)$ between any two vertices *u* and *v* of a graph *G* is equal to the length of (number of edges in) a shortest path connecting them. For a vertex v ∈ V (G) and a positive integer *k*, the open *k*- neighbourhood of *v* in *G* is denoted by $N_k(v)$ and is defined as $N_k(v = \{u \in V(G) : d_G(u,v) = k\}$. The *k*-distance degree of a vertex *v* in *G* is denoted by $d_k(v)$ and is defined as the number of *k*-neighbours of the vertex *v* in *G*, i.e., $d_k(v) = |N_k(v)|$. It is clear that $d_1(v) = d(v)$ for every $v \in V(G)$. For a vertex *v* of *G*, the eccentricity of *v* is $e(v) = max\{d_G(v,u) : u \in V(G)\}$. The diameter of *G* is $diam(G) = max\{e(v): v \in$

$V(G)\}$ and the radius of G is $rad(G) = min\{e(v) : v \in V(G)\}$. The line graph $L(G)$ of G has the edges of G as its vertices which are adjacent in $L(G)$ if and only if the corresponding edges are adjacent in G. The complement $\bar{G}$ of a simple graph G is the simple graph with vertex set $V(G)$ defined by $u, v \in E(\bar{G})$ iff $uv \notin E(G)$ [8]. Let G be a simple graph, that is graph without multiple, directed, or weighted edges, and without self-loops, with vertex set $V(G)$ and edge set $E(G)$, where $|V(G)| = n$ and $|E(G)| = m$. In this paper, motivated by forgotten topological index, the authors introduced the Forgotten eccentric topological index of complement of line graph $F\xi(\overline{L(G)})$ of a graph G which is defined as the forgotten topological index of a graph G is defined as the sum of the cube of the degrees of its vertices and the eccentricity of every vertex in G. The exact values of $F\xi(\overline{L(G)})$ for some well-known graphs are obtained. Define the Forgotten eccentric topological index of complement of line graph and it is denoted by $F\xi(\overline{L(G)})$, i.e.

$$F\xi(\overline{L(G)}) = \sum_{e \in V(G)} d^3 . e(v),$$

where e = v ∈ V (G). The study of forgotten topological index of the chemical structure in drugs is carried out in [4]. Recently, Gao et al. [4] have obtained the forgotten topological index of some drug structures.

2. Main results

The aim of this paper is to present the exact value of forgotten eccentric topological index of complement of line graph of some standard graphs.

Theorem 2.1. For $n - 1 \geq 5$, $\overline{L(P_n)}$ be a complement of line of path. Then

$$F\xi(\overline{L(G)}) = 4(n-2)^3 + 2((n-3)^3(n-2)).$$

Proof: Let $\overline{L(P_n)}$ be a complement of line of path $n - 1 \geq 5$. The first and last edge vertex degree $g(\overline{L(P_n)} = n-2,\ deg(\overline{L(P_i)})) = n-3$, where $i = 2, 3, \ldots, n-2$ and $e(\overline{L(P_n)}) = 2$ as show in Fig. 3. Therefore, by the definition of forgotten eccentric topological index of complement of line graph, we get,

$$F\xi\left(\overline{L(P_n)}\right) = \sum_{e \in V(G)} d3(v).e(v)$$

$$= d_G^3(v_1).e(v_1) + \sum_{i=2}^{n-2} d_G^3(v_i).e(v_i) + d^3(v_n).e(v_n)$$

$$= (n-2)^3.2 + \sum_{i=2}^{n-2} (n-3)^3.2 + (n-2)^2.2$$

$$= 4(n-2)^3 + 2((n-3)^3(n-2)).$$

Figure 1: P_n

Figure 2: L(P_n)

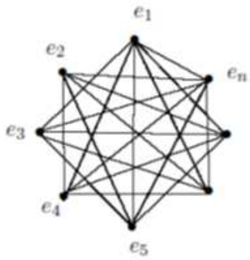

Figure 3: $\overline{\mathbf{L}(Pn)}$

Theorem 2.2. For $n \geq 4$, $\overline{L(C_n)}$ be a complement of line of cycle. Then

$$F\xi\left(\overline{L(C_n)}\right) = 2n.(n - 3)^3.$$

Proof: $\overline{L(C_n)}$ be a complement of line of cycle, $n \geq 4$, is a regular graph. The edge vertex of degree is $d_G(\overline{L(C_n)}) = n-3$ and edge vertex of eccentricity is $e(\overline{L(C_n)}) = 2$. Therefore, by the definition of forgotten eccentric topological index of complement of line graph, we get

$$F\xi(\overline{L(C_n)}) = \sum_{v \in V(G)} d^3(v).e(v)$$

$$= \sum_{i=1}^{n} (n-3)^2.2 = 2n.(n-3)^3.$$

Corollary 2.3. For $n \geq 2$, $\overline{L(K_{1,n})}$ be a complement of line of star is a null edge vertex graph.
Proof: Let $n \geq 2$, $L(K_{1,n})$ be a complement of line of star is a null edge vertex graph.

Theorem 2.4. For $n, m \geq 4$, $\overline{L(K_{m,n})}$, be a complement of line of complete bipartite graph. Then

$$F\xi(\overline{L(K_{m,n})}) = 2nm(n^2 - (2n - 1))^3.$$

Proof: Let $\overline{L(K_{m,n})}$ be a complement of line of complete bipartite graph, $n, m \geq 4$. The edge vertex of degree is $d_G(\overline{L(K_{m,n})}) = n^2 - (2n-1)$ and edge vertex of eccentricity is $e(\overline{L(K_{m,n})}) = 2$, where $i = j = 4, 5, 6 \ldots, m, n$. Therefore, by the definition of forgotten eccentric topological index of complement of line graph, we get,

$$F\xi\left(\overline{L(K_{m,n})}\right) = \sum_{e \in V(G)} d^3(\mathrm{v}).\mathrm{e}(\mathrm{v})$$

$$= \sum_{i,j=4,4}^{m,n} (n^2 - (2n-1))^3.2$$

$$= 2\mathrm{mn}(\mathrm{n}^2 - (2\mathrm{n} - 1))^3.$$

Theorem 2.5. For $p \geq 5$, $\overline{L(K_p)}$ be a complement of line of complete graph. Then

$$F\xi(\overline{L(K_p)}) = \frac{p(p-1)}{2}(2p-4)^3.2.$$

Proof: Let $\overline{L(K_p)}$ be a complement of line of complete graph, $p \geq 5$. The edge vertex of degree is $d_G\overline{(L(K_p)} = 2n - 4$ and edge vertex of eccentricity is $e(\overline{L(K_p)}) = 2$. Therefore by the definition of forgotten eccentric topological index of complement of line graph, we get,

$$F\xi(\overline{L(K_p)}) = \sum_{e \in V(G)} d^3(v).e(v) = \sum_{i=5}^{\frac{p(p-1)}{2}} (2n-4)^3.2 = \frac{p(p-1)}{2}(2p-4)^3.2.$$

3. Bounds for forgotten eccentric topological index of complement of line graphs

Theorem 3.1. Let any r-regular graph G with n-edge vertex. Then

$$F\xi\left(\overline{L(K_p)}\right) \geq (r-3)\theta(G)$$

and equality holds iff G is a (C_4, C_5) -free graph.

Proof: Let G be an r-regular graph with n-edge vertex. Then by corollary 1.2 we get,

$$F\xi(\overline{L(K_p)}) = \sum_{e \in V(G)} d^3(v).e(v)$$

$$\geq \sum_{e \in V(G)} (r-3).e(v) \qquad (1)$$

$$\geq (r-3) \sum_{e \in V(G)} e(v)$$

$$F\xi\left(\overline{L(K_p)}\right) \geq (r-3)\theta(G). \qquad (2)$$

Again by Corollary 1.2, we have that $d^3(v) = r-3$, iff G is a (C_4, C_5) -free graph. This implies that the equality in Equation (1) holds iff G is a (C_4, C_5) -free graph.

Theorem 3.2. Let G be a connected with n-edge vertex. Then

$$F\xi(\overline{L(G)}) \geq diam(G)(M_1(G) - 2m).$$

Equality hold iff G is a self centred (C_5, C_6)-free graph.

Proof: Let G be a connected with n-edge vertex. Since $e(v) \leq diam(G)$ for every $v \in V(G)$, by Lemma 1.1 we get that $d^3(v) \geq \sum_{u \in N(V)} d(u) - d(v)$. Then

$$F\xi(\overline{L(G)}) = \sum_{e \in V(G)} d^3(v).e(v)$$

$$\geq \sum_{e \in V(G)} d^3(v).diam(v) \qquad (3)$$

$$\geq diam(v) \sum_{e \in V(G)} d^3(v)$$

$$\geq diam(v) \sum_{e \in V(G)} \left(\sum_{u \in N(V)} d(u) - d(v) \right)$$

$$F\xi\left(\overline{L(G)}\right) \geq diam(G)(M_1(G) - 2m). \qquad (4)$$

Equality holds in (3) iff G is a self-centred graph and by Lemma 1.1 equality holds in (4) iff G is a (C_5, C_6) −free graph.

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