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F-Sombor and Modified *F*-Sombor Indices of Certain Nanotubes

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Abstract. In this study, we introduce the *F*-Sombor index, modified *F*-Sombor index and their exponentials of a graph. Furthermore, we present exact expressions for these *F*-Sombor indices and their exponentials of certain nanotubes.

Keywords: *F*-Sombor index, modified *F*-Sombor index, nanotube

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*40, 05*C*92

1. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. For definitions and notations, we refer to the book [1].

In Chemistry, topological indices have been found to be useful in discrimination, chemical documentation, structure-property relationships, structure-activity relationships and pharmaceutical drug design. There has been considerable interest in the general problem of determining topological indices, see [2].

The first *F*-index [3] and second *F*-index [4] of a graph *G* are defined as

$$F_1(G) = \sum_{uv \in E(G)} \left[d_G(u)^2 + d_G(v)^2 \right], \qquad F_2(G) = \sum_{uv \in E(G)} d_G(u)^2 d_G(v)^2.$$

Recently some *F*-indices were studied in [5, 6, 7].

We put forward the *F*-Sombor index of the graph *G* and defined it as

$$FSO(G) = \sum_{uv \in E(G)} \sqrt{\left(d_G(u)^2\right)^2 + \left(d_G(v)^2\right)^2}.$$

We propose the modified *F*-Sombor index of the graph *G*, defined as

$$^{m}FSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\left(d_G(u)^2\right)^2 + \left(d_G(v)^2\right)^2}}.$$

Recently, some Sombor indices were studied in [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

We introduce the *F*-Sombor exponential of the graph *G* and defined it as

$$ {}^{m}FSO(G,x)=\sum_{uv\in E(G)} x^{\sqrt{\left(d_G(u)^2\right)^2+\left(d_G(v)^2\right)^2}}. $$

We define the modified *F*-Sombor exponential of a graph *G* as

$$ {}^{m}FSO(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{\left(d_G(u)^2\right)^2+\left(d_G(v)^2\right)^2}}}. $$

In this paper, the *F*-Sombor and modified *F*-Sombor indices and their corresponding exponential versions of certain nanotubes are determined.

2. HC_5C_7 [*p, q*] Nanotubes

The chemical graphs *G* of nanotube $HC_5C_7[p, q]$ structure have $4pq$ vertices and $6pq-p$ edges are shown in Figure 1.

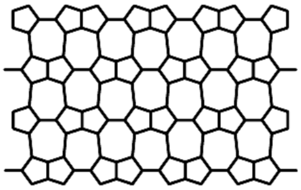

Figure 1: 2-D lattice of nanotube HC_5C_7 [*p,q*]

In the above structure, we obtain that $\{d_G(u), d_G(v): uv\in E(G)\}$ has two edge set partitions.

$E_1=\{uv\in E(G) \mid d_G(u)=2, d_G(v)=3\}$, $|E_1|=4p$,
$E_2=\{uv\in E(G) \mid d_G(u)=3, d_G(v)=3\}$, $|E_2|=6pq-5p$.

Theorem 1. The *F*-Sombor index of a nanotube $HC_5C_7[p, q]$ is

$$ FSO(G)=54\sqrt{2}pq+\left(4\sqrt{97}-45\sqrt{2}\right)p. $$

Proof: Applying definition and edge set partition of *G*, we conclude

$$ FSO(G)=\sum_{uv\in E(G)}\sqrt{\left(d_G(u)^2\right)^2+\left(d_G(v)^2\right)^2} $$

$$ =\left(\sqrt{2^4+3^4}\right)4p+\left(\sqrt{3^4+3^4}\right)(6pq-5p) $$

gives the desired result after simplification.

Theorem 2. The modified *F*-sombor index of a nanotube $HC_5C_7[p, q]$ is given by

$$ {}^{m}FSO(G)=\frac{2}{3\sqrt{2}}pq+\left(\frac{4}{\sqrt{97}}-\frac{5}{9\sqrt{2}}\right)p. $$

Proof: Applying definition and edge set partition of G_1, we conclude

$$ {}^{m}FSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{(d_G(u)^2)^2+(d_G(v)^2)^2}} $$

$$= \left(\frac{1}{\sqrt{2^4+3^4}}\right) 4p + \left(\frac{1}{\sqrt{3^4+3^4}}\right) (6pq - 5p)$$

After simplification, we get the desired result.

By using definitions and cardinalities of the edge partitions of *G*, we obtain the F-Sombor and modified F-Sombor exponentials of *G* as follows:

Theorem 3. The *F*-Sombor exponential of G_1 is

$$FSO(G,x) = 4px^{\sqrt{97}} + (6pq - 5p)x^{9\sqrt{2}}.$$

Theorem 4. The modified *F*-Sombor exponential of a G_1 is

$$^{m}FSO(G,x) = 4px^{\frac{1}{\sqrt{97}}} + (6pq - 5p)x^{\frac{1}{9\sqrt{2}}}.$$

3. $SC_5C_7[p, q]$ nanotubes

The chemical graphs *H* of nanotube $SC_5C_7[p, q]$ structure have $4pq$ vertices and $6pq-p$ edges are shown in Figure 2.

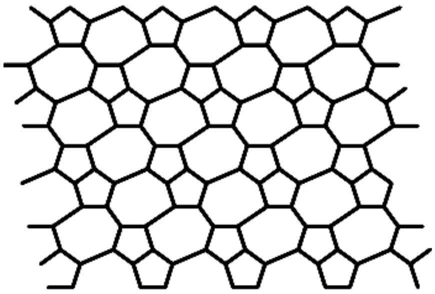

Figure 2: 2-D lattice of nanotube SC_5C_7 [*p*, *q*]

In the above structure, we obtain that $\{d_H(u), d_H(v): uv \in E(H)\}$ has three edge set partitions.

$E_1=\{uv \in E(H) \mid d_H(u)=2, d_H(v)=2\}$, $|E_1|=q$,
$E_2=\{uv \in E(H) \mid d_H(u)=2, d_H(v)=3\}$, $|E_2|=6q$,
$E_3=\{uv \in E(H) \mid d_H(u)=3, d_H(v)=3\}$, $|E_2|=6pq- p-7q$.

Theorem 5. The *F*-Sombor index of *H* is

$$FSO(H) = 54\sqrt{2}pq - 9\sqrt{2}p + (6\sqrt{97} - 50\sqrt{2})q.$$

Proof: Applying definition and edge set partition of *H*, we conclude

$$FSO(H) = \sum_{uv\in E(H)} \sqrt{\left(d_H(u)^2\right)^2 + \left(d_H(v)^2\right)^2}$$

$$= \left(\sqrt{2^4 + 2^4}\right)q + \left(\sqrt{2^4 + 3^4}\right)6q + \left(\sqrt{3^4 + 3^4}\right)(6pq - p - 7q)$$

gives the desired result after simplification.

Theorem 6. The modified F-sombor index of a nanotube $SC_5C_7[p, q]$ is given by

$$^{m}FSO(H)=\frac{2}{3\sqrt{2}}pq-\frac{1}{9\sqrt{2}}p+\left(\frac{1}{4\sqrt{2}}+\frac{6}{\sqrt{97}}-\frac{7}{9\sqrt{2}}\right)q$$

Proof: Applying definition and edge set partition of H, we conclude

$$^{m}FSO(H)=\sum_{uv\in E(G)}\frac{1}{\sqrt{(d_G(u)^2)^2+(d_G(v)^2)^2}}$$

$$=\left(\frac{1}{\sqrt{2^4\times 2^4}}\right)q+\left(\frac{1}{\sqrt{2^4\times 3^4}}\right)6q+\left(\frac{1}{\sqrt{3^4\times 3^4}}\right)(6pq-p-7q).$$

After simplification, we obtain the desired result.

By using definitions and cardinalities of the edge partitions of H, we obtain the F-Sombor and modified F-Sombor exponentials of G_1 as follows:

Theorem 7. The F-Sombor exponential of H is

$$FSO(H,x)=qx^{4\sqrt{2}}+6qx^{\sqrt{97}}+(6pq-p-7q)x^{9\sqrt{2}}.$$

Theorem 8. The modified F-Sombor exponential of H is

$$^{m}FSO(H,x)=qx^{\frac{1}{4\sqrt{2}}}+6qx^{\frac{1}{\sqrt{97}}}+(6pq-p-7q)x^{\frac{1}{9\sqrt{2}}}.$$

4. Conclusion

In this study, we have computed the F-Sombor and modified F-Sombor indices and their corresponding exponentials of certain nanotubes.

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