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Note on Wing Symmetric *n*-Sigraphs

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Abstract. In this paper, we introduced a new notion wing symmetric *n*-sigraph of a symmetric *n*-sigraph andits properties are obtained. Further, we discuss structural characterization of wing symmetric *n*-sigraph.

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1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory, the reader is referred to [2]. We consider only finite, simple graphs free from self-loops.

Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2,\ldots ,a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1,a_2,\ldots,a_n) : a_k \in \{+,-\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric *n*-tuples. Note that H_n is a group under coordinate-wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G,\sigma)$ ($S_n = (G,\mu)$), where $G = (V,E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \rightarrow H_n$ ($\mu : V \rightarrow H_n$) is a function.

In this paper, by an *n-tuple/n-sigraph/n-marked graph*, we always mean a symmetric *n*-tuple/symmetric *n*-sigraph/symmetric *n*-marked graph.

An n-tuple $(a_1, a_2, \ldots, a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise, it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*; otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [10], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also Rangarajan and Reddy [6]):

Definition 1.1. Let $S_n = (G,\sigma)$ be an n-sigraph. Then,

(i) S_n is *identity balanced* (or *i-balanced*), if the product of n-tuples on each cycle of S_n is the identity n-tuple, and

(ii) S_n is *balanced* if every cycle in S_n contains an even number of non-identity edges.

Note: An i-balanced n-sigraph need not be balanced and conversely.

The following characterization of i-balanced n-sigraphs is obtained in [10].

Theorem 1.1. (**E. Sampathkumar et al. [10]**) An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S_n defined as follows: each vertex $v \in V$, $\mu(v)$ is the n-tuple which is the product of the n-tuples on the edges incident with v. *The complement* of S_n is an *n-sigraph* $\overline{S_n} = (\bar{G}, \sigma^c)$, where for any edge e $= uv \in \bar{G}$, $\sigma^c(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ is defined here as an i-balanced *n-sigraph due to Theorem 1.1.*

In [10], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [1, 4, 5, 7–9, 12–23])

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic* if there exists an isomorphism $\phi : G \rightarrow G'$ such that if uv is an edge in S_n with label $(a_1, a_2, \ldots, a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, \ldots, a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. Then$n$-sigraph obtained in this way is denoted by $S_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $S_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G,\sigma)$ and $S'_n = (G', \sigma')$ are said to be a cycle *isomorphic*, if there exists an isomorphism $\phi : G \rightarrow G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\boldsymbol{\sigma}(\Phi(C))$ in S'_n.

We use the following known result (see [10]).

Theorem 1.2. (**E. Sampathkumar et al. [10]**) *Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.*

2. Wing *n*-sigraph of an *n*-sigraph

The wing graph W(G) of $G = (V,E)$ is a graph with V (W(G)) = $E(G)$ and any two vertices e_1 and e_2 in W(G) are joined by an edge if they are non-incident edges of some induced 4-vertex path in G. This concept was introduced by Hoang [3]. Wing graphs have been introduced in connection with perfect graphs.

By the motivation of complement of an n-sigraph and balance in an n-sigraph, we now extend the notion of wing graphs to n-sigraphs as follows: The *wing n-sigraph* W(S_n) of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph whose underlying graph are W(G) and the n-tuple of any edge e_1e_2 in W(S_n) is $\sigma(e_1)\sigma(e_2)$. Further, an n-sigraph $S_n = (G,\sigma)$ is called wing n-sigraph, if $S_n \cong$ W(S_n') for some n-sigraph S_n'. The following result restricts the class of wing graphs.

Theorem 2.1. *For any n-sigraph $S_n = (G,\sigma)$, its wing n-sigraph* W(S_n) *is i-balanced.*

Proof: Let σ' denote the n-tuple of W(S_n) and let the n-tuple σ of S_n be treated as an n-marking of the vertices of W(S_n). Then by definition of W(S_n) we see that $\sigma'(e_1e_2) = \sigma(e_1)\sigma(e_2)$, for every edge e_1e_2 of W(S_n) and hence, by Theorem 1.1, W(S_n) is i-balanced.

For any positive integer k, the k^{th} iterated wing n-sigraph, $W^k(S_n)$ of S_n is defined as follows:

$$W^0(S_n) = S,\ W^k(S_n) = W(W^{k-1}(S_n)).$$

Corollary 2.2. *For any n-sigraph $S_n = (G,\sigma)$ and for any positive integer k, $W^k(S_n)$ is i-balanced.*

The following result characterize signed graphs which are wing n-sigraphs.

Theorem 2.3. *An n-sigraph $S_n = (G,\sigma)$ is a wing n-sigraph if, and only if, S_n is i-balanced n-sigraph and its underlying graph G is a wing graph.*

Proof: Suppose that S_n is i-balanced and G is a wing graph. Then there exists a graph G^I such that W(G') $\cong G$. Since S_n is i-balanced, by Theorem 1.1, there exists a marking ζ of G such that each edge $e = uv$ in S_n satisfies $\sigma(uv) = \zeta(u)\zeta(v)$. Now consider the n-sigraph $S_n' = (G', \sigma')$, where for any edge e in G', $\sigma'(e)$ is the n-marking of the corresponding vertex in G. Then clearly, W(S_n') $\cong S_n$. Hence S_n is a wing n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a wing n-sigraph. Then there exists an n-sigraph $S_n' = (G', \sigma')$ such that W(S_n') $\cong S_n$. Hence G is the wing graph of G' and by Theorem 2.1, S_n is i-balanced.

Theorem 2.4. *For any two n-sigraphs S_n and S_n' with the same underlying graph, their wang n-sigraphs are switching equivalent.*

Proof: Suppose $S_n = (G, \sigma)$ and $S_n' = (G', \sigma')$) be two n-sigraphs with $G \cong G'$. By Theorem 2.1, W(S_n) and W(S_n') are i-balanced and hence, the result follows from Theorem 1.2.

For any $m \in H_n$, the *m-complement* of $a = (a_1, a_2,\dots ,a_n)$ is: $a^m = am$. For any $M \subseteq H_n$, and $m \in H_n$, the *m-complement* of M is $M^m = \{a^m : a \in M\}$.
For any $m \in H_n$, the *m- complement* of an n-sigraph $S_n = (G,\sigma)$, written $({S_n}^m)$, is the same graph but with each edge label $a = (a_1, a_2,\dots ,a_n)$ replaced by a^m.

For an n-sigraph $S_n = (G, \sigma)$, the $W(S_n)$ is i-balanced. We now examine, the condition under which m-complement of $W(S_n)$ is i-balanced, where for any $m \in H_n$.

Theorem 2.5. *Let $S_n = (G, \sigma)$ be an n-sigraph. Then, for any $m \in H_n$, if $W(G)$ is bipartite then $(W(S_n))^m$ is i-balanced.*

Proof: Since, by Theorem 2.1, $W(S_n)$ is i-balanced, for each k, $1 \le k \le n$, the number of n-tuples on any cycle C in $W(S_n)$ whose k^{th} co-ordinate are − is even. Also, since $W(G)$ is bipartite, all cycles have even length; thus, for each k, $1 \le k \le n$, the number of n-tuples on any cycle C in $W(S_n)$ whose k^{th} co-ordinate are + is also even. This implies that the same thing is true in any m-complement, where for any $m \in H_n$. Hence $(W(S_n))^t$ is i-balanced.

In [3], the author proved that, the graph G and its wing graph $W(G)$ are isomorphic, if $G \cong C_{2k+1}$. In view of this, we have the following result:

Theorem 2.6. *For any n-sigraph $S_n = (G, \sigma)$, $S_n \sim W(S_n)$ if, and only if, S_n is an i-balanced n-sigraph and $G \cong C_{2k+1}$.*

Proof: Suppose $S_n \sim W(S_n)$. This implies $G \cong W(G)$, and hence G is isomorphic to C_{2k+1}. Now, if S_n is any n-sigraph with underlying graph G is C_{2k+1}, Theorem 2.1 implies that $W(S_n)$ is i-balanced, and hence if S_n is i-unbalanced and its $W(S_n)$ being i-balanced cannot be switching equivalent to S_n in accordance with Theorem 1.2. Therefore, S_n must be i-balanced.

Conversely, suppose that S_n is an i-balanced n-sigraph and G is isomorphic to C_{2k+1}. Then, since $W(S_n)$ is i-balanced as per Theorem 2.1 and since $G \cong W(G)$, the result follows from Theorem 1.2 again.

Theorem 2.4 and 2.6 provides easy solutions to other n-sigraph switching equivalence relations, which are given in the following results.

Corollary 2.7. *For any two n-sigraphs S_n and ${S_n}'$ with the same underlying graph, $W(S_n)$ and $W(({S_n}')^m)$ are switching equivalent.*

Corollary 2.8. *For any two n-sigraphs S_n and ${S_n}'$ with the same underlying graph, $W((S_n)^m)$ and $W({S_n}')$ are switching equivalent.*

Corollary 2.9. *For any two n-sigraphs S_n and ${S_n}'$ with the same underlying graph, $W((S_n)^m)$ and $W(({S_n}')^m)$ are switching equivalent.*

Corollary 2.10. *For any two n-sigraphs $S_n = (G, \sigma)$ and ${S_n}' = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, $(W(S_n))^m$ and $W({S_n}')$ are switching equivalent.*

Corollary 2.11. *For any two n-sigraphs* $S_n = (G,\sigma)$ *and* $S_n' = (G',\sigma')$ *with the* $G \cong G'$ *and* G, G' *are bipartite,* $W(S_n)$ *and* $W((S_n')^m)$ *are switching equivalent.*

Corollary 2.12. *For any two n-sigraphs* $S_n = (G,\sigma)$ *and* $S_n' = (G',\sigma')$ *with the* $G \cong G'$ *and* G, G' *are bipartite,* $(W(S_1))^m$ *and* $(W(S_2))^m$ *are switching equivalent.*

Corollary 2.13. *For any n-sigraph* $S_n = (G, \sigma)$, $S_n \sim W((S_n)^m)$ *if, and only if,* S_n *is an i-balanced n-sigraph and* $G \cong C_{2k+1}$.

3. Conclusion

We have introduced a new notion for *n*-signed graphs called wing *n*-sigraph of an *n*-signed graph. We have proved some results and presented the structural characterization of the wing *n*-signed graph. There is no structural characterization of the wing graph, but we have obtained the structural characterization of the wing *n*-signed graph.

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