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Common Minimal Common Neighborhood Dominating Symmetric *n*-Sigraphs

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Abstract. In this paper, we define the common minimal common neighborhood dominating symmetric *n*-sigraph (or common minimal *CN*-dominating symmetric *n*-sigraph) of a given symmetric *n*-sigraph and offer a structural characterization of common minimal common neighborhood dominating symmetric *n*-sigraphs. In the sequel, we also obtained switching equivalence characterization: $\overline{S_n} \sim CMCN(S_n)$, where $\overline{S_n}$ and $CMCN(S_n)$ are complementary symmetric *n*-sigraph and common minimal *CN*-dominating symmetric *n*-sigraph of a symmetric *n*-sigraph S_n respectively.

Keywords: Symmetric *n*-sigraphs, Symmetric *n*-marked graphs, Balance, Switching, Common minimal *CN*-dominating symmetric *n*-sigraphs, Complementation.

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1. Introduction

Unless mentioned or defined otherwise, the reader is referred to for all terminology and notions in graph theory [3]. We consider only finite, simple graphs free from self-loops.

Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, \ldots, a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, \ldots, a_n) : a_k \in \{+,-\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric *n*-tuples. Note that H_n is a group under coordinate-wise multiplication, and the order of H_n is 2^m, where $m = \left\lceil \frac{n}{2} \right\rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ ($S_n = (G, \mu)$), where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \rightarrow H_n$ ($\mu : V \rightarrow H_n$) is a function.

In this paper by an *n-tuple/n-sigraph/n-marked graph*, we always mean a symmetric *n*-tuple/symmetric *n*-sigraph/symmetric *n*-marked graph.

An *n*-tuple $(a_1, a_2, \ldots, a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise, it is a *non-identity n-tuple*. In an *n*-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity *n*-tuple is called an *identity edge*, otherwise, it is a *non-identity edge*.

Further, in an *n*-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the *n*-tuples on the edges of *A*.

In [10], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P. S. K. Reddy [6]):

Definition 1.1. Let $S_n = (G, \sigma)$ be an n-sigraph. Then,

(i) S_n is *identity balanced* (or *i-balanced*), if the product of n-tuples on each cycle of S_n is the identity n-tuple, and

(ii) S_n is *balanced* if every cycle in S_n contains an even number of non-identity edges.

Note 1.1: An i-balanced n-sigraph need not be balanced and conversely.
The following characterization of i-balanced n-sigraphs is obtained in [10].

Theorem 1.1. (**E. Sampathkumar et al. [10]**) An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S_n defined as follows: each vertex $v \in V$, $\mu(v)$ is the n-tuple which is the product of the n-tuples on the edges incident with v. *The complement* of S_n is an n-sigraph $\overline{S_n}=(\bar{G}, \sigma^c)$, where for any edge e $= uv \in \bar{G}$, $\sigma^c(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ is defined here as an i-balanced n-sigraph due to Theorem 1.1.

In [10], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [4-9, 11–25])

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic* if there exists an isomorphism $\phi : G \to G'$ such that if uv is an edge in S_n with label $(a_1, a_2, \ldots, a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, \ldots, a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n-sigraph obtained in this way is denoted by $S_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $S_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ are said to be *cycle isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\boldsymbol{\sigma}(\Phi(C))$ in S'_n.

We make use of the following known result (see [10]).

Theorem 1.2. (**E. Sampathkumar et al. [10]**) *Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.c*

2. Common minimal common neighborhood dominating *n*-sigraph of an *n*-sigraph

Let $G=(V, E)$ be a graph. A set $D \subseteq V$ is a dominating set of G, if every vertex in V-D is adjacent to some vertex in D. A dominating set D of G is minimal, if for any vertex $v \in D$, D-$\{v\}$ is not a dominating set of G.

Let $G=(V, E)$ be a simple graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. For $i \neq j$, the common neighborhood of the vertices v_i and v_j is the set of vertices different from vi and vj which are adjacent to both v_i and v_j and is denoted by $\Upsilon(v_i, v_j)$. Further, a subset *D* of *V* is called the common neighborhood dominating set (or *CN*-dominating set) if every $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\Upsilon(u, v)| \geq 1$, where $|\Upsilon(u, v)|$ is the number of common neighborhoods between u and v. This concept was introduced by Alwardi et al. [1].

A common neighborhood dominating set *D* is said to be minimal common neighborhood dominating set if no proper subset of *D* is common neighborhood dominating set (See [1]).

Alwardi and Soner [2] introduced a new class of intersection graphs in the field of domination theory. The commonality minimal *CN*-dominating graph is denoted by *CMCN*(*G*) is the graph which has the same vertex set as *G* with two vertices are adjacent if and only if there exist minimal *CN*-dominating in *G* containing them.

In this paper, we introduce a natural extension of the notion of common minimal *CN*-dominating graphs to the realm of *n*-sigraphs.

Motivated by the existing definition of complement of an *n*-sigraph, we extend the notion of common minimal *CN*-dominating graphs to *n*-sigraphs as follows: The common minimal *CN*-dominating *n*-sigraph *CMCN*(*G*) of an *n*-sigraph $S_n = (G, \sigma)$ is an *n*-sigraph whose underlying graph is *CMCN*(*G*) and the *n*-tuple of any edge *uv* is $CMCN(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical *n*-marking of S_n. Further, an *n*-sigraph $S_n = (G, \sigma)$ is called common minimal *CN*-dominating *n*-sigraph, if $S_n \cong CMCN(S_n')$ for some *n*-sigraph S_n'. The purpose of this paper is to initiate a study of this notion.

The following result indicates the limitations of the notion $CMCN(S_n)$ as introduced above, since the entire class of *i*-unbalanced *n*-sigraphs is forbidden to be common minimal *CN*-dominating *n*-sigraphs.

Theorem 2.1. *For any n-sigraph $S_n = (G, \sigma)$, its common minimal CN-dominating n-sigraph $CMCN(S_n)$ is i-balanced.*

Proof: Since the *n*-tuple of any edge *uv* in $CMCN(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical *n*-marking of S_n, by Theorem 1.1, $CMCN(S_n)$ is *i*-balanced.

For any positive integer *k*, the k^{th} iterated common minimal *CN*-dominating *n*-sigraph, $CMCN^k(S_n)$ of S_n is defined as follows:

$$CMCN^0(S_n) = S_n,\ CMCN^k(S_n) = CMCN(CMCN^{k-1}(S_n)).$$

Corollary 2.2. *For any n-sigraph $S_n = (G, \sigma)$ and for any positive integer k, $CMCN^k(S_n)$ is i-balanced.*

The following result characterizes *n*-sigraphs which are common minimal *CN*-dominating *n*-sigraphs.

Theorem 2.3. *An n-sigraph $S_n = (G, \sigma)$ is a common minimal CN-dominating n-sigraph if, and only if, S_n is i-balanced n-sigraph and its underlying graph G is a common minimal CN-dominating graph.*

Proof: Suppose that S_n is *i*-balanced and *G* is a common minimal *CN*-dominating graph. Then there exists a graph *H* such that $CMCN(H) \cong G$. Since S_n is *i*-balanced, by Theorem 1.1, there exists a marking ζ of *G* such that each edge $e = uv$ in S_n satisfies $\sigma(uv) = \zeta(u)\zeta(v)$. Now consider the *n*-sigraph $S_n' = (H, \sigma')$, where for any edge *e* in *H*, $\sigma'(e)$ is the *n*-

marking of the corresponding vertex in G. Then clearly, $CMCN(S_n{}') \cong S_n$. Hence S_n is a common minimal CN-dominating n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a common minimal CN-dominating n-sigraph. Then there exists an n-sigraph $S_n{}' = (H, \sigma')$ such that $CMCN(S_n{}') \cong S_n$. Hence G is the common minimal CN-dominating graph of H and by Theorem 2.1, S_n is i-balanced.

In [2], the authors characterized graphs for which $CMCN(G) \cong \bar{G}$.

Theorem 2.4. (Anwar Alwardi et al. [2])
For any graph $G=(V, E)$, $CMCN(G) \cong \bar{G}$ if and only if every minimal CN-dominating set of G is independent.

We now characterize n-sigraphs whose common minimal CN-dominating n-sigraphs and complementary n-sigraphs are switching equivalent.

Theorem 2.5. *For any n-sigraph $S_n = (G, \sigma)$, $\overline{S_n} \sim CMCN(S_n)$ if, and only if, every minimal CN-dominating set of G is independent.*
Proof: Suppose $\overline{S_n} \sim CMCN(S_n)$. This implies, $CMCN(G) \cong \bar{G}$ and hence by Theorem 2.4, every minimal CN-dominating set of G is independent.
Conversely, suppose that every minimal CN-dominating set of G is independent. Then $CMCN(G) \cong \bar{G}$ by Thorem 2.4. Now, if S_n is an n-sigraph with underlying graph G satisfies the conditions of Theorem 2.4, by the definition of complementary n-sigraph and Theorem 2.1, $\overline{S_n}$ and $CMCN(S_n)$ are i-balanced and hence, the result follows from Theorem 1.2.

Theorem 2.6. *For any two n-sigraphs S_n and $S_n{}'$ with the same underlying graph, their common minimal CN-dominating n-sigraphs are switching equivalent.*
Proof. Suppose $S_n = (G, \sigma)$ and $S_n{}' = (G', \sigma')$) be two n-sigraphs with $G \cong G'$. By Theorem 2.1, $CMCN(S_n)$ and $CMCN(S_n{}')$ are i-balanced and hence, the result follows from Theorem 1.2.

For any $m \in H_n$, the *m-complement* of $a = (a_1, a_2,.., a_n)$ is: $a^m = am$. For any $M \subseteq H_n$, and $m \in H_n$, the *m-complement* of M is $M^m = \{a^m : a \in M\}$.
For any $m \in H_n$, the *m- complement* of an n-sigraph $S_n = (G, \sigma)$, written $(S_n{}^m)$, is the same graph but with each edge label $a = (a_1, a_2,... ,a_n)$ replaced by a^m.

For an n-sigraph $S_n = (G, \sigma)$, the $CMCN(S_n)$ is i-balanced. We now examine, the condition under which m-complement of $CMCN(S_n)$ is i-balanced, where for any $m \in H_n$.

Theorem 2.7. *Let $S_n = (G, \sigma)$ be an n-sigraph. Then, for any $m \in H_n$, if $CMCN(G)$ is bipartite then $(CMCN(S_n))^m$ is i-balanced.*
Proof: Since, by Theorem 2.1, $CMCN(S_n)$ is i-balanced, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $CMCN(S_n)$ whose k^{th} co-ordinate are $-$ is even. Also, since $CMCN(G)$ is bipartite, all cycles have even length; thus, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $CMCN(S_n)$ whose k^{th} co-ordinate are $+$ is also even. This implies that the same thing is true in any m-complement, where for any $m \in H_n$. Hence $(CMCN(S_n))^t$ is i-balanced.

Theorem 2.6 provides easy solutions to other *n*-sigraph switching equivalence relations, which are given in the following results.

Corollary 2.8. *For any two n-sigraphs S_n and S_n' with the same underlying graph, CMCN(S_n) and CMCN($(S_n')^m$) are switching equivalent.*

Corollary 2.9. *For any two n-sigraphs S_n and S_n' with the same underlying graph, CMCN($(S_n)^m$) and CMCN(S_n') are switching equivalent.*

Corollary 2.10. *For any two n-sigraphs S_n and S_n' with the same underlying graph, CMCN($(S_n)^m$) and CMCN($(S_n')^m$) are switching equivalent.*

Corollary 2.11. *For any two n-sigraphs $S_n = (G, \sigma)$ and $S_n' = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, $(CMCN(S_n))^m$ and CMCN(S_n') are switching equivalent.*

Corollary 2.12. *For any two n-sigraphs $S_n = (G,\sigma)$ and $S_n' = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, CMCN(S_n) and CMCN($(S_n')^m$) are switching equivalent.*

Corollary 2.13. *For any two n-sigraphs $S_n = (G, \sigma)$ and $S_n' = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, $(CMCN(S_1))^m$ and $(CMCN(S_2))^m$ are switching equivalent.*

3. Conclusion

We have introduced a new notion for *n*-signed graphs called common minimal *CN*-dominating *n*-sigraph of an *n*-signed graph. We have proved some results and presented the structural characterization of a common minimal *CN*-dominating *n*-signed graph. There is no structural characterization of a common minimal *CN*-dominating graph, but we have obtained the structural characterization of a common minimal *CN*-dominating *n*-signed graph.

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