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# On the Exponential Diophantine Equation $p.3^x + p^y = z^2$ with $p$ a Prime Number

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***Abstract.*** In this paper, we find non-negative integer solutions for exponential Diophantine equations of the type $p3^x + p^y = z^2$ where $p$ is a prime number. We prove that such equation has a unique solution $(x, y, z) = (\log_3(p-2), 0, p-1)$ if $2 \neq p \equiv 2(mod3)$ and $(x, y, z) = (0,1,2)$if $p = 2$. We also display the infinite solution set of that equation in the case $p = 3$. Finally, a brief discussion of the case $p \equiv 1(mod3)$ is made, where we display an equation that does not have a non-negative integer solution and leave some open questions. The proofs are based on the use of the properties of the modular arithmetic.

***Keywords:*** Congruences; exponential diophantine equations

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## 1. Introduction

Diophantine equations of the form $a^x + b^y = c^z$ have been studied by numerous mathematicians for many decades and by a variety of methods. One of the first references to these equations was given by Fermat-Euler [4], showing that $(a, c) = (5,3)$ is the unique positive integer solution of the equation $a^2 + 2 = c^3$. Several works on exponential Diophantine equations have been developed in recent years. In 2011, Suvarnamani [4] studied the Diophantine equation $2^x + p^y = z^2$. Rabago [5] studied the equations $3^x + 19^y = z^2$ and $3^x + 91^y = z^2$. The solution sets are $(1,0,2), (4,1,10)$ and $(1,0,2), (2,1,10)$, respectively. A. Suvarnamani *et al.* [7] found solutions of two Diophantine equations $4^x + 7^y = z^2$ and $4^x + 11^y = z^2$. Sroysang (see [6]) studied the Diophantine equation $3^x + 17^y = z^2$. Chotchaisthit (see [3]) showed that the Diophantine equation $p^x + (p+1)^y = z^2$ has unique solutions $(p, x, y, z) = (7,0,1,3)$ and $(p, x, y, z) = (3,2,2,5)$ if $(x, y, z) \in \mathbb{N}^3$ and $p$ is a Mersenne prime. In 2019, Thongnak *et al.* (see [9]) found exactly two non-trivial solutions for the equation $2^x - 3^y = z^2$, namely $(1,0,1)$ and $(2,1,1)$. Buosi *et al.* (see [1] and [2]) studied some exponential Diophantine equations that generalized the work of Thongnak *et al.* (see [9]). Several other similar and recent works can be found in Thongnak et al. [10], [11] and [12].

In this work we show that when $p > 3$ is a prime integer such that $p \equiv 2(mod3)$, there is an ordered triple $(x, y, z)$of non-negative integers that solves the equation $p.3^x + p^y = z^2$ if and only if $p - 2$ is a non-trivial power of 3. In the affirmative case, there exists only one solution which is given by $(x, y, z) = (\log_3(p - 2), 0, p - 1)$.

This result generalizes the theorem obtained in Thongnak *et al*. [10] when $p = 11$. In other words, Thongnak et al. [10] found the only solution (2,0,10) for the Diophantine equation $11.3^x + 11^y = z^2$ using modular arithmetic. We also determine the unique solution of the case $p = 2$ and the infinite set of solutions when $p = 3$. The case where $p$ is congruent to 1 modulo 3 has not been solved completely because it is not understood why there are situations whose equation has a solution and others that do not. At the end of the article, a brief discussion of the case $p \equiv 1(mod3)$ is made, showing an example of an equation with no solution and suggesting some open questions.

## 2. Some notations

Denote by $\mathbb{Z}$ be the set of integer numbers and let $\mathbb{N}$ be the set of all positive integers together with the number 0, that is, $\mathbb{N} = \{0,1,2,3, \dots\}$, such a set will be called the set of *natural numbers*. Define $\mathbb{N}^*=\mathbb{N} - \{0\}$ and $\mathbb{N}^q = \mathbb{N} \times \mathbb{N} \times \cdots \times \mathbb{N}$ as the *cartesian product* of $q$ copies of $\mathbb{N}$. When $a$ divides $b$ we will use the symbol $a \mid b$. When $a$ is congruent to $b$ module $m$ we will write $a \equiv b(modm)$. Let $a, m$ be integers with $a > 0$ and $m > 2$. The smallest positive integer $k$ such that $a^k \equiv 1(modm)$ will be said the *order* of $a$ modulo $m$ and will be denoted as $|a|_m$. The set of all non-negative integer solutions of the equation $p3^x + p^y = z^2$ will be said simply the *solution set of the equation*, i.e., the set $\{(x, y, z) \in \mathbb{N}^3 : p3^x + p^y = z^2\}$.

## 3. Results

In this section, we will find the solution set for the equation

$$p3^x + p^y = z^2, (x, y, z) \in \mathbb{N}^3, \quad (1)$$

for several prime integers. We will divide the results into four sections: case $p = 3$, general results for $p > 3$, case $p \equiv 2(mod3)$ and finally, we will make a brief explanation of the case $p \equiv 1(mod3)$ since in this case the general problem still remains open. The motivation for this work is the paper Thongnak *et al*. [10] where the authors solved the above equation in the particular case $p = 11$. The result of Thongnak *et al.* is an immediate consequence of Theorem 1.13 proved in this article.

**Case** $p = 3$

In this subsection, we present all the non-negative integer solutions of the equation $p3^x + p^y = z^2$ in the particular case when $p = 3$.

**Theorem 1.1.** The solution set of the Diophantine exponential equation

$$3 \cdot 3^x + 3^y = z^2 \quad (2)$$

in $\mathbb{N}^3$ is $\{(2n, 2n, 2.3^n); n \in \mathbb{N}\} \cup \{(1 + 2n, 3 + 2n, 2.3^{n+1}); n \in \mathbb{N}\}$.

The proof is based on the combination of the results of the following six lemmas.

**Lemma 1.2.** If $(y, z) \in \mathbb{N}^2$ is a solution of the equation $3^{y+1} + 1 = z^2$ then $y = 0$.
**Proof:** $3^{y+1} + 1 = z^2 \Rightarrow 3^{y+1} = (z + 1)(z - 1) \Rightarrow z - 1 = 1, z + 1 = 3 \Rightarrow y = 0$. ***Q.E.D.***

**Lemma 1.3.** If $(x,y,z) \in \mathbb{N}^3$ is a solution of the equation $3^x(3^{y+1}+1) = z^2$ then $y = 0$.
**Proof:** Suppose there exists $(x,y,z) \in \mathbb{N}^3$ such that $3^x(3^{y+1}+1) = z^2$ and $y > 0$. By Lemma 1.2, $3^{y+1}+1$ it is not a perfect square. Thus there is a prime integer $q$that appears an odd number of times in the prime factorization of $3^{y+1}+1$. Since $q \mid z^2$ we have two possibilities:

$x = 0 \Rightarrow 3^{y+1}+1$ is a perfect square;
$x > 0 \Rightarrow q \mid 3^x \Rightarrow q = 3 \Rightarrow 3 \mid 1$.

In both cases we have an absurd. Therefore $y = 0$. ***Q.E.D.***

**Lemma 1.4.** If $(x,y,z) \in \mathbb{N}^3$ is a solution of the equation (2) then $y - x \in \{0,1,2\}$.
**Proof:** If $y < x$ then there exists an integer $k > 0$ such that x=y+k. Replacing $x$in (2) with $y + k$ we obtain
$3 \cdot 3^{y+k} + 3^y = z^2$ if and only if $3^y(3^{k+1}+1) = z^2$.
which contradicts Lemma 1.3. Therefore $x \leqslant y$.

If $y - x \geq 3$ then $y = x + k$ for some integer $k \geq 3$. Replacing $y$ in (2) with $x + k$ we obtain $3 \cdot 3^x + 3^{x+k} = z^2$ if and only if $3^{x+1}(3^{k-1}+1) = z^2$.
which is a contradiction with Lemma 1.3. Therefore $y - x \in \{0,1,2\}$. ***Q.E.D.***

**Lemma 1.5.** If $(x,y,z) \in \mathbb{N}^3$ is a solution of the equation (2) then $y = x$ or $y = x + 2$.
**Proof:** By Lema 1.3, $y - x \in \{0,1,2\}$. Suppose $y = x + 1$. Replacing $y$ in (2) with $x + 1$we obtain $3^{x+1} + 3^{x+1} = z^2 \Rightarrow 2.3^{x+1} = z^2 \Rightarrow 2 \mid z^2$ and 4 does not divide $z^2$,
which is an absurd. Therefore $y - x \in \{0,1,2\}$. ***Q.E.D.***

**Lemma 1.6.** If $(x,y,z) \in \mathbb{N}^3$ is a solution of the equation (2) and$y = x$, then there exists $n \in \mathbb{N}$ such that $x = y = 2n$ and $z = 2 \cdot 3^n$.
**Proof:** Making $y = x$ in equation (2) we get
$3^{x+1} + 3^x = z^2 \Rightarrow 4 \cdot 3^x = z^2 \Rightarrow x$ is even.

Henceforth there exists $n \in \mathbb{N}$ such that $y = x = 2n$ and $z = \sqrt{4 \cdot 3^{2n}} = 2 \cdot 3^n$. ***Q.E.D.***

**Lemma 1.7.** If $(x,y,z) \in \mathbb{N}^3$ is a solution of the equation (2) and $y - x = 2$ then there exists $n \in \mathbb{N}$ such that
x=$1 + 2n$, y=$3 + 2n$ and $z = 3^{n+1}$.
**Proof:** Making $y = x + 2$ in equation (2) we get
$3^{x+1} + 3^{x+2} = z^2 \Rightarrow 4 \cdot 3^{x+1} = z^2 \Rightarrow x$ is odd.
Henceforth there exists $n \in \mathbb{N}$ such that x=$1 + 2n$, y=$3 + 2n$ and $z = \sqrt{4 \cdot 3^{2n+2}} = 2 \cdot 3^{n+1}$.***Q.E.D.***

**General results for a prime** $p \neq 3$

**Lemma 1.8.** Let $p \neq 3$ be a prime integer. If $(x,y,z) \in \mathbb{N}^3$ is a solution of

$$p3^x + p^y = z^2, \tag{3}$$

then $y = 0$ or $y = 1$.

**Proof:** Let $(x, y, z)$ be a solution of (3). Assume $y \geq 2$. It is clear that $z \neq 0$. In this case, $p$ divides $z$ because $p3^x + p^y = z^2 \Rightarrow p(3^x + p^{y-1}) = z^2$.
Let $m \in \mathbb{N}^*$ such that $z = m.p$. Substitute $m.p$ for $z$ in the above equation to obtain
$3^x = p(m^2 - p^{y-2})$.
If $x = 0$ the above equation is an absurd for all prime integer $p \geq 2$. If $x > 0$ we have $3 \mid p$, which is absurd for all prime integer $p \neq 3$. Therefore $y = 0$ or $y = 1$.***Q.E.D.***

If one substitute $0$ for $y$ in the equation (3) one obtain $p3^x + 1 = z^2$ which is equivalent to the following equation

$$p3^x = z^2 - 1 = (z-1)(z+1). \tag{4}$$

**Lemma 1.9.** Let $p \neq 3$ be a prime integer. If $(x, z) \in \mathbb{N}^2$ is a solution of (4), then $x > 0$ and $z$ it is not equivalet to $0$ module $3$.
**Proof:** Let $(x, z)$ be a solution of (4). If $x = 0$ then
$p = (z-1)(z+1) \Rightarrow z = 2$ and $p = 3$,
which is a contradiction. Hence $x > 0$. If $3$ divides $z$ then $3$ divides $z^2 - p3^x = 1$, which is an absurd. Therefore it is not equivalet to $0$ module $3$. ***Q.E.D.***

We say that $h$ is a non-trivial power of $3$ if $h = 3^x$ with $x \in \mathbb{N}^*$.

**Lemma 1.10.** Let $p > 3$ be a prime integer. The equation (4) has a solution in $\mathbb{N}^2$ if and only if $p - 2$ is a non-trivial power of $3$ or $p + 2$ is a non-trivial power of $3$. In the affirmative case, the equation (4) has a unique solution in $\mathbb{N}^2$ given by
$(\log_3(p-2), p-1)$ if $p - 2$ is a non-trivial power of $3$;
$(\log_3(p+2), p+1)$ if $p + 2$ is a non-trivial power of $3$.
**Proof:** Let $(x, z) \in \mathbb{N}^2$ be a solution of (4). By Lemma 1.9, $x > 0$ and $z$ it is not equivalent to $0$ module $3$.

If $z \equiv 1(mod3)$ then $z - 1 \equiv 0(mod3)$. Since $p3^x = (z-1)(z+1)$ it follows that

$$\begin{matrix} z+1 = p \\ z-1 = 3^x \end{matrix} \Rightarrow \begin{matrix} z = p-1 \\ 3^x = z-1 = p-2 \end{matrix} \Rightarrow \begin{matrix} z = p-1 \\ x = \log_3(p-2) \end{matrix}.$$

If $z \equiv 2(mod3)$ then $z + 1 \equiv 0(mod3)$. Since $p3^x = (z-1)(z+1)$ it follows that

$$\begin{matrix} z-1 = p \\ z+1 = 3^x \end{matrix} \Rightarrow \begin{matrix} z = p+1 \\ 3^x = z+1 = p+2 \end{matrix} \Rightarrow \begin{matrix} z = p+1 \\ x = \log_3(p+2) \end{matrix}.$$

The converse is straightforward and will be omitted. ***Q.E.D.***

Making $y = 1$ in the equation (3) one obtain

$$p3^x + p = z^2. \tag{5}$$

**Lemma 1.11.** Let $p > 3$ be a prime integer. If $(x, y) \in \mathbb{N}^2$ is a solution of (5) then $x > 0$ and $z$ it is not equivalent to $0$ module $3$.
**Proof:** Suppose $(x, y) \in \mathbb{N}^2$ is a solution of (5). If $x = 0$ then $2p = z^2$, which is an absurd. It follows that $x > 0$.

If $z \equiv 0 (mod 3)$ then 3 divides $z^2 = p3^x + p$ and therefore 3 divides $p$ which is a contradiction. ***Q.E.D.***

**Case** $p \equiv 2(mod\ 3)$

The Theorem 1.13 below presents all the non-negative integer solutions of the equation $p3^x + p^y = z^2$ in the particular case where $p \equiv 2(mod 3)$ and $p \neq 2$. This result generalizes Theorem 2.1 of [10] where $p = 11$.

**Lemma 1.12.** There is no $z \in \mathbb{Z}$ such that $z^2 \equiv 2(mod 3)$.

**Proof:** If $z \equiv 0(mod 3)$, then $z^2 \equiv 0(mod 3)$. If $z \equiv 1(mod 3)$ or $z \equiv 2(mod 3)$, then $z^2 \equiv 1(mod 3)$. ***Q.E.D.***

**Theorem 1.13.** Let $p > 3$ be a prime integer such that $p \equiv 2(mod 3)$. The equation

$$p3^x + p^y = z^2, \tag{6}$$

admits a solution in $\mathbb{N}^3$ if and only if $p - 2$ is a non-trivial power of 3. In the affirmative case, the unique solution is $(x, y, z) = (\log_3(p - 2), 0, p - 1)$.

**Proof:** Let $(x, y, z)$ be a solution of (6). By Lemma 1.8 we must have $y = 0$ or $y = 1$. If $y = 0$, it follows from Lemma 1.10 that $(\log_3(p - 2), 0, p - 1)$ is the unique solution in $\mathbb{N}^3$ of the equation $p3^x + 1 = z^2$, since $\log_3(p - 2)$ is an integer. Now consider $y = 1$. By Lemma 1.11, $x \geq 1$. So we get

$2 \equiv p \equiv z^2 - p3^x \equiv z^2(mod 3)$,

which is a contradiction by Lemma 1.12. ***Q.E.D.***

**Remark 1.14.** For example, for $p = 17,23,41,53,59,71$ the equation of the previous theorem has no non-negative integer solutions. For $p = 5,11,29,83$ the solutions are respectively $(1,0,4)$, $(2,0,10)$, $(3,0,28)$ and $(4,0,82)$.

**Theorem 1.15.** The unique solution of the Diophantine exponential equation

$$2 \cdot 3^x + 2^y = z^2, (x, y, z) \in \mathbb{N}^3, \tag{7}$$

is the ordered triple $(x, y, z) = (0,1,2)$.

**Proof:** Let $(x, y, z)$ be a solution of (7). By Lemma 1.8 we must have $y = 0$ or $y = 1$. If $y = 0$, it follows from Lemma 1.9 that $x > 0$ and $z$ it is not equivalent to 0 module 3. In this case we have the following equivalence for equation (7)

$2 \cdot 3^x + 1 = z^2$ if and only if $2 \cdot 3^x = (z - 1)(z + 1) = z^2 - 1$.

If $z \equiv 1(mod 3)$ then $z - 1 \equiv 0(mod 3)$ and $z + 1 \equiv 2(mod 3)$, then we have

$$\begin{matrix} z + 1 = p \\ z - 1 = 3^x \end{matrix} \Rightarrow \begin{matrix} z = 1 \\ 3^x = 0 \end{matrix},$$

an absurd.

If $z \equiv 2(mod 3)$ then $z - 1 \equiv 1(mod 3)$ and $z + 1 \equiv 0(mod 3)$, then we have

$$\begin{matrix} z + 1 = 3^x \\ z - 1 = 2 \end{matrix} \Rightarrow \begin{matrix} z = 3 \\ 3^x = 4 \end{matrix},$$

an absurd.

Now consider $y = 1$. In this case equation (7) reduces to equation

$2 \cdot 3^x + 2 = 2(3^x + 1) = z^2$.
If $x = 0$ we have $z^2 = 4$, so $z = 2$. Therefore $(x, y, z) = (0,1,2)$ is a solution to equation (7) in $\mathbb{N}^3$. If $x > 0$ then $z^2 \equiv 2(3^x + 1) \equiv 2(mod 3)$, a contradiction by Lemma 1.12. Therefore $(x, y, z) = (0,1,2)$ is the only solution of equation (7). ***Q.E.D.***

**4.** When $p \equiv 1(mod\ 3)$, the equation

$$p3^x + p^y = z^2, (x, y, z) \in \mathbb{N}^3 \tag{8}$$

has not yet been completely solved, that is, the behavior of the solutions of these equations is not known, whether they have a solution and whether the solutions, if any, are finite or infinite.

Let $(x, y, z)$ be a solution of (8). By Lemma 1.8, $y \in \{0,1\}$. By Lemma 1.10 we can say whether equation (8) will have a solution as long as $p + 2$ is a non-trivial power of 3. Furthermore, that lemma determines the unique solution in this case. However, for the case $y = 1$ we do not have a conclusive result for the time being. For example, equations with $p = 7,61$ and $547$ respectively have the following solutions $(3,1,14)$, $(5,1,122)$ and $(7,1,1094)$. We do not know if those three equations have other solutions.

**Remark 1.16.** Note that $(q, 2) \in \mathbb{N}^2$ is a solution of $3^x + 1 = p \cdot w^2$ if and only if $(q, 1,2p)$ is a solution of $p3^x + p^y = z^2, (x, y, z) \in \mathbb{N}^3$.

In the next theorem we will show an example whose given equation does not have non-negative integer solutions.

**Theorem 1.17.** The exponential Diophantine equation

$$13 \cdot 3^x + 13^y = z^2, (x, y, z) \in \mathbb{N}^3, \tag{9}$$

has no solutions.

**Proof:** Let $(x, y, z)$ be a solution of (9). By Lemma 1.8, $y \in \{0,1\}$. First consider $y = 0$. By Lemma 1.10 there are no solutions to the equation in this case, since $p + 2 = 15$ is not a non-trivial power of 3.

Suppose there is a solution $(x, 1, z) \in \mathbb{N}^3$ of (9). In this case equation (9) reduces to $13 \cdot 3^x + 13 = z^2$. Note that 13 divides $z$ and therefore $z = 13w, w \in \mathbb{N}^*$. So we have the following equivalence of equations
$13 \cdot 3^x + 13 = z^2 = 13^2 \cdot w^2$ if and only if $3^x + 1 = 13 \cdot w^2 \equiv 0(mod 13)$.
On the other hand, notice that $3^2 \equiv 9(mod 13)$ and $3^3 \equiv 1(mod 13)$. Therefore the order of 3 modulo 13 is equal to 3, that is $|3|_{13} = 3$. So write $x = 3m + r$, where $m \in \mathbb{N}$ and $r \in \{0,1,2\}$. So we have the following equation

$$3^x + 1 = 3^{3m+r} + 1 = 27^m \cdot 3^r + 1 \equiv 3^r + 1(mod 13) \equiv \begin{matrix} 2(mod\ 13) \\ 4(mod\ 13) \\ 10(mod\ 13) \end{matrix}, \text{if } \begin{matrix} r = 0 \\ r = 1, \\ r = 2 \end{matrix}$$

an absurd. ***Q.E.D.***

## 5. Open questions

The following questions refer to the equation

$$p3^x + p^y = z^2, (x, y, z) \in \mathbb{N}^3 \text{ with } p \equiv 1(mod\ 3). \tag{10}$$

- When $p \equiv 1(mod3)$, what additional conditions must exist on $p$ for the equation (11) to have a solution?
- If there is a solution for equation (10), how do you know if the number of solutions is finite or infinite?
- What additional conditions must be imposed on $p$ for there to be a unique solution?

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