Annals of Pure and Applied Mathematics
Vol. 29, No. 1, 2024, 9-17
ISSN: 2279-087X (P), 2279-0888(online)
Published on 14 January 2024
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v29n1a02928

Annals of
Pure and Applied
Mathematics

Some New Results in Cubic Graphs

Farshid Mofidnakhaei

Department of Physics, Sari Branch, Islamic Azad University, Sari, Iran
Email: Farshid.Mofidnakhaei@gmail.com

Received 24 November 2023; accepted 5 January 2024

Abstract. In this paper, two new operations on cubic graphs, namely, maximal product and residue product were presented, and some results concerning their degrees were introduced. Likewise, we presented certain types of cubic graphs, including totally irregular, strongly irregular, and strongly totally irregular cubic graphs, which are described for the first time here.

Keywords: Cubic set, cubic graph, maximal product, strongly irregular.

AMS Mathematics Subject Classification (2010): 05C99, 03E72

1. Introduction

Graph theory serves as an exceptionally beneficial tool in solving combinatorial problems in various fields, such as geometry, algebra, number theory, topology, and social systems. A graph basically holds a model of relations, and it is used to depict the real-life problems encompassing the relationships among objects. To represent the objects and the relations between objects, the graph vertices and edges are employed, respectively. Fuzzy graph models are helpful mathematical tools in order to address the combinatorial problems in various fields incorporating research, optimization, algebra, computing, and topology. Due to the natural existence of vagueness and ambiguity, fuzzy graphical models are noticeably better than graphical models. Originally, fuzzy set theory was required to deal with many multifaceted issues, which are replete with incomplete information. In 1965, fuzzy set theory was first suggested by Zadeh [42]. Fuzzy set theory is a highly powerful mathematical tool for solving approximate reasoning-related problems. Jun et al. [11] introduced cubic sets. Later on, Muhiuddin et al. [14-16] applied the notion cubic sets on different aspects. The first description of fuzzy graphs was proposed by Kafmann [12] in 1993, taken from Zadeh's fuzzy relations [43-44] . However, Rosenfeld [31] described another detailed definition, including fuzzy vertex and fuzzy edges and various fuzzy analogs of graphical theoretical concepts, including paths, cycles, connectedness, etc. Akram et al. [1,2] presented new definitions of fuzzy graphs. Rashmanlou et al. [22-29] investigated different concepts on cubic graphs, vague graphs, and bipolar fuzzy graphs. Samanta et al. [32,33] introduced fuzzy competition graphs and some properties of irregular bipolar fuzzy graphs. Borzooei and Rashmanlou [3-6] studied new concepts on vague graphs. Gani and Radha [17,18] recommended regular fuzzy graphs and totally regular fuzzy graphs. Kumaravel and Radha [30] described the concepts of the edge degree and the total edge degree in regular fuzzy graphs. Latha and Gani defined neighborly irregular fuzzy graphs and highly irregular fuzzy graphs. Sunitha et al. [34,35] presented new concepts for fuzzy graphs. Talebi et al. [37-41] represented several concepts on interval-valued fuzzy graphs, intuitionistic fuzzy graphs, and bipolar fuzzy graphs. Shoaib et al. [36] given some results on pythagorean fuzzy graphs. Ghorai et al. [7-10] studied novel concepts in fuzzy graphs. Pal et al. [19-21] investigated several results on fuzzy graphs.

In this research, two new operations on cubic graphs, namely, maximal product and residue product were presented, and some results concerning their degrees were introduce. Also, we presented certain types of cubic graphs, including totally irregular, strongly irregular and strongly totally irregular cubic graphs.

2. Preliminaries

A fuzzy graph is of the from $G=(\psi,\phi)$ which is a pair of mappings $\psi: V\to[0,1]$ and $\phi: V\times V\to[0,1]$ as is defined as $\phi(m,n)\le\psi(m)\wedge\psi(n)$, $\forall m,n\in V$, and ϕ is a symmetric fuzzy relation on ψ and $\wedge$ denotes minimum.

Let X be a non-empty set. A function $A: X\to[I]$ is called an interval-valued fuzzy set (shortly, IVF set) in X. Let $[I]^X$ stands for the set of all IVF sets in X. For every $A\in[I]^X$ and $x\in X$, $A(x)=[A^-(x),A^+(x)]$ is called the degree of membership of an element $x\in A$, where $A^-: X\to I$ and $A^+: X\to I$ are fuzzy sets in X which are called a lower fuzzy set and upper fuzzy set in X, respectively. For simplicity, we denote $A=[A^-,A^+]$. For every $A,B\in[I]^X$, we define $A\subseteq B$ if and only if $A(x)\le B(x)$, for all $x\in X$.

Definition 2.1. *Let $A=[A^-,A^+]$, and $B=[B^-,B^+]$ be two interval valued fuzzy sets in X. Then, we define*

$$r\min\{A(x),B(x)\}=[\min\{A^-(x),B^-(x)\},\min\{A^+(x),B^+(x)\}],$$
$$r\max\{A(x),B(x)\}=[\max\{A^-(x),B^-(x)\},\max\{A^+(x),B^+(x)\}].$$

Definition 2.2. *Let x be a non-empty set. By a cubic set in X, we mean a structure $A=\{\langle x,A(x),\lambda(x): x\in X\rangle\}$ in which A is an interval-valued fuzzy sets in X and λ is a fuzzy set in X. A cubic set $A=\{\langle m,A(m),\lambda(m): m\in X\rangle\}$ is simply denoted by $A=\langle A,\lambda\rangle$. The collection of all cubic sets in X is denoted by $CP(X)$.*

Definition 2.3. *A cubic graph is a triple $\zeta=(G^*,P,Q)$ where $G^*=(V,E)$ is a graph, $P=(\widetilde{\mu_P},\lambda_P)$ is a cubic set on V and $Q=(\widetilde{\mu_Q},\lambda_Q)$ is a cubic set on $V\times V$ so that $\widetilde{\mu_Q}(mn)\le r\min\{\widetilde{\mu_P}(m),\widetilde{\mu_P}(n)\}$ and $\lambda_Q(mn)\ge max\{\lambda_P(m),\lambda_P(n)\}$.*

3. New concepts of cubic graphs

Definition 3.1. *Let $\zeta_1=(P_1,Q_1)$ and $\zeta_2=(P_2,Q_2)$ be two cubic graphs with underlying crisp graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$, respectively. $\zeta_1*\zeta_2=(P,Q)$ is called maximal cubic graph with underlying crisp graph $G=(V,E)$, where $V=V_1\times V_2$ and $E=\{(m_1,n_1)(m_2,n_2)|\, m_1=m_2, n_1n_2\in E_2 \text{ or } n_1=n_2, m_1m_2\in E_1\}$.*

$$(i)\ \begin{cases}(\widetilde{\mu_{P_1}} * \widetilde{\mu_{P_2}})(m,n) = r\max\{\widetilde{\mu_{P_1}}(m), \widetilde{\mu_{P_2}}(n)\},\\ (\lambda_{P_1} * \lambda_{P_2})(m,n) = \min\{\lambda_{P_1}(m), \lambda_{P_2}(n)\},\\ for\ all\ (m,n) \in V = V_1 \times V_2\end{cases}$$

$$(ii)\ \begin{cases}(\widetilde{\mu_{Q_1}} * \widetilde{\mu_{Q_2}})\big((m_1,n_1)(m_2,n_2)\big) = r\max\left(\widetilde{\mu_{P_1}}(m_1), \widetilde{\mu_{Q_2}}(n_1n_2)\right),\\ (\lambda_{Q_1} * \lambda_{Q_2})\big((m_1,n_1)(m_2,n_2)\big) = \min\left(\lambda_{P_1}(m_1), \lambda_{Q_2}(n_1n_2)\right),\\ m_1 = m_2,\ n_1n_2 \in E_2,\end{cases}$$

$$(iii)\ \begin{cases}(\widetilde{\mu_{Q_1}} * \widetilde{\mu_{Q_2}})\big((m_1,n_1)(m_2,n_2)\big) = r\max\left(\widetilde{\mu_{P_2}}(n_1), \widetilde{\mu_{Q_1}}(m_1m_2)\right),\\ (\lambda_{Q_1} * \lambda_{Q_2})\big((m_1,n_1)(m_2,n_2)\big) = \min\left(\lambda_{P_2}(n_1), \lambda_{Q_1}(m_1m_2)\right),\\ m_1m_2 \in E_1, n_1 = n_2.\end{cases}$$

Theorem 3.2. *The maximal product of two cubic graphs ζ_1 and ζ_2, is a cubic graph, too.*

Proof: Let $\zeta_1 = (P_1, Q_1)$ and $\zeta_2 = (P_2, Q_2)$ be two cubic graphs and $\big((m_1, m_2)(n_1, n_2)\big) \in E_1 \times E_2$. Then, by Definition 3, we have two cases:

$$(i)\ m_1 = n_1 = m$$

$$\begin{aligned}(\widetilde{\mu_{Q_1}} * \widetilde{\mu_{Q_2}})\big((m,m_2)(m,n_2)\big) &= r\max\left(\widetilde{\mu_{P_1}}(m), \widetilde{\mu_{Q_2}}(m_2n_2)\right)\\ &\le r\max\left\{\widetilde{\mu_{P_1}}(m), r\min\{\widetilde{\mu_{P_2}}(m_2), \widetilde{\mu_{Q_2}}(n_2)\}\right\}\\ &= r\min\left\{r\max\{\widetilde{\mu_{P_1}}(m), \widetilde{\mu_{P_2}}(m_2)\}, r\max\{\widetilde{\mu_{P_1}}(m), \widetilde{\mu_{P_2}}(n_2)\}\right\}\\ &= r\min\{(\widetilde{\mu_{P_1}} * \widetilde{\mu_{P_2}})(m,m_2), (\widetilde{\mu_{P_1}} * \widetilde{\mu_{P_2}})(m,n_2)\},\end{aligned}$$

$$\begin{aligned}(\lambda_{Q_1} * \lambda_{Q_2})\big((m,m_2)(m,n_2)\big) &= \min\{\lambda_{P_1}(m), \lambda_{P_2}(m_2n_2)\}\\ &\ge \min\left\{\lambda_{P_1}(m), \max\{\lambda_{P_2}(m_2), \lambda_{P_2}(n_2)\}\right\}\\ &= \max\left\{\min\{\lambda_{P_1}(m), \lambda_{P_2}(m_2)\}, \min\{\lambda_{P_1}(m), \lambda_{P_2}(n_2)\}\right\}\\ &= \max\{(\lambda_{P_1} * \lambda_{P_2})(m,m_2), (\lambda_{P_1} * \lambda_{P_2})(m,n_2)\}.\end{aligned}$$

(i) if $m_2 = n_2 = z$

$$\begin{aligned}(\widetilde{\mu_{Q_1}} * \widetilde{\mu_{Q_2}})\big((m_1,z)(n_1,z)\big) &= r\max\{\widetilde{\mu_{Q_1}}(m_1n_1), \widetilde{\mu_{P_2}}(z)\}\\ &\le r\max\{r\min\{\widetilde{\mu_{P_1}}(m_1), \widetilde{\mu_{P_1}}(n_1)\}, \widetilde{\mu_{P_2}}(z)\}\\ &= r\min\left\{r\max\{\widetilde{\mu_{P_1}}(m_1), \widetilde{\mu_{P_2}}(z)\}, r\max\{\widetilde{\mu_{P_1}}(n_1), \widetilde{\mu_{P_2}}(z)\}\right\}\\ &= r\min\{(\widetilde{\mu_{P_1}} * \widetilde{\mu_{P_2}})(m_1,z), (\widetilde{\mu_{P_1}} * \widetilde{\mu_{P_2}})(n_1,z)\},\end{aligned}$$

$$\begin{aligned}(\lambda_{Q_1} * \lambda_{Q_2})((m_1,z)(n_1,z)) &= \min\{\lambda_{Q_1}(m_1n_1), \lambda_{P_2}(z)\}\\ &\geq \min\{\max\{\lambda_{P_1}(m_1), \lambda_{P_1}(n_1)\}, \lambda_{P_2}(z)\}\\ &= \max\left\{\min\{\lambda_{P_1}(m_1), \lambda_{P_2}(z)\}, \min\{\lambda_{P_1}(n_1), \lambda_{P_2}(z)\}\right\}\\ &= \max\{(\lambda_{P_1} * \lambda_{P_2})(m_1,z), (\lambda_{P_1} * \lambda_{P_2})(n_1,z)\}.\end{aligned}$$

Hence, $\zeta_1 * \zeta_2$ is a cubic graph. □

Definition 3.3. *A Cubic graph $\zeta = (P, Q)$ is strong if: $\widetilde{\mu_Q}(mn) = \widetilde{\mu_P}(m) \wedge \widetilde{\mu_P}(n)$, $\lambda_Q(mn) = \lambda_P(m) \vee \lambda_P(n)$, for all $mn \in E$.*

Theorem 3.4. *The maximal product of two strong cubic graphs ζ_1 and ζ_2, is a strong cubic graph.*

Proof: Let $\zeta_1 = (P_1, Q_1)$ and $\zeta_2 = (P_2, Q_2)$ be two strong cubic graphs. Then $\widetilde{\mu_{Q_1}}(m_1m_2) = r\min\left(\widetilde{\mu_{P_1}}(m_1), \widetilde{\mu_{P_1}}(m_2)\right)$, $\lambda_{Q_1}(m_1m_2) = \max\left(\lambda_{P_1}(m_1), \lambda_{P_2}(m_2)\right)$, for any $m_1m_2 \in E_1$ and $\widetilde{\mu_{Q_2}}(n_1n_2) = r\min\left(\widetilde{\mu_{P_1}}(n_1), \widetilde{\mu_{P_2}}(n_2)\right)$, $\lambda_{Q_2}(n_1n_2) = \max\left(\lambda_{P_1}(n_1), \lambda_{P_2}(n_2)\right)$, for any $n_1n_2 \in E_2$. Then, proceeding according to the definition of maximal product,

(i) if $n_1 = n_2$ and $m_1m_2 \in E_2$. Then,

$$\begin{aligned}(\widetilde{\mu_{Q_1}} * \widetilde{\mu_{Q_2}})((n_1,m_1)(n_2,m_2)) &= r\max\{\widetilde{\mu_{P_1}}(n_1), \widetilde{\mu_{Q_2}}(m_1m_2)\}\\ &= r\max\left\{\widetilde{\mu_{P_1}}(n_1), r\min\{\widetilde{\mu_{P_2}}(m_1), \widetilde{\mu_{P_2}}(m_2)\}\right\}\\ &= r\min\left\{r\max\{\widetilde{\mu_{P_1}}(n_1), \widetilde{\mu_{P_2}}(m_1)\}, r\max\{\widetilde{\mu_{P_1}}(n_1), \widetilde{\mu_{P_2}}(m_2)\}\right\}\\ &= r\min\{(\widetilde{\mu_{P_1}} * \widetilde{\mu_{P_2}})(n_1,m_1), (\widetilde{\mu_{P_1}} * \widetilde{\mu_{P_2}})(n_1,m_2)\},\end{aligned}$$

$$\begin{aligned}(\lambda_{Q_1} * \lambda_{Q_2})((n_1,m_1)(n_2,m_2)) &= \min\{\lambda_{P_1}(n_1), \lambda_{Q_2}(m_1m_2)\}\\ &= \min\left\{\lambda_{P_1}(n_1), \max\{\lambda_{P_2}(m_1), \lambda_{P_2}(m_2)\}\right\}\\ &= \max\left\{\min\{\lambda_{P_1}(n_1), \lambda_{P_2}(m_1)\}, \min\{\lambda_{P_1}(n_1), \lambda_{P_2}(m_2)\}\right\}\\ &= \max\{(\lambda_{P_1} * \lambda_{P_2})(n_1,m_1), (\lambda_{P_1} * \lambda_{P_2})(n_1,m_2)\}.\end{aligned}$$

(ii) if $m_1 = m_2$ and $n_1n_2 \in E_1$. Then

$$\begin{aligned}(\widetilde{\mu_{Q_1}} * \widetilde{\mu_{Q_2}})((n_1,m_1)(n_2,m_2)) &= r\max\{\widetilde{\mu_{Q_1}}(n_1n_2), \widetilde{\mu_{P_2}}(m_2)\}\\ &= r\max\{r\min\{\widetilde{\mu_{P_1}}(n_1), \widetilde{\mu_{P_2}}(n_2)\}, \widetilde{\mu_{P_2}}(m_2)\}\\ &= r\min\left\{r\max\{\widetilde{\mu_{P_1}}(n_1), \widetilde{\mu_{P_2}}(m_2)\}, r\max\{\widetilde{\mu_{P_1}}(n_2), \widetilde{\mu_{P_2}}(m_2)\}\right\}\\ &= r\min\{(\widetilde{\mu_{P_1}} * \widetilde{\mu_{P_2}})(n_1,m_2), (\widetilde{\mu_{P_1}} * \widetilde{\mu_{P_2}})(n_2,m_2)\},\end{aligned}$$

$$\begin{aligned}(\lambda_{Q_1} * \lambda_{Q_2})\big((n_1,m_1)(n_2,m_2)\big) &= \min\{\lambda_{Q_1}(n_1n_2), \lambda_{P_2}(m_2)\}\\ &= \min\{\max\{\lambda_{P_1}(n_1), \lambda_{P_1}(n_2)\}, \lambda_{P_2}(m_2)\}\\ &= \max\Big\{\min\{\lambda_{P_1}(n_1), \lambda_{P_2}(m_2)\}, \min\{\lambda_{P_1}(n_2), \lambda_{P_2}(m_2)\}\Big\}\\ &= \max\{(\lambda_{P_1} * \lambda_{P_2})(n_1,m_2), (\lambda_{P_1} * \lambda_{P_2})(n_2,m_2)\}.\end{aligned}$$

Therefore, $\zeta_1 * \zeta_2$ is a strong cubic graph. ☐

Remark 3.5. *If the maximal product of two cubic graphs* $(\zeta_1 * \zeta_2)$ *is a strong, then* ζ_1 *and* ζ_2 *need not to be strong, in general.*

Definition 3.6. *A cubic graph* ζ *is called complete if:* $\widetilde{\mu_Q}(mn) = \widetilde{\mu_P}(m) \wedge \widetilde{\mu_P}(n)$,

$\lambda_Q(mn) = \lambda_P(m) \vee \lambda_P(n)$, *for all* $m, n \in V$.

Remark 3.7. *The maximal product of two complete cubic graphs is not a complete cubic graph, in general. Because we do not include the case* $(m_1, m_2) \in E_1$ *and* $(n_1, n_2) \in E_2$ *in the definition of the maximal product of two cubic graphs.*

Remark 3.8. *The maximal product of two complete cubic graphs is strong cubic graph.*

Definition 3.9. *The residue product* $\zeta_1 \bullet \zeta_2$ *of two cubic graphs* $\zeta_1 = (P_1, Q_1)$ *and* $\zeta_2 = (P_2, Q_2)$ *is defined as:*

$$(i)\begin{cases}(\widetilde{\mu_{P_1}} \bullet \widetilde{\mu_{P_2}})\big((m_1,m_2)\big) = r\max\{\widetilde{\mu_{P_1}}(m_1), \widetilde{\mu_{P_2}}(m_2)\},\\ (\lambda_{P_1} \bullet \lambda_{P_2})\big((m_1,m_2)\big) = \min\{\lambda_{P_1}(m_1), \lambda_{P_2}(m_2)\},\\ for\ all\ (m_1,m_2) \in V_1 \times V_2\end{cases}$$

$$(ii)\begin{cases}(\widetilde{\mu_{Q_1}} \bullet \widetilde{\mu_{Q_2}})\big((m_1,m_2)(n_1,n_2)\big) = \widetilde{\mu_{Q_1}}(m_1n_1),\\ (\lambda_{Q_1} \bullet \lambda_{Q_2})\big((m_1,m_2)(n_1,n_2)\big) = \lambda_{Q_1}(m_1n_1),\\ for\ all\ m_1n_1 \in E_1, m_2 \neq n_2.\end{cases}$$

Example 3.10. *Consider the cubic graphs* ζ_1 *and* ζ_2 *as in Figure 1. The residue product of* ζ_1 *and* ζ_2 *(*$\zeta_1 \bullet \zeta_2$*) shown in Figure 2.*

Definition 3.11. *Let* ζ *be a cubic graph. The degree of a vertex* m *in* ζ *is defined by:*

$$d_\zeta(m) = \left(\sum_{\substack{m\neq n\\ n\in V}} \widetilde{\mu_Q}(mn), \sum_{\substack{m\neq n\\ n\in V}} \lambda_Q(mn)\right) = \left(d^P(m), d^Q(n)\right)$$

Definition 3.12. *An cubic graph* ζ *is said to be an irregular cubic graph if there is a vertex which is adjacent to vertices with distinct degrees.*

Definition 3.13. *A cubic graph* ζ *is said to be a totally irregular cubic graph if* $\exists$ *a vertex which is adjacent to vertices with different total degree.*

Definition 3.14. *A cubic graph* ζ *is said to be a strongly irregular cubic graph if every vertex has a different degree.*

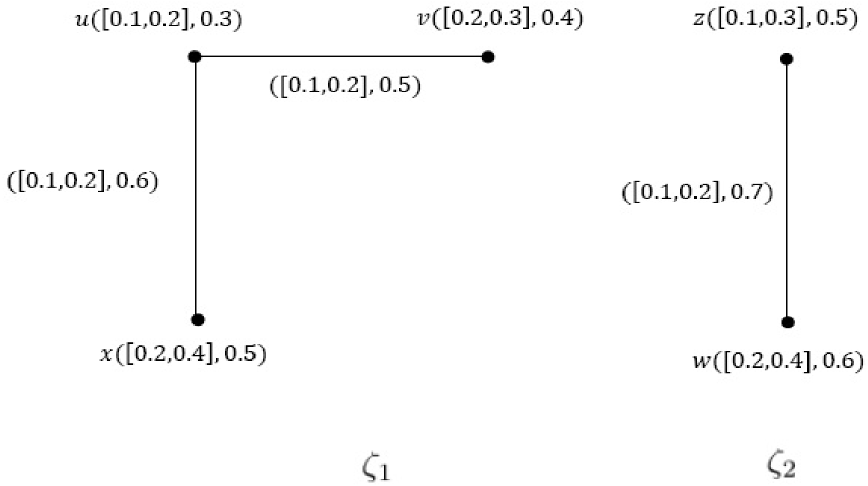

Figure 1: *Cubic graphs ζ_1 and ζ_2*

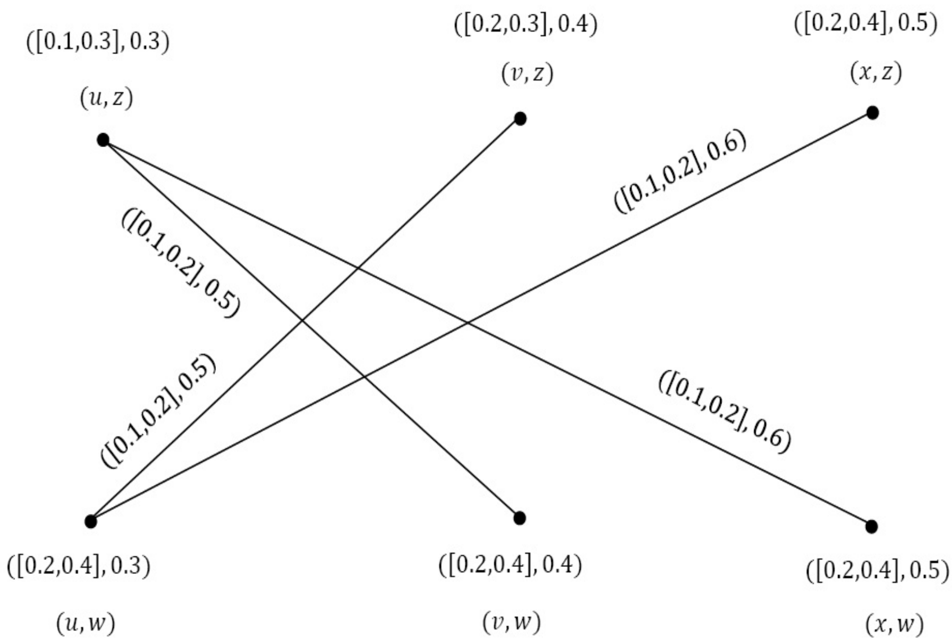

Figure 2: *Residue product of two cubic graphs.*

Definition 3.15. *A cubic graph ζ is said to be a strongly totally irregular cubic graph if every vertex has a different total degree.*

Proposition 3.16. *The residue product of two cubic graphs ζ_1 and ζ_2 is a cubic graph.*

Proof. Let $\zeta_1 = (P_1, Q_1)$ and $\zeta_2 = (P_2, Q_2)$ be two cubic graphs and $\big((m_1, m_2)(n_1, n_2)\big) \in E_1 \times E_2$. If $m_1 n_1 \in E_1$ and $m_2 \neq n_2$, then we have:

$$\begin{aligned}
\big(\widetilde{\mu_{Q_1}} \bullet \widetilde{\mu_{Q_2}}\big)\big((m_1, m_2)(n_1, n_2)\big) &= \widetilde{\mu_{Q_1}}(m_1 n_1) \\
&\leq r\min\{\widetilde{\mu_{P_1}}(m_1), \widetilde{\mu_{P_1}}(n_1)\} \\
&\leq r\max\big\{r\min\{\widetilde{\mu_{P_1}}(m_1), \widetilde{\mu_{P_1}}(n_1)\}, r\min\{\widetilde{\mu_{P_2}}(m_2), \widetilde{\mu_{P_2}}(n_2)\}\big\} \\
&= r\min\big\{r\max\{\widetilde{\mu_{P_1}}(m_1), \widetilde{\mu_{P_1}}(n_1)\}, r\max\{\widetilde{\mu_{P_2}}(m_2), \widetilde{\mu_{P_2}}(n_2)\}\big\} \\
&= \min\big\langle\big(\widetilde{\mu_{P_1}} \bullet \widetilde{\mu_{P_2}}\big)(m_1, m_2), \big(\widetilde{\mu_{P_1}} \bullet \widetilde{\mu_{P_2}}\big)(n_1, n_2)\big\},
\end{aligned}$$

$$\begin{aligned}
\big(\lambda_{Q_1} \bullet \lambda_{Q_2}\big)\big((m_1, m_2)(n_1, n_2)\big) &= \lambda_{Q_1}(m_1 n_1) \\
&\geq \max\{\lambda_{P_1}(m_1), \lambda_{P_1}(n_1)\} \\
&\geq \min\big\{\max\{\lambda_{P_1}(m_1), \lambda_{P_1}(n_1)\}, \max\{\lambda_{P_2}(m_2), \lambda_{P_2}(n_2)\}\big\} \\
&= r\max\big\{\min\{\lambda_{P_1}(m_1), \lambda_{P_1}(n_1)\}, \min\{\lambda_{P_2}(m_2), \lambda_{P_2}(n_2)\}\big\} \\
&= \max\big\langle\big(\lambda_{P_1} \bullet \lambda_{P_2}\big)(m_1, m_2), \big(\lambda_{P_1} \bullet \lambda_{P_2}\big)(n_1, n_2)\big\}.
\end{aligned}$$

□

4. Conclusion

Cubic graphs are highly practical tools for the study of different computational intelligence and computer science domains. Cubic graphs have many applications in different sciences such as topology, natural networks, and operation research. Operations are conveniently used in many combinatorial applications; hence; in this paper, two new operations on cubic graphs, namely, maximal product, and residue product were presented, and some results concerning their degrees were introduced. In our future work, we will discuss several types of domination in cubic graphs.

Acknowledgements. The author is also thankful to the referees for their valuable comments for improvement of the comments.

Conflicts of Interest: This is a single-authored paper. There is no conflict of Interest.

Author's Contributions: This is the authors' sole contribution.

REFERENCES

1. M.Akram, Interval-valued fuzzy graphs, *Computers and Mathematics with Applications*, 61 (2) (2011) 289-299.
2. M. Akram and M. Sitara, Certain concepts in intuitionistic neutrosophic graph structures, *Information*, (2017) 8, 154, doi:10.3390/info8040154.
3. R.A. Borzooei and H. Rashmanlou, Ring sum in product intuitionistic fuzzy graphs, *J. Adv. Res. Pure Math*, 7 (2015) 16-31.
4. R.A. Borzooei and H. Rashmanlou, Domination in vague graphs and its applications, *J. Intel-Ligent Fuzzy Syst*, 29 (2015) 1933-1940.

5. R.A. Borzooei and H. Rashmanlou, Degree of vertices in vague graphs, *J. Appl. Math. Inform*, 33 (2015) 545-557.
6. R.A. Borzooei, H. Rashmanlou, S. Samanta and M. Pal, Regularity of vague graphs, *J. Intell. Fuzzy Syst,* 30 (2016) 3681-3689.
7. G. Ghorai and M. Pal, Planarity in vague graphs with application, *Acta Mathematica*, 33(2) (2017) 147-164.
8. G. Ghorai and M. Pal, On some operations and density of m-polar fuzzy graphs, *Pacific Science Review A: Natural Science and Engineering,* 17 (1) (2015) 14-22.
9. G. Ghorai and M. Pal, A note on "Regular bipolar fuzzy graphs" Neural Computing and Applications 21 (1)(2012) 197–205, *Neural Computing and Applications*, 30 (2018) 1569-1572.
10. G. Ghorai and M. Pal, Novel concepts of strongly edge irregular m-polar fuzzy graphs, *International Journal of Applied and Computational Mathematics,* 3 (2017) 3321-3332.
11. Y.B. Jun, Ch.S. Kim and K.O. Yang, Cubic sets, *Annals of Fuzzy Mathematics and Informatics,* 4 (1) (2012) 83-98.
12. A. Kauffman, Introduction a la theories des sous-emsembles 503 ous. Masson Cie 1973, 1, 607-618.
13. P.K. Kishore Kumar, Yahya Talebi, H. Rashmanlou, A.A. Talebi and F. Mofidnakhaei, New concept of cubic graph with application, *J. of Mult.-Valued Logic & Soft Computing*, 33 (2019) 135-154.
14. G. Muhiuddin, S.S. Ahn, C.S. Kim and Y.B. Jun, Stable cubic sets, *Journal of Computational Analysis and Applications*, 23(5) (2017) 802-819.
15. G. Muhiuddin, F. Feng and Y.B. Jun, Subalgebras of BCK/BCI-algebras based on cubic soft sets, *The Scientific World Journal*, 2014 (2014) Article ID 458638, 9.
16. G. Muhiuddin and A.M. Al-roqi, Cubic soft sets with applications in BCK/BCI-algebras, *Annals of Fuzzy Mathematics and Informatics,* 8 (2) (2014) 291-304.
17. A. Nagoorgani and S.R. Latha, On irregular fuzzy graphs, *Appl. Math. Sci*., 6 (2012) 517-523.
18. A. Nagoorgani and K. Radha, On regular fuzzy graphs, *J. Phys. Sci*,, 12 (2008) 33-44.
19. M. Pal, S.K.Khan and A.K.Shyamal, Intuitionistic fuzzy matrices, *Notes on Intuitionistic Fuzzy Sets*, 8 (2) (2002) 51-62.
20. M. Pal, Fuzzy matrices with fuzzy rows and columns, *Journal of Intelligent & Fuzzy System,* 30 (1) (2016) 561-573.
21. M.Pal, S. Samanta and G. Ghorai, *Modern Trends in Fuzzy Graph Theory,* Springer, 2020.
22. H.Rashmanlou and R.A.Borzooei, Product vague and its applications, *Journal of Intelligent & Fuzzy Systems*, (2016) 30, 371-382.
23. H.Rashmanlou and R.A.Borzooei, Vague graphs with application, *J. Intell. Fuzzy Syst*,, 30 (2016) 3291-3299.
24. H.Rashmanlou, S.Samanta, M.Pal and R.A.Borzooei, A study on bipolar fuzzy graphs. *J. Intell. Fuzzy Syst*,, 2015, 28, 571-580.
25. H. Rashmanlou, Y.B. Jun and R.A. Borzooei, More results on highly irregular bipolar fuzzy graphs, *Ann. Fuzzy Math. Inform,* 8 (2014) 149-168.

26. H. Rashmanlou, R.A. Borzooei, M. Shoaib, Yahya Talebi, M. Taheri and F. Mofidnakhaei, New way for finding shortest path problem in a network, *J. of Mult.-Valued Logic & Soft Computing*, 34 (2020) 451-460.
27. H.Rashmanlou, S.Samanta, M.Pal and R.A.Borzooei, Bipolar fuzzy graphs with Categorical properties, *International Journal of Computational Intelligence Systems*, 8(5) (2015) 808-818.
28. H.Rashmanlou, G.Muhiuddin, S.K.Amanathulla, F.Mofidnakhaei and M.Pal, A study on cubic graph with novel application, *Journal of Intelligent and Fuzzy Systems*, 40 (2021) 89-101.
29. H.Rashmanlou, R.A.Borzooei, S.Samanta and M.Pal, Properties of interval valued intuitionistic (s,t)-fuzzy graphs, *Pacific Science Review A: Natural Science and Engineering*, 18 (1) (2016) 30-37.
30. H.Rashmanlou and R.A.Borzooei, New concepts of interval-valued intuitionistic (S, T)-fuzzy graphs, *Journal of Intelligent & Fuzzy Systems*, 30 (4) (2016) 1893-1901.
31. K.Radha and N.Kumaravel, Some properties of edge regular fuzzy graphs, *Jamal Acad. Res. J,* (2014) 121-127.
32. A.Rosenfeld, *Fuzzy graphs, Fuzzy Sets and their Applications*; Zadeh, L.A., Fu, K.S., Shimura, M., Eds.; Academic Press: New York, NY, USA, 1975; 77-95.
33. S.Samanta and M.Pal, Irregular bipolar fuzzy graphs, *Int. J. Appl. Fuzzy Sets,* 2 (2012) 91-102.
34. S.Samanta and M.Pal, Fuzzy k-competition graphs and pcompetition fuzzy graphs, *Fuzzy Inf. Eng*, 5 (2013) 191-204.
35. M.S.Sunitha and S.Mathew, Fuzzy graph theory: A survey, *Ann. Pure Appl. Math.*, 4 (2013) 92-110.
36. M.S.Sunitha and A.Vijayakumar, Complement of a fuzzy graph, *Indian J. Pure Appl. Math,* 33 (2002) 1451-1464.
37. M.Shoaib, S.Kosari, H.Rashmanlou, M.Aslam Malik, Y.Rao, Yahya Talebi and F.Mofidnakhaei, Notion of complex Pythagorean fuzzy graph with properties and application, *J. Mult.-Valued Logic & Soft Computing,* 34 (2020) 553-586.
38. A.A.Talebi, Janusz Kacprzyk, H.Rashmanlou and S.H.Sadati, A new concept of an intuitionistic fuzzy graph with applications, *J. Mult.-Valued Logic & Soft Computing*, 35 (2020) 431-454.
39. A.A.Talebi, H.Rashmanlou and S.H.Sadati, Interval-valued intuitionistic fuzzy competition graph, *J. Mult.-Valued Logic & Soft Computing,* 34 (2020) 335-364.
40. A.A.Talebi, H.Rashmanlou and S.H.Sadati, New concepts on m-polar interval-valued intuitionistic fuzzy graph, *TWMS J. App. and Eng. Math,* 10 (3) (2020) 806-818.
41. A.A.Talebi and H.Rashmanlou, Isomorphic on interval-valued fuzzy graphs, *Annals of fuzzy Mathematics and Informatics*, 6 (1) (2013) 47-58.
42. A.A.Talebi, H.Rashmanlou and N.Mehdipoor, Isomorphism on vague graphs, *Annals of Fuzzy Mathematics and Informatics*, 6 (3) (2013) 575-588.
43. L.A.Zadeh, Fuzzy sets, *Information and Control,* 8 (1965) 338-353.
44. L.A.Zadeh, Similarity relations and fuzzy ordering, *Information Sciences*, 3 (1971) 177-200.
45. L.A.Zadeh, Is there a need for fuzzy logic? *Information Sciences*, 178 (2008) 2751-2779.