Annals of Pure and Applied Mathematics
Vol. 32, No. 2, 2025, 83-86
ISSN: 2279-087X (P), 2279-0888(online)
Published on 28 December 2025
www.researchmathsci.org
DOI: http://doi.org/10.22457/apam.v32n2a06981

Annals of Pure and Applied Mathematics

Short Communication

Augmented Nirmala Index of Certain Nanotubes

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Received 1 November 2025; accepted 23 December 2025

Abstract. In this study, we introduce the augmented Nirmala and reciprocal augmented Nirmala indices of a graph. Furthermore, we compute these augmented Nirmala indices for certain nanotubes.

Keywords: augmented Nirmala index, reciprocal augmented Nirmala index, nanotube.

AMS Mathematics Subject Classification (2010): 05*C*10, 05*C*69

1. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer [1], for other undefined notations and terminologies.

Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [2].

The Nirmala index [3] of a graph is defined as

$$N(G)=\sum_{uv\in E(G)}\sqrt{d_G(u)+d_G(v)}.$$

We define the augmented Nirmala index of a graph G as

$$AN(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)}{d_G(u)+d_G(v)-2}}.$$

We define the reciprocal augmented Nirmala index of a graph G as

$$RAN(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)+d_G(v)}}.$$

Recently, some Nirmala indices were studied in [4, 5, 6, 7, 8].

In this paper, the augmented Nirmala and reciprocal augmented Nirmala indices of certain nanotubes are computed.

2. Results for $HC_5C_7[p,q]$ nanotubes

We consider $HC_5C_7[p,q]$ nanotubes in which p is the number of heptagones in the first row and q rows of pentagones repeated alternately. The 2-D lattice of nanotube $HC_5C_7[8,4]$ is shown in Figure 1.

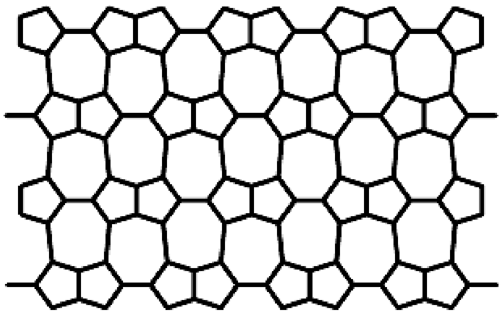

Figure 1: 2-D lattice of $HC_5C_7[8,4]$ nanotube

Let G be the graph of $HC_5C_7[p,q]$ nanotubes. We see that the vertices of G are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By algebraic method, we obtain that G has $4pq$ vertices and $6pq - p$ edges. In G, there are two types of edges based on the degree of end vertices of each edge as follows:

$E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_{23}| = 4p$.

$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 6pq - 5p$.

Theorem 1. Let $G = HC_5C_7[p,q]$ be the nanotubes. Then

$$AN(G) = 6\sqrt{\frac{3}{2}}pq + \left(4\sqrt{\frac{5}{3}} - 5\sqrt{\frac{3}{2}}\right)p.$$

Proof: We have

$$AN(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v)}{d_G(u) + d_G(v) - 2}}$$

$$= 4p\sqrt{\frac{2+3}{2+3-2}} + (6pq - 5p)\sqrt{\frac{3+3}{3+3-2}}$$

$$= 4p\sqrt{\frac{5}{3}} + (6pq - 5p)\sqrt{\frac{3}{2}} = 6\sqrt{\frac{3}{2}}pq + \left(4\sqrt{\frac{5}{3}} - 5\sqrt{\frac{3}{2}}\right)p.$$

Theorem 2. Let $G = HC_5C_7[p,q]$ be the nanotubes. Then

$$RAN(G) = 6\sqrt{\frac{2}{3}}pq + \left(4\sqrt{\frac{3}{5}} - 5\sqrt{\frac{2}{3}}\right)p.$$

Proof: We have

$$RAN(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u) + d_G(v)}}$$

$$= 4p\sqrt{\frac{2+3-2}{2+3}} + (6pq - 5p)\sqrt{\frac{3+3-2}{3+3}} = 6\sqrt{\frac{2}{3}}pq + \left(4\sqrt{\frac{3}{5}} - 5\sqrt{\frac{2}{3}}\right)p.$$

3. Results for $SC_5C_7[p,q]$ nanotubes

We consider $SC_5C_7[p,q]$ nanotubes in which p is the number of heptagones in the first row and q rows of vertices and edges are repeated alternately. The 2-*D* lattice of nanotube $SC_5C_7[8,4]$ is depicted in Figure 2.

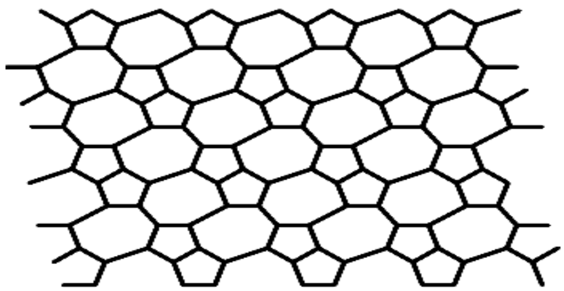

Figure 2: 2-*D* lattice of nanotube $SC_5C_7[8,4]$

Let H be the graph of $SC_5C_7[p,q]$ nanotubes. We see that the vertices of H are either of degree 2 or 3. Thus $\Delta(H) = 3$ and $\delta(H) = 2$. By algebraic method, we obtain that H has $4pq$ vertices and $6pq - p$ edges. In H, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_{22} = \{uv \in E(H) \mid d_G(u) = d_G(v) = 2\}$ $|E_{22}| = q$.
$E_{23} = \{uv \in E(H) \mid d_G(u) = 2, d_G(v) = 3\}$ $|E_{23}| = 6q$.
$E_{33} = \{uv \in E(H) \mid d_G(u) = d_G(v) = 3\}$ $|E_{33}| = 6pq - p - 7q$.

Theorem 3. Let $H=SC_5C_7[p,q]$ be the nanotubes. Then

$$AN(H) = 6\sqrt{\frac{3}{2}}pq - \sqrt{\frac{3}{2}}p + \left(\sqrt{2} + 6\sqrt{\frac{5}{3}} - 7\sqrt{\frac{3}{2}}\right)p.$$

Proof: We have

$$AN(H) = \sum_{uv \in E(H)} \sqrt{\frac{d_H(u) + d_H(v)}{d_H(u) + d_H(v) - 2}}$$

$$= q\sqrt{\frac{2+2}{2+2-2}} + 6q\sqrt{\frac{2+3}{2+3-2}} + (6pq - p - 7q)\sqrt{\frac{3+3}{3+3-2}}$$

$$= 6\sqrt{\frac{3}{2}}pq - \sqrt{\frac{3}{2}}p + \left(\sqrt{2} + 6\sqrt{\frac{5}{3}} - 7\sqrt{\frac{3}{2}}\right)p.$$

Theorem 4. Let $H=SC_5C_7[p,q]$ be the nanotubes. Then

$$RAN(H)=6\sqrt{\frac{2}{3}}pq-\sqrt{\frac{2}{3}}p+\left(\sqrt{\frac{1}{2}}+6\sqrt{\frac{3}{5}}-7\sqrt{\frac{2}{3}}\right)p.$$

Proof: We have

$$RAN(H)=\sum_{uv\in E(H)}\sqrt{\frac{d_H(u)+d_H(v)-2}{d_H(u)+d_H(v)}}$$

$$=q\sqrt{\frac{2+2-2}{2+2}}+6q\sqrt{\frac{2+3-2}{2+3}}+(6pq-p-7q)\sqrt{\frac{3+3-2}{3+3}}$$

$$=6\sqrt{\frac{2}{3}}pq-\sqrt{\frac{2}{3}}p+\left(\sqrt{\frac{1}{2}}+6\sqrt{\frac{3}{5}}-7\sqrt{\frac{2}{3}}\right)p.$$

4. Conclusion

In this paper, we have introduced the augmented Nirmala and reciprocal augmented Nirmala indices of a graph. Also the augmented Nirmala and reciprocal augmented Nirmala indices of certain nanotubes are determined.

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