Fuzzy Semi-Open Sets and Fuzzy Pre-Open Sets in Fuzzy Quad Topological Space

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Abstract. The aim of this paper is to introduce two new types of fuzzy open sets namely fuzzy q-semi-open sets and fuzzy q-pre-open sets in fuzzy q-topological spaces and also defined the fuzzy continuity namely fuzzy q-continuity, fuzzy q-semi-continuity and fuzzy q-pre-continuity in fuzzy q-topological spaces.

Keywords: fuzzy q-semi-open sets, fuzzy q-pre-open sets, fuzzy q-continuity, fuzzy q-semi-continuity and fuzzy q-pre-continuity.

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1. Introduction

The study of tri-topological space was first initiated by Kovar [9]. Palaniammal [15] studied tri topological space and introduced semi and pre-open sets in tri topological space and he also introduced fuzzy tri topological space. Hameed and Moh. Abid [10] gives the definition of 123 open set in tri topological spaces. We [17] studied properties of tri semi-open sets and tri pre-open sets in tri topological space. Mukundan [5] introduced the concept on topological structures with four topologies, quad topology) and defined new types of open (closed) set. We have [18] introduced semi and pre-open sets in quad topological spaces.

in fuzzy topological spaces was studied by Azad [6]. Bin [2] defined the concept of pre-open sets in fuzzy topological space. Thakur and Malviya [16] introduced semi-open sets, semi continuity in fuzzy bitopological spaces. Sampath Kumar [13] defined a $(\tau_i, \tau_j)$ fuzzy pre-open set and characterized a fuzzy pair wise pre continuous mappings on a fuzzy bitopological space. We have [12] introduced fuzzy tri semi-open sets and fuzzy tri pre-open sets, fuzzy tri continuous function, fuzzy tri semi-continuous function and fuzzy tri pre-continuous functions and their basic properties.

In this paper, we introduce fuzzy q-semi-open sets, fuzzy q-pre-open sets, fuzzy q-continuity, fuzzy q-semi-continuity and fuzzy q-pre-continuity and their fundamental properties in fuzzy q-topological space.

2. Preliminaries

Definition 2.1. [12] Consider two fuzzy tri topological spaces $(X, \tau_1, \tau_2, \tau_3)$, $(Y, \tau'_1, \tau'_2, \tau'_3)$. A fuzzy function $f : I^X \rightarrow I^Y$ is called a fuzzy tri continuous function if $\chi^{\tau}_{\lambda}$ is fuzzy tri open in $X$, for every tri open set $\chi^{\tau}_{\lambda}$ Y.

Definition 2.2. [12] Let $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space then a fuzzy subset $\chi^{\tau}_{\lambda}$ of $X$ is said to be fuzzy tri semi-open set if $\chi^{\tau}_{\lambda} \leq \text{cl} (\text{int} \chi^{\tau}_{\lambda})$ and complement of fuzzy tri semi-open set is fuzzy tri semi-closed. The collection of all fuzzy tri semi-open sets of $X$ is denoted by $\text{tri-FSO}(X)$.

Definition 2.3. [12] Let $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space then a fuzzy subset $\chi^{\tau}_{\lambda}$ of $X$ is said to be fuzzy tri pre-open set if $\chi^{\tau}_{\lambda} \leq \text{int} (\text{tri-cl} \chi^{\tau}_{\lambda})$ and complement of fuzzy tri pre-open set is fuzzy tri pre-closed. The collection of all fuzzy tri semi-open sets of $X$ is denoted by $\text{tri-FPO}(X)$.

Definition 2.4. [5] Let $X$ be a nonempty set and $\tau_1, \tau_2, \tau_3, and \tau_4$ are general topologies on $X$. Then a subset $A$ of space $X$ is said to be quad-open(q-open) set if $A = \tau_1 \vee \tau_2 \vee \tau_3 \vee \tau_4$ and its complement is said to be q-closed and set $X$ with four topologies called q-topological spaces $(X, \tau_1, \tau_2, \tau_3, \tau_4)$.

Definition 2.5. [5] Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a quad topological space and let $A \subset X$. The intersection of all q-closed sets containing $A$ is called the q-closure of $A$ & denoted by $q-\text{cl}A$. We will denote the q-interior (resp. q-closure) of any subset, say of $A$ by $q-\text{int} A$ (resp. $q-\text{cl} A$) where $q-\text{cl} A$ is the union of all q-open sets contained in $A$, and $q-\text{cl} A$ is the intersection of all q-closed sets containing $A$. 

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3. Fuzzy q-semi-open sets and fuzzy q-pre-open sets in fuzzy q-topological space

**Definition 3.1.** Let \( \tau_1, \tau_2, \tau_3, \tau_4 \) are fuzzy topologies on \( X \). Then a fuzzy subset \( \chi \subseteq X \) is said to be fuzzy q-open if
\[
\chi \subseteq \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4
\]
and its complement is said to be fuzzy q-closed and set \( X \) with four fuzzy topologies called fuzzy q-topological spaces \((X, \tau_1, \tau_2, \tau_3, \tau_4)\).

**Definition 3.2.** Let \((X, \tau_1, \tau_2, \tau_3, \tau_4)\) be a fuzzy quad topological space and let \( \chi \subseteq X \). The intersection of all fuzzy q-closed sets containing \( \chi \) is called the fuzzy q-closure of \( \chi \) and denoted by \( Fqcl(\chi) \). We will denote the fuzzy q-interior (resp. fuzzy q-closure) of any fuzzy subset, say of \( \chi \), by fuzzy \( Fqint(\chi) \)(Fqcl(\chi)), where \( Fqint(\chi) \) is the union of all fuzzy q-open sets contained in \( \chi \), and \( Fqcl(\chi) \) is the intersection of all fuzzy q-closed sets containing \( \chi \).

**Definition 3.3.** Let \((X, \tau_1, \tau_2, \tau_3, \tau_4)\) be a fuzzy q-topological space then a fuzzy subset \( \chi \subseteq X \) is said to be fuzzy q-semi-open set if
\[
\chi \subseteq Fqcl(Fqint(\chi)).
\]
Complement of fuzzy q-semi-open set is called fuzzy q-semi-closed set. The collection of all fuzzy q-semi-open sets of \( X \) are denoted by \( FqSO(X) \).

**Example 3.4.** Let \( X = \{a, b, c, d\} \) be a non-empty fuzzy set. Consider four fuzzy topologies on \( X \)
\[
\tau_1 = \{\emptyset, X, X_{\{a\}}\},
\tau_2 = \{\emptyset, X, X_{\{a,d\}}\},
\tau_3 = \{\emptyset, X, X_{\{c,d\}}\},
\tau_4 = \{\emptyset, X, X_{\{a,c,d\}}\}.
\]
Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies. Fuzzy q-open sets of \( X \) are denoted by
\[
FqSO(X) = \{\emptyset, X, X_{\{a\}}, X_{\{a,d\}}, X_{\{c,d\}}, X_{\{a,c,d\}}\}.
\]
Definition 3.5. Let \((X, \tau_1, \tau_2, \tau_3, \tau_4)\) be a fuzzy q-topological space then a fuzzy subset \(\mathcal{A}\) of \(X\) is said to be fuzzy q-pre-open set if

\[
\bigwedge_{g \neq 1} \bigwedge_{g \neq 2} \bigwedge_{g \neq 3} \bigwedge_{g \neq 4} \mathcal{A}. 
\]

Complement of fuzzy q-pre-open set is called fuzzy q-pre-closed set. The collection of all fuzzy q-pre-open sets of \(X\) is denoted by \(FqPO(X)\).

Example 3.6. Let \(X = \{a, b, c, d\}\) be a non-empty fuzzy set.

Consider four fuzzy topologies on \(X\)

\[
\begin{align*}
\tau_1 &= \{\bar{1}X, \bar{0}X, X(a)\}, \\
\tau_2 &= \{\bar{1}X, \bar{0}X, X(a,d)\}, \\
\tau_3 &= \{\bar{1}X, \bar{0}X, X(b)\}, \\
\tau_4 &= \{\bar{1}X, \bar{0}X, X(a,c,d)\}.
\end{align*}
\]

Fuzzy open sets in fuzzy q-topological space are union of all four fuzzy topologies.

Then fuzzy q-open sets of \(X\) denoted by

\[
FqSO(X) = \{\bar{1}X, \bar{0}X, X(a), X(a,d), X(b), X(a,c,d)\}.
\]

Definition 3.7. Let \((X, \tau_1, \tau_2, \tau_3, \tau_4)\) be a fuzzy q-topological space. The intersection of all fuzzy q-semi-closed sets of \(X\) containing a fuzzy subset \(\mathcal{A}\) of \(X\) is called fuzzy q-semi closure of \(\mathcal{A}\) and is denoted by \(Fqsc(\mathcal{A})\).

Definition 3.8. Let \((X, \tau_1, \tau_2, \tau_3, \tau_4)\) be a fuzzy q-topological space. The intersection of all fuzzy q-pre-closed sets of \(X\) containing a fuzzy subset \(\mathcal{A}\) of \(X\) is called fuzzy q-pre closure of \(\mathcal{A}\) and is denoted by \(Fqpc(\mathcal{A})\).

Definition 3.9. Let \((X, \tau_1, \tau_2, \tau_3, \tau_4)\) be a fuzzy q-topological space. The intersection of all fuzzy q-semi-closed sets of \(X\) containing a fuzzy subset \(\mathcal{A}\) of \(X\) is called fuzzy q-semi closure of \(\mathcal{A}\) and is denoted by \(Fqsc(\mathcal{A})\).

Definition 3.10. Let \((X, \tau_1, \tau_2, \tau_3, \tau_4)\) be a fuzzy q-topological space. The intersection of all fuzzy q-pre closed sets of \(X\) containing a fuzzy subset \(\mathcal{A}\) of \(X\) is called fuzzy q-pre closure of \(\mathcal{A}\) and is denoted by \(Fqpc(\mathcal{A})\).

Theorem 3.11. \(\mathcal{A}\) is fuzzy q-semi open if and only if \(\mathcal{A} = Fqsint(\mathcal{A})\).
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**Theorem 3.12.** \( Fqsint(\chi_\alpha \lor \chi_\delta) \supseteq Fqsint(\chi_\alpha) \lor Fqsint(\chi_\delta). \)

**Theorem 3.13.** \( \chi_\alpha \) is a fuzzy q-semi closed set if and only if 
\[
\chi_\alpha = Fqscl(\chi_\alpha).
\]

**Theorem 3.14.** Let \( \chi_\alpha \) and \( \chi_\delta \) be two fuzzy subsets of \( (X, \tau_1, \tau_2, \tau_3, \tau_4) \) and \( X_\{x\} \leq \tilde{1}_X \)
a) \( \chi_\alpha \) is fuzzy q-pre closed if and only if \( \chi_\alpha = Fqpcl(\chi_\alpha) \)
b) If \( \chi_\alpha \leq \chi_\delta \), then \( Fqpcl(\chi_\alpha) \subset Fqpcl(\chi_\delta) \);
c) \( \chi_\{x\} \subset Fqpcl(\chi_\delta) \) if and only if \( \chi_\alpha \land \chi_\delta \neq \tilde{0}_X \) for every fuzzy q-pre-open set 
\( f(\chi_\alpha) \) containing \( f(\chi_\{x\}) \).

**Theorem 3.15.** Let \( \chi_\alpha \) be a fuzzy subset of \( (X, \tau_1, \tau_2, \tau_3, \tau_4) \), if there exist a fuzzy q-pre-open set \( \chi_\delta \) such that \( \chi_\alpha \subset \chi_\delta \subset \chi_\alpha \land \chi_\delta \), then \( \chi_\alpha \) is fuzzy q-pre-open.

**Theorem 3.16.** In a fuzzy q-topological space \( (X, \tau_1, \tau_2, \tau_3, \tau_4) \) the union of any two fuzzy q-semi-open sets is always a fuzzy q-semi-open set.

**Proof:** Let \( \chi_\alpha \) and \( \chi_\delta \) be any two fuzzy q-semi-open sets in \( X \). Now
\[
\chi_\alpha \lor \chi_\delta \leq Fqcl(Fqoint(\chi_\alpha) \lor Fqcl(Fqoint(\chi_\delta)) \Rightarrow \chi_\alpha \lor \chi_\delta \leq Fqcl(Fqoint(\chi_\alpha \lor \chi_\delta))
\]

Hence, \( \chi_\alpha \lor \chi_\delta \) fuzzy q-semi-open sets.

**Remark 3.17.** The intersection of any two fuzzy q-semi-open sets may not be fuzzy q-semi-open sets as show in the following example

**Example 3.18.** Let \( X = \{a, b, c, d\} \) be a non-empty fuzzy set.

Consider four fuzzy topologies on \( X \)
\[
\tau_1 = \{1X, 0X, \chi_{\{a\}}\},
\tau_2 = \{1X, 0X, \chi_{\{a,d\}}\},
\tau_3 = \{1X, 0X, \chi_{\{c,d\}}\},
\tau_4 = \{1X, 0X, \chi_{\{a,c,d\}}\}.
\]

Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies.

Then Fuzzy q-open sets of
Fuzzy q-semi-open set of \( X \) is denoted by

\[ FqSO(X) = \{ \bar{1}X, \bar{\partial}X, \chi_{(a)}, \chi_{(a,d)}, \chi_{(c,d)}, \chi_{(a,c,d)} \}. \]

Here

\[ \chi_{(a,d)} \land \chi_{(c,d)} = \chi_{(d)} > FqSO(X). \]

**Theorem 3.19.** If \( \chi_\lambda \) is fuzzy q-open sets then \( \chi_\lambda \) is fuzzy q-semi-open set.

**Proof:** Let \( \chi_\lambda \) is a fuzzy q-open set.

Therefore \( \chi_\lambda = Fqint(\chi_\lambda) \).

Now \( \chi_\lambda \prec Fqcl(\chi_\lambda) = Fqcl(Fqint(\chi_\lambda)) \) hence \( \chi_\lambda \) is fuzzy q-semi-open set.

**Theorem 3.20.** Let \( \chi_\lambda \) and \( \chi_\delta \) be two fuzzy subsets of \( X \) such that \( \chi_\delta \prec \chi_\lambda \prec Fqcl(\chi_\delta) \).

If \( \chi_\delta \) is a fuzzy q-semi-open set then \( \chi_\lambda \) is also fuzzy q-semi-open set.

**Proof:** Given \( \chi_\delta \) is fuzzy q-semi-open set.

So, we have \( \chi_\delta \leq Fqcl(Fqint(\chi_\delta)) \leq Fqcl(Fqint(\chi_\lambda)) \).

Thus \( Fqcl(\chi_\delta) \leq Fqcl(Fqint(\chi_\lambda)) \) hence \( \chi_\lambda \) is also fuzzy q-semi-open set.

**4. Fuzzy q-semi-continuity and fuzzy q-pre-continuity in fuzzy q-topological space**

**Definition 4.1.** A fuzzy function \( f \) from a fuzzy q-topological space \((X, \tau_1, \tau_2, \tau_3, \tau_4)\) into another fuzzy q-topological space \((Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)\) is called fuzzy q-semi-continuous if \( f^{-1}(\chi_\lambda) \) is fuzzy q-semi-open set in \( X \) for each fuzzy q-open set \( \chi_\lambda \) in \( Y \).

**Example 4.2.** Let \( X = \{a, b, c, d\} \) be a non-empty fuzzy set.

Consider four fuzzy topologies on \( X \)

\[
\begin{align*}
\tau_1 &= \{ \bar{1}X, \bar{\partial}X, \chi_{(a)} \}, \\
\tau_2 &= \{ \bar{1}X, \bar{\partial}X, \chi_{(a,d)} \}, \\
\tau_3 &= \{ \bar{1}X, \bar{\partial}X, \chi_{(c,d)} \}, \\
\tau_4 &= \{ \bar{1}X, \bar{\partial}X, \chi_{(a,c,d)} \}.
\end{align*}
\]

Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies.

Then fuzzy q-open sets of

\[ X = \{ \bar{1}X, \bar{\partial}X, \chi_{(a)}, \chi_{(a,d)}, \chi_{(c,d)}, \chi_{(a,c,d)} \}. \]

Fuzzy q-semi-open set of \( X \) is denoted by

\[ FqSO(X) = X = \{ \bar{1}X, \bar{\partial}X, \chi_{(a)}, \chi_{(a,d)}, \chi_{(c,d)}, \chi_{(a,c,d)} \}. \]
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Let \( Y = \{1, 2, 3, 4\} \) be a non-empty fuzzy set.

Consider four fuzzy topologies on \( Y \)
\[
\tau'_1 = \{ \bar{1}Y, \bar{0}Y, \chi(1,4) \}, \quad \tau'_2 = \{ \bar{1}Y, \bar{0}Y, \chi(4) \}, \\
\tau'_3 = \{ \bar{1}Y, \bar{0}Y, \chi(1,2) \}, \quad \tau'_4 = \{ \bar{1}Y, \bar{0}Y, \chi(1,2,4) \}.
\]

Fuzzy q-open sets of
\[
Y = \{ \bar{1}Y, \bar{0}Y, \chi(4), \chi(1,2), \chi(1,4), \chi(1,2,4) \}.
\]

Fuzzy q-semi-open set of \( Y \) is
\[
FqSO(Y) = \{ \bar{1}Y, \bar{0}Y, \chi(4), \chi(1,2), \chi(1,4), \chi(1,2,4) \}.
\]

Consider the fuzzy function \( f: I^X \rightarrow I^Y \) is defined as
\[
f^{-1}(\chi(4)) = \chi(a), \quad f^{-1}(\chi(2)) = \chi(c,d), \quad f^{-1}(\chi(4)) = \chi(a,d),
\]
\[
f^{-1}(\chi(1,2,4)) = \chi(a,c,d), \quad f^{-1}(\bar{0}_Y) = (\bar{0}_X), \quad f^{-1}(\bar{1}_Y) = (\bar{1}_X).
\]

Since the inverse image of each fuzzy q-open set in \( Y \) under \( f \) is fuzzy q-semi-open set in \( X \). Hence \( f \) is fuzzy q-semi-continuous function.

**Definition 4.2.** A fuzzy function \( f \) defined from a fuzzy q-topological space \( (X, \tau', \tau', \tau', \tau') \) into another fuzzy q-topological space \( (Y, \tau', \tau', \tau', \tau', \tau') \) is called fuzzy q-pre-continuous function if \( f^{-1}(\chi_A) \) is fuzzy q-pre-open set in \( X \) for each fuzzy q-open set \( \chi_A \) in \( Y \).

**Example 4.3.** Let \( X = \{a, b, c, d\} \) be a non-empty fuzzy set.

Consider four fuzzy topologies on \( X \)
\[
\tau_1 = \{ \bar{1}X, \bar{0}X, \chi(a) \}, \quad \tau_2 = \{ \bar{1}X, \bar{0}X, \chi(a,d) \},
\]
\[
\tau_3 = \{ \bar{1}X, \bar{0}X, \chi(b,d) \}, \quad \tau_4 = \{ \bar{1}X, \bar{0}X, \chi(a,b,d) \}.
\]

Fuzzy open-sets in fuzzy q-topological spaces are union of all four fuzzy topologies.

Then fuzzy q-open sets of
Fuzzy q-pre-open set of $X$ is denoted by

$$FqPO(X) = \{ I_X, \emptyset X, X[a], X(a,d), X(b,d), X[a,b,d] \}.$$  

Let $Y = \{1,2,3,4\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on $Y$

$$\tau'_1 = \{ I_Y, \emptyset Y, X(1,4) \}, \quad \tau'_2 = \{ I_Y, \emptyset Y, X(4) \},$$

$$\tau'_3 = \{ I_Y, \emptyset Y, X(1,2) \}, \quad \tau'_4 = \{ I_Y, \emptyset Y, X(1,2,4) \}.$$  

Fuzzy q-open sets of $Y = \{ I_Y, \emptyset Y, X(4), X(1,2), X(1,4), X(1,2,4) \}$.

Fuzzy q-pre-open set of $Y$ is denoted by

$$FqPO(Y) = \{ I_Y, \emptyset Y, X(4), X(1,2), X(1,4), X(1,2,4) \}.$$  

Consider the fuzzy function $f : I^X \to I^Y$ is defined as

$$f^{-1}(X(4)) = X[a], \quad f^{-1}(X(1,2)) = X[a,d], \quad f^{-1}(X(1,4)) = X[a,d],$$

$$f^{-1}(X(1,2,4)) = X[a,b,d], \quad f^{-1}(\emptyset Y) = (\emptyset_X), \quad f^{-1}(\bar{I}_Y) = (\bar{I}_X).$$

Since the inverse image of each fuzzy q-open set in $Y$ under $f$ is fuzzy q-pre-open set in $X$. Hence $f$ is fuzzy q-pre-continuous function.

**Theorem 4.4.** Let $f : (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ be a fuzzy q-pre-continuous open function. If $\chi_\lambda$ is a fuzzy q-pre-open set of $X$, then $f(\chi_\lambda)$ is a fuzzy q-pre-open set in $Y$.

**Proof:** First, let $\chi_\lambda$ be fuzzy q-pre-open set in $X$. There exist a fuzzy q-open set $\chi_\delta$ in $X$ such that $\chi_\lambda < \chi_\delta < Fqcl(\chi_\lambda)$. Since $f$ is a fuzzy q-open function then $f(\chi_\delta)$ is a fuzzy q-open set in $Y$. Since $f$ is a fuzzy q-continuous function, we have

$$f(\chi_\lambda) < f(\chi_\delta) < f(Fqcl(\chi_\lambda)) < Fqcl(f(\chi_\lambda)).$$

This show that $f(\chi_\lambda)$ is fuzzy q-pre-open in $Y$.

Let $\chi_\lambda$ be a fuzzy q-pre-open in $X$. There exist a fuzzy q-pre-open set $\chi_\delta$ such that $\chi_\lambda < \chi_\delta < Fqcl(\chi_\delta)$. Since $f$ is a fuzzy q-continuous function, we have by the proof of first part, $f(\chi_\delta)$ is a fuzzy q-pre-open in $X$. Therefore $f(\chi_\lambda)$ is a fuzzy q-pre-open in $Y$.  

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**Theorem 4.5.** Let \( f : (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4) \) be a fuzzy q-pre continuous open function. If \( \chi_A \) is a fuzzy q-pre-open set of \( Y \), then \( f^{-1}(\chi_A) \) is a fuzzy q-pre-open in \( X \).

**Proof:** First, let \( \chi_A \) be a fuzzy q-pre-open set of \( Y \). There exist a fuzzy q-open set \( \chi_\delta \) in \( Y \). Such that \( \chi_\delta < \chi_\delta < Fqcl(\chi_A) \). Since \( f \) is a fuzzy q-open, we have

\[
f^{-1}(\chi_A) < f^{-1}(\chi_\delta) < f^{-1}(Fqcl(\chi_A)) < Fqcl(f^{-1}(\chi_A)).
\]

Since \( f \) is a fuzzy q-pre continuous function, \( f^{-1}(\chi_\delta) \) is a fuzzy q-pre-open set in \( X \). By theorem 3.13, \( f^{-1}(\chi_A) \) is a fuzzy q-pre-open set in \( X \).

**Theorem 4.6.** The following are equivalent for a function \( f : (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4) \)

a) \( f \) is a fuzzy q-pre continuous function;

b) The inverse image of each fuzzy q-closed set of \( Y \) is fuzzy q-pre closed in \( X \);

c) \( Fqpc(f^{-1}(\chi_A)) < f^{-1}(Fqpc(f(\chi_\delta))) \) for every subset \( \chi_A \) of \( Y \).

d) \( f(Fqpc(\chi_\delta)) < Fqcl(f(\chi_\delta)) \) for every subset \( \chi_\delta \) of \( X \).

**Theorem 4.7.** If \( f : (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4) \) and

\[
g : (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4) \to (Z, \tau''_1, \tau''_2, \tau''_3, \tau''_4)
\]

be two fuzzy q-semi continuous function then

\[
fog : (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Z, \tau''_1, \tau''_2, \tau''_3, \tau''_4)
\]

may not be fuzzy q-semi continuous function.

**Theorem 4.8.** Every fuzzy q-continuous function is a fuzzy q-semi continuous function.

**Theorem 4.9.** Let \( f : (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4) \) be bijective. Then the following conditions are equivalent:

i) \( f \) is a fuzzy q-semi-open continuous function.

ii) \( f \) is a fuzzy q-semi closed continuous function and

iii) \( f^{-1} \) is a fuzzy q-semi continuous function.

**Proof:** (i) \( \to \) (ii) Suppose \( \chi_A \) is a fuzzy q-closed set in \( X \). Then \( \bar{I}_X - \chi_A \) is a fuzzy q-open set in \( X \). Now by (i) \( f(\bar{I}_X - \chi_A) \) is a fuzzy q-semi-open set in \( Y \). Now since \( f^{-1} \) is fuzzy bijective function so \( f(\bar{I}_X - \chi_A) = Y - f(\chi_A) \). Hence \( f(\chi_A) \) is a fuzzy q-semi closed set in \( Y \). Therefore \( f \) is a fuzzy q-semi closed continuous function.
(ii)$\Rightarrow$(iii) Let $f$ is a fuzzy q-semi closed map and $\chi_{\lambda}$ be a fuzzy q-closed set of $X$. Since $f^{-1}$ is bijective so $(f^{-1})^{-1}\chi_{\lambda}$ which is a fuzzy q-semi-closed set in $Y$. Hence $f^{-1}$ is a fuzzy q-semi-continuous function.

(iii)$\Rightarrow$(i) Let $\chi_{\lambda}$ be a fuzzy q-open set in $X$. Since $f^{-1}$ is a fuzzy q-semi continuous function so $(f^{-1})^{-1}\chi_{\lambda} = f(\chi_{\lambda})$ is a fuzzy q-semi open set in $Y$. Hence $f$ is fuzzy q-semi-open continuous function.

**Theorem 4.10.** Let $X$ and $Y$ are two fuzzy q-topological spaces. Then $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \rightarrow (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ is fuzzy q-semi-continuous function if one of the followings holds:

i) $f^{-1}(Fqsint(\chi_{\lambda})) \leq Fqsint(f^{-1}(\chi_{\lambda}))$, for every fuzzy q-open set $\chi_{\lambda}$ in $Y$.

ii) $Fqscl(f^{-1}(\chi_{\lambda})) \leq f^{-1}(Fqscl(\chi_{\lambda}))$, for every fuzzy q-open set $\chi_{\lambda}$ in $Y$.

**Proof:** Let $\chi_{\lambda}$ be any fuzzy q-open set in $Y$ and if condition (i) is satisfied then

$$f^{-1}(Fqsint(\chi_{\lambda})) \leq Fqsint(f^{-1}(\chi_{\lambda})).$$

We get $f^{-1}(\chi_{\lambda}) \leq Fqsint(f^{-1}(\chi_{\lambda}))$.

Therefore $f^{-1}(\chi_{\lambda})$ is a fuzzy q-semi-open set in $X$. Hence $f$ is a fuzzy q-semi-continuous function. Similarly we can prove (ii).

**Theorem 4.11.** A function $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \rightarrow (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ is called fuzzy q-semi open continuous function if and only if

$$f(Fqsint(\chi_{\lambda})) \leq Fqsint(f(\chi_{\lambda})).$$

for every quad open set $\chi_{\lambda}$ in $X$.

**Proof:** Suppose that $f$ is a quad semi open continuous function. Since $Fqsint(f(\chi_{\lambda})) \leq \chi_{\lambda}$ so $f(Fqsint(f(\chi_{\lambda}))) \leq f(\chi_{\lambda})$.

By hypothesis $Fqsint(f(\chi_{\lambda}))$ is a fuzzy q-semi-open set and $Fqsint(f(\chi_{\lambda}))$ is largest fuzzy q-semi-open set contained in $f(\chi_{\lambda})$ so $f(Fqsint(\chi_{\lambda})) \leq Fqsint(f(\chi_{\lambda}))$.

Conversely, suppose $\chi_{\lambda}$ is a fuzzy q-open set in $X$. So $f(Fqsint(\chi_{\lambda})) \leq Fqsint(f(\chi_{\lambda}))$.

Now since $\chi_{\lambda} = Fsint(\chi_{\lambda})$ so $f(\chi_{\lambda}) \leq Fqsint(f(\chi_{\lambda}))$

Therefore $f(\chi_{\lambda})$ is a fuzzy q-semi-open set in $Y$ and $f$ is a fuzzy q-semi-open continuous function.

**Theorem 4.12.** A function $f: (X, \tau_1, \tau_2, \tau_3, \tau_4) \rightarrow (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ is called fuzzy q-semi closed continuous function if and only if $f(Fqscl(\chi_{\lambda})) \leq Fqscl(f(\chi_{\lambda}))$ for every fuzzy q-closed set $\chi_{\lambda}$ in $X$. 

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**Proof:** Suppose that $f$ is a fuzzy q-semi closed continuous function. Since $\chi_\lambda \leq Fqscl(\chi_\lambda)$ so $f(\chi_\lambda) \leq f(Fqscl(\chi_\lambda))$. By hypothesis, $f(Fqscl(\chi_\lambda))$ is a fuzzy q-semi closed set and $f(Fqscl(\chi_\lambda))$ is smallest fuzzy q-semi closed set containing $f(\chi_\lambda)$ so $f(Fqscl(\chi_\lambda)) \leq Fqscl(f(\chi_\lambda))$.

Conversely, suppose $\chi_\lambda$ is a fuzzy q-closed set in $X$. So $f(Fqscl(\chi_\lambda)) \leq Fqscl(f(\chi_\lambda))$.

Since $\chi_\lambda = Fqscl(\chi_\lambda)$ so $Fqscl(f(\chi_\lambda)) \leq f(\chi_\lambda)$.

Therefore $f(\chi_\lambda)$ is a fuzzy q-semi closed set in $Y$ and $f$ is fuzzy q-semi closed continuous function.

**Theorem 4.13.** Every fuzzy q-semi continuous function is fuzzy q-continuous function.

5. **Conclusion**

In this paper the idea of fuzzy q-semi-open sets, fuzzy q-pre-open sets, fuzzy q-continuous function, fuzzy q-semi continuous function and fuzzy q-pre continuous function in fuzzy q-topological spaces were introduced and studied.

**REFERENCES**