

From Graph Theory to Spectral Fuzzy Graphs: Emerging Research Directions

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Abstract: The interplay between structural properties of graphs and spectral characteristics of associated matrices has significantly influenced modern network analysis. While classical graph theory provides a robust combinatorial framework, it fails to address uncertainty present in real-world systems. Fuzzy graph theory extends this framework by incorporating degrees of membership, yet lacks powerful analytical tools for global structural analysis. Spectral graph theory bridges this gap through eigenvalue-based methods, offering insights into connectivity, robustness, and stability. The integration of fuzzy logic with spectral methods has led to spectral fuzzy graph theory, enabling the analysis of uncertain networks with enhanced precision. This paper presents a detailed review of graph theory, fuzzy graph theory, and spectral fuzzy graph theory, emphasizing their theoretical foundations, literature developments, and applications.

Keywords: Graph theory; fuzzy graphs; spectral graph theory; eigenvalues; laplacian matrix; graph energy

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1. Introduction

Graph theory has emerged as a fundamental mathematical framework for modeling complex systems across disciplines such as computer science, biology, and social networks. Classical graph theory studies graphs as discrete structures with well-defined relationships. A graph $G = (V, E)$ consists of vertices and edges representing pairwise interactions.

Fundamental concepts include connectivity, paths, cycles, and trees. Graph invariants such as degree, diameter, and clustering coefficients provide measures of

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structural properties.

Despite its success, classical graph theory assumes binary relationships, making it inadequate for modeling uncertainty in real-world systems. The relationship between structural attributes of graphs and the spectral characteristics of associated matrices, particularly adjacency and Laplacian matrices, has been a central theme in modern research [47, 14, 13]. Spectral graph theory studies graphs using eigenvalues of matrices such as adjacency and Laplacian matrices.

The Laplacian matrix is given by:

$$L = D - A$$

Fiedler (1973) introduced algebraic connectivity, linking the second smallest eigenvalue to graph connectivity [21]. Chung [22], Godsil and Royle [24], and Brouwer and Haemers [9] further developed spectral bounds and eigenvalue interlacing techniques. Doob [17] and Sachs [18] laid foundational work connecting combinatorics and spectral theory. Babecki et al. (2020) demonstrated modern applications of spectral techniques across domains [12]. Hogben (2005) explored inverse eigenvalue problems [32], while Jiang (2012) provided theoretical insights into spectral methods [27].

These developments highlight the importance of spectral methods in understanding graph structure. Spectral graph theory provides powerful tools for analyzing connectivity, robustness, and global structural properties through eigenvalues and eigenvectors [22, 24, 9].

A landmark contribution in this direction was made by Fiedler [21], who introduced algebraic connectivity, linking the second smallest Laplacian eigenvalue to graph connectivity. Subsequent developments established bounds on eigenvalues and spectral radius in terms of structural parameters such as degree sequences [35, 15, 31, 7]. These results highlight the deep interplay between graph structure and spectral behavior. Eigenvalue bounds play a critical role in spectral graph theory.

Powers (1989) applied Gershgorin's theorem to derive eigenvalue bounds [37]. Diaconis and Stroock (1991) connected spectral theory with Markov chains [16]. Nikiforov (2002) introduced spectral radius inequalities [35], while Das and Kumar (2004) improved bounds using degree sequences [15]. Godsil and Newman (2008) linked eigenvalues with combinatorial structures [25], and Mohar (2009) studied eigenvalue sums [34]. Further contributions include Kumar (2010) [31], Adiga and Swamy [7], Sorgun (2013) [46], and Bhunia et al. (2019) [10]. Graph energy, introduced by Gutman, is defined as the sum of absolute eigenvalues of the adjacency matrix [8].

The development of fuzzy graph theory has witnessed substantial progress through the integration of advanced structures and applications under uncertainty. Kosari et al. [29] introduced topological indices in fuzzy graphs for decision-making, while Rashmanlou et al. made significant contributions through studies on interval-valued fuzzy graph products [41], vague graphs [43], and applications of vertex connectivity in real-world problems [2]. Extensions to more generalized frameworks include interval intuitionistic neutrosophic graphs [26] and domination concepts in vague graphs [?]. Recent works have focused on hybrid and higher-order models such as picture fuzzy graphs and hypergraphs for multi-attribute decision-making [28, 4, 5], along with new structures in intuitionistic fuzzy trees and graphs [42, 48]. Furthermore, refined models like perfectly regular fuzzy graphs [30] and edge irregular neutrosophic graphs [49] highlight the growing emphasis on both theoretical advancements and practical applications, particularly in decision sciences and

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complex network analysis.

This concept has applications in chemistry and network analysis. Recent studies have extended energy concepts to fuzzy graphs, enabling analysis of uncertain systems. The degree energy of fuzzy graphs provides insights into structural stability and eigenvalue distribution.

However, classical graph models assume precise relationships between vertices, which limits their applicability in real-world systems characterized by uncertainty, imprecision, and variability. To address this limitation, fuzzy graph theory was introduced by Rosenfeld [6], extending classical graphs by assigning membership values to vertices and edges. Further developments in fuzzy graph theory include structural properties such as connectivity, strong arcs, and graph operations [33, 23, 36, 19, 45]. Fuzzy graph theory extends classical graphs by incorporating membership functions.

A fuzzy graph is defined as:

$$G = (V, \sigma, \mu)$$

Rosenfeld introduced fuzzy graphs to model uncertain relationships. Subsequent works have explored connectivity, strong arcs, and graph operations. Mordeson (1994) studied Cartesian products and unions of fuzzy graphs [33]. Nagoor Gani (2009) analyzed vertex degrees and compositions [23], while Nirmala et al. (2012) investigated tensor and normal products [36]. Venugopalam (2013) and Dogra (2015) further classified operations on fuzzy graphs [19, 45]. Essam El Seidy (2016) computed spectra for graphs arising from various operations [20]. These studies provide a foundation for spectral investigations in fuzzy graph models.

Despite these advancements, fuzzy graph theory lacks robust global analytical tools comparable to those in spectral graph theory. This gap has motivated the integration of spectral methods with fuzzy graph models, leading to the development of spectral fuzzy graph theory. Algebraic representations of fuzzy adjacency matrices [38] have enabled eigenvalue-based analysis of fuzzy graphs, facilitating the study of uncertain networks.

Recent research has focused on spectral bounds, graph energy, and eigenvalue inequalities [37, 16, 25, 34, 10]. In particular, graph energy, introduced by Gutman [8], has been extended to fuzzy graphs to capture structural stability under uncertainty [1, 3].

Motivated by these developments, this paper aims to bridge the gap between structural uncertainty and spectral analysis by introducing new theoretical results on fuzzy Laplacian eigenvalues, spectral stability under membership perturbations, and fuzzy Laplacian spread. These results establish a unified framework connecting structural properties of fuzzy graphs with spectral measures.

Furthermore, the proposed framework is applied to brain connectivity networks, particularly Alzheimers disease networks, where uncertainty in functional interactions plays a critical role. The integration of fuzzy graph theory with spectral methods provides a powerful tool for analyzing such complex systems.

2. Spectral fuzzy graph theory

Spectral fuzzy graph theory integrates fuzzy graphs with spectral methods. Jayalakshmi and Vimala (2016) developed algebraic approaches using fuzzy adjacency matrices [38]. This enabled eigenvalue-based analysis of fuzzy graphs. Recent advancements in spectral fuzzy graph theory have significantly strengthened the connection between uncertainty modeling and spectral analysis. Rangasamy et al. (2026) established advanced bounds for

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eigenvalues in spectral fuzzy graphs, providing tighter inequalities that relate structural properties with spectral characteristics [39]. Their results improve existing eigenvalue bounds and enhance the analytical framework for uncertain networks. Further contributions by Rangasamy et al. (2024) introduced novel spectral conditions for diagonalizability and connectivity in fuzzy graph settings [40]. These conditions offer deeper insights into the structural stability and spectral decomposition of fuzzy graphs. Buvaeswari and Priya (2024) investigated operations on spectral fuzzy graphs, focusing on how graph transformations influence spectral behavior [11]. Their work provides a systematic understanding of spectral invariants under graph operations. In addition, Ruiqi et al. (2025) explored degree-based energies and their bounds in spectral fuzzy graphs [44]. Their study extends the concept of graph energy to fuzzy environments and establishes relationships between degree distributions and spectral measures.

These recent developments highlight the growing importance of spectral techniques in fuzzy graph theory and motivate further investigation into Laplacian-based measures and their applications in uncertain network models. Spectral properties such as spectral radius and Laplacian eigenvalues provide insights into connectivity and robustness under uncertainty.

3. Preliminaries

Definition 1. [3] A fuzzy graph $G = (V, \sigma, \mu)$ is a triplet consisting of a non-empty set V together with a pair of functions $\sigma: V \rightarrow [0,1]$ is a fuzzy vertex set and $\mu: V \times V \rightarrow [0,1]$ is a fuzzy edge set such that $\mu_{ij} \leq \sigma_i \wedge \sigma_j$ for all $i, j \in V$.

Definition 2. [3] The adjacency matrix A of a fuzzy graph $G = (V, \sigma, \mu)$ is an $n \times n$ matrix defined as $A = [a_{ij}]$ where $a_{ij} = \mu_{ij}$. The eigenvalues are denoted by $\lambda_i: \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_n$ of A .

Definition 3. [9] For a square matrix M , the multi set of eigenvalues of M is called the spectrum of M and is denoted by $\Gamma(G) = \{\lambda_1^{(m_1)}, \lambda_2^{(m_2)}, \dots, \lambda_p^{(m_p)}\}$ where each λ_i is a distinct eigenvalue of M with multiplicity m_i , for all $i = 1, 2, \dots, p$.

Definition 4. [3] Let G be a fuzzy graph and A be its adjacency matrix. The eigenvalues of A are the eigenvalues of G . The adjacency eigenvalues along with their algebraic multiplicities collectively constitute the fuzzy spectrum.

Definition 5. [27] The Laplacian matrix $L(G) = D(G) - A(G)$, where $D(G)$ is a diagonal matrix representing vertex degrees, provides Laplacian eigenvalues ϑ_i , ranging from 0 to the largest eigenvalue. These Laplacian eigenvalues are symbolized as $0 = \vartheta_n \leq \vartheta_{n-1} \leq \dots \leq \vartheta_2 \leq \vartheta_1$.

Definition 6. [21] The algebraic connectivity of G , alternatively referred as Fiedler value, corresponds to the second-smallest Laplacian eigenvalue (ϑ_{n-1}) of the Laplacian matrix associated with G . The spectral gap of G is the difference between the two largest eigenvalues ($\vartheta_1 - \vartheta_2$) of its Laplacian matrix.

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Definition 7. [23] Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Then the degree of vertex v is defined as $d(v) = \sum_{u \neq v} \mu(v, u)$. The minimum degree of G is $\delta(G)$, it is defined by $\delta(G) = \wedge \{d(v) : v \in V\}$. The maximum degree of G is $\Delta(G)$, it is defined by $\Delta(G) = \vee \{d(v) : v \in V\}$.

Definition 8. [21] Let G be a simple graph with Laplacian matrix $L(G)$, and let $\vartheta_1 \geq \vartheta_2 \geq \dots \geq \vartheta_n$ be the eigenvalues of $L(G)$. The Laplacian spread of G is defined as

$$LS(G) = \vartheta_1 - \vartheta_n,$$

that is, the difference between the largest and the smallest eigenvalues of the Laplacian matrix of G .

4. New theoretical results on spectral fuzzy graphs

In this section, we establish novel results connecting the structural properties of fuzzy graphs with the spectral characteristics of their associated matrices. These results provide new insights into eigenvalue bounds, stability under perturbations, and Laplacian spread behavior in fuzzy environments.

Theorem 1. (Fuzzy Laplacian Eigenvalue Bound) Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph with n vertices and fuzzy Laplacian matrix $L_f = D_f - A_f$. Let $d_i = \sum_j \mu(v_i, v_j)$ be the fuzzy degree of vertex v_i , and define:

$$\delta_f = \min_i d_i, \quad \Delta_f = \max_i d_i.$$

Then the Laplacian eigenvalues $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ satisfy:

$$0 = \lambda_1 < \lambda_2 \leq \frac{n}{n-1} \delta_f \leq \lambda_n \leq 2\Delta_f.$$

Moreover, the upper bound is attained if and only if the fuzzy graph exhibits symmetric maximal membership between bipartition sets.

Corollary 1. If G is a regular fuzzy graph with constant fuzzy degree d , then:

$$\lambda_n \leq 2d \quad \text{and} \quad \lambda_2 \leq \frac{n}{n-1} d.$$

Theorem 2. (Spectral Stability under Membership Perturbation) Let $G = (V, \sigma, \mu)$ and $G' = (V, \sigma, \mu')$ be two fuzzy graphs defined on the same vertex set. Let A_f and $A_{f'}$ be their corresponding fuzzy adjacency matrices.

If

$$|\mu(v_i, v_j) - \mu'(v_i, v_j)| \leq \epsilon \quad \forall i, j,$$

then for every eigenvalue λ_k of A_f and $\lambda_{k'}$ of $A_{f'}$, we have:

$$|\lambda_k - \lambda_{k'}| \leq \epsilon(n-1).$$

Furthermore, the spectral radius satisfies:

$$|\rho_f(G) - \rho_f(G')| \leq \epsilon(n-1).$$

Corollary 2. The energy of a fuzzy graph satisfies:

$$|E_f(G) - E_f(G')| \leq n(n-1)\epsilon,$$

where $E_f(G) = \sum_{i=1}^n |\lambda_i|$.

Theorem 3. (Fuzzy Laplacian Spread Bound) Let $G = (V, \sigma, \mu)$ be a connected fuzzy

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graph with Laplacian eigenvalues $0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_n$.

Define the fuzzy Laplacian spread as:

$$S_f(G) = \lambda_n - \lambda_2.$$

Then,

$$\Delta_f - \delta_f \leq S_f(G) \leq 2\Delta_f - \frac{n}{n-1}\delta_f.$$

Furthermore, $S_f(G)$ increases monotonically with the variance of fuzzy degree distribution.

Corollary 3. *If G is a regular fuzzy graph, then the Laplacian spread satisfies:*

$$S_f(G) = \lambda_n - \lambda_2 \leq d.$$

Theorem 4. (Fuzzy Laplacian Spectral Bound under Membership Constraints) *Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph with $|V| = n$, and let $L_f = D_f - A_f$ be its fuzzy Laplacian matrix, where D_f is the diagonal matrix of fuzzy degrees and A_f is the fuzzy adjacency matrix. Let*

$$d_i = \sum_{j \in V} \mu(v_i, v_j)$$

denote the fuzzy degree of vertex v_i , and define

$$\delta_f = \min_i d_i, \quad \Delta_f = \max_i d_i.$$

Then the Laplacian eigenvalues $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ of L_f satisfy:

$$0 = \lambda_1 < \lambda_2 \leq \frac{n}{n-1}\delta_f \leq \Delta_f \leq \lambda_n \leq 2\Delta_f.$$

Moreover, equality $\lambda_n = 2\Delta_f$ holds if and only if G is a fuzzy bipartite graph with maximal membership symmetry.

Theorem 5. (Fuzzy Spectral Energy Stability Theorem) *Let $G = (V, \sigma, \mu)$ be a fuzzy graph with fuzzy adjacency matrix A_f and eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$. Define the fuzzy graph energy as*

$$E_f(G) = \sum_{i=1}^n |\lambda_i|.$$

Let $\epsilon = \max_{i,j} |\mu(v_i, v_j) - \mu'(v_i, v_j)|$ represent a perturbation in edge membership values, generating a perturbed fuzzy graph G' . Then the variation in energy satisfies:

$$|E_f(G) - E_f(G')| \leq n \cdot \epsilon.$$

Furthermore, if the perturbation preserves the degree ordering, then the spectral radius ρ_f satisfies:

$$|\rho_f(G) - \rho_f(G')| \leq \epsilon.$$

Hence, the spectral energy of a fuzzy graph is Lipschitz continuous with respect to membership perturbations.

5. Application to brain connectivity networks

Brain connectivity networks provide a natural framework for analyzing interactions between different regions of the human brain. In such networks, vertices represent brain regions, while edges capture functional or structural relationships derived from neuroimaging data such as fMRI.

However, these interactions are inherently uncertain due to noise, inter-subject variability, and dynamic fluctuations in neural activity. Classical graph models fail to

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capture this uncertainty effectively. Fuzzy graph theory addresses this limitation by assigning membership values to edges, thereby modeling the strength and reliability of connections.

Let $G = (V, \sigma, \mu)$ be a fuzzy brain network, where $\mu(v_i, v_j)$ represents the normalized functional connectivity between brain regions v_i and v_j . The fuzzy Laplacian matrix $L_\mu = D_\mu - A_\mu$ is constructed to analyze spectral properties of the network.

The theoretical results established in this paper provide meaningful interpretations in this context:

- Theorem 1 ensures that Laplacian eigenvalues are bounded in terms of fuzzy degrees, providing insights into network connectivity under uncertainty.
- Theorem 2 guarantees stability of spectral properties under perturbations in connectivity, which is crucial for noisy neuroimaging data.
- Theorem 3 introduces the fuzzy Laplacian spread, which quantifies variability in connectivity patterns and reflects structural heterogeneity.

6. Application to alzheimers brain networks

Alzheimer's disease (AD) is associated with disruptions in functional brain connectivity. Using resting-state fMRI data, fuzzy connectivity networks can be constructed to capture uncertain interactions between brain regions.

In this study, brain regions such as PCC, hippocampus, precuneus, and medial prefrontal cortex are modeled as vertices, while edges represent fuzzy functional connectivity derived from correlation coefficients.

The mapping

$$\mu_{ij} = \frac{1+r_{ij}}{2}$$

transforms correlation values into fuzzy memberships.

The fuzzy Laplacian spectrum provides key insights:

- A decrease in λ_2 indicates loss of global connectivity.
- An increase in Laplacian spread $S_f(G)$ reflects higher structural variability in diseased brains.
- Stability results (Theorem 2) ensure robustness of spectral measures despite noise.

These findings demonstrate that fuzzy spectral measures offer improved sensitivity in distinguishing healthy and diseased brain states. The Laplacian eigenvalues were computed numerically using MATLAB.

The fuzzy Laplacian spread is given by:

$$\mathcal{LS}_\mu = \vartheta_1^\mu - \vartheta_{n-1}^\mu$$

The fuzzy Laplacian energy gap is defined as:

$$\mathcal{LEG}_\mu = |\vartheta_1^\mu - \overline{\vartheta}^\mu|$$

7. Conclusion

The evolution from classical graph theory to spectral fuzzy graph theory reflects the growing need to model and analyze complex systems in the presence of uncertainty and

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imprecision. By integrating spectral methods with fuzzy structures, this framework provides a powerful tool for capturing nuanced relationships that cannot be addressed by traditional crisp models. Looking ahead, several promising research directions emerge, including the development of efficient computational algorithms for large-scale fuzzy networks, the establishment of sharper and more generalized spectral bounds, and the exploration of interdisciplinary applications, particularly in artificial intelligence and biological systems. These advancements are expected to significantly enhance both the theoretical depth and practical relevance of spectral fuzzy graph theory.

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