Intern. J. Fuzzy Mathematical Archive
Vol. 10, No. 1, 2016, 27-33
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 11 January 2016
www.researchmathsci.org

International Journal of
Fuzzy Mathematical Archive

On Edge Neighborhood Transformation Graphs

V.R. Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585 106, India
e-mail: vrkulli@gmail.com

Received 1 January 2016; accepted 9 January 2016

Abstract. Let S be the set of all open edge neighborhood sets of edges of $G = (V, E)$. Let x, y, z be three variables each taking value + or –. The edge neighborhood transformation graph N_eG^{xyz} is the graph having $E\cup S$ as the vertex set and for any two vertices u and v in $E \cup S$, u and v are adjacent in N_eG^{xyz} if and only if one of the following conditions holds: (i) $u, v \in E$. $x = +$ if $u, v \in F$ where F is an open edge neighborhood set of G. $x = –$ if $u, v \notin F$ where F is an open edge neighborhood set of G. (ii) $u, v \in S$. $y = +$ if $u \cap v \neq \phi$. $y = –$ if $u \cap v = \phi$. (iii) $u \in E$ and $v \in S$. $z = +$ if $u \in v$. $z = –$ if $u \notin v$. In this paper, we initiate a study of edge neighborhood transformation graphs. Also characterizations are given for graphs for which (i) N_eG^{+++} is totally disconnected (ii) $N_e(G) = N_eG^{+++}$ and (iii)$N_{se}(G) = N_eG^{+++}$.

Keywords: edge neighborhood graph, middle edge neighborhood graph, semientire edge neighborhood graph, entire edge neighborhood graph

AMS Mathematics Subject Classification (2010): 03E72, 05C72

1. Introduction

By a graph $G = (V, E)$, we mean a finite, undirected without loops, multiple edges or isolated vertices. Any undefined term in this paper may be found in Kulli [1].

Let G be a graph with $|V| = p$ vertices and $|E| = q$ edges. For any edge $e \in E$, the open edge neighborhood set $N(e)$ of e is the set of edges adjacent to e. Let $E = \{e_1, e_2,\ldots,e_q\}$. Let $S = \{N(e_1), N(e_2), \ldots, N(e_q)\}$ be the set of all open edge neighborhood sets of edges of G.

The edge neighborhood graph $N_e(G)$ of a graph $G = (V, E)$ is the graph with the vertex set $E\cup S$ in which two vertices u and v are adjacent if $u \in E$ and v is an open edge neighborhood set containing u. This concept was introduced in [2]. Many other graph valued functions in graph theory were studied, for example, in [3 -16] and also graph valued functions in domination theory were studied, for example, in [17- 28].

The open edge neighborhood graph $N_{oe}(G)$ of a graph G is the graph with the vertex set S in which two vertices u and v are adjacent if $u \cap v \neq \phi$. This concept was introduced in [29].

The common edge neighborhood graph $N_{ce}(G)$ of a graph G is the graph having the vertex set E and with two vertices adjacent in $N_{ce}(G)$ if there exists an open edge neighborhood set in G containing them. This concept was introduced in [29].

The middle edge neighborhood graph $M_{en}(G)$ of a graph $G = (V, E)$ is the graph with the vertex set $E \cup S$ where S is the set of all open edge neighborhood sets of G and with two vertices u and v in $M_{en}(G)$ adjacent if $u, v \in S$ and $u \cap v \neq \phi$ or $u \in E$ and v is an open edge neighborhood set containing u. This concept was introduced by Kulli in [30].

The semientire edge neighborhood graph $N_{se}(G)$ of a graph $G = (V, E)$ is the graph with the vertex set $E \cup S$ where S is the set of all open edge neighborhood sets of G and with two vertices u, v in $N_{se}(G)$ adjacent if $u, v \in N$ where N is an open edge neighborhood set in G or $u \in E$ and v is an open edge neighborhood set containing u. This concept was introduced in [31].

The entire edge neighborhood graph $N_{ee}(G)$ of a graph $G = (V, E)$ is the graph with the vertex set $E \cup S$ where S is the set of all open edge neighborhood sets of G with two vertices u, v in $N_{ee}(G)$ adjacent if $u, v \in N$ where N is an open edge neighborhood set in G or $u, v \in S$ and $u \cap v \neq \phi$ or $u \in E$ and v is an open edge neighborhood containing u. This concept was introduced by Kulli in [32].

Let $\overline{G}$ be the complement of G.

Recently many new transformation graphs were studied, for example, in [33 - 38]. In this paper, we introduce edge neighborhood transformation graphs.

2. Edge neighborhood transformation graphs

The entire edge neighborhood graph of a graph inspired us to introduce edge neighborhood transformation graphs. We now define edge neighborhood transformation graphs G^{xyz} when x or y or z is either + or –.

Definition 1. Let $G = (V, E)$ be a graph. Let S be the set of all open edge neighborhood sets of edges of G. Let x, y, z be three variables each taking value + or –. The edge neighborhood transformation graph N_eG^{xyz} is the graph having $E \cup S$ as the vertex set and for any two vertices u and v in $E \cup S$, u and v are adjacent if and only if one of the following conditions holds:

(i) $u, v \in E$. $x = +$ if $u, v \in F$ where F is an open edge neighborhood set of G. $x = -$ if $u, v, \notin F$ where F is an open edge neighborhood set of G.

(ii) $u, v \in S$. $y = +$ if $u \cap v \neq \phi$. $y = -$ if $u \cap v = \phi$.

(iii) $u \in E$ and $v \in S$. $z = +$ if $u \in v$. $z = -$ if $u \notin v$.

Using the above edge neighborhood transformation, we find eight distinct edge neighborhood transformation graphs: N_eG^{+++}, N_eG^{+-+}, N_eG^{++-}, N_eG^{-++}, N_eG^{+--}, N_eG^{-+-}, N_eG^{--+}, N_eG^{---}.

Example 2. In Figure 1, a graph G, its edge neighborhood graphs N_eG^{+++}, N_eG^{---}, N_eG^{-++}, and N_eG^{+--} are shown.

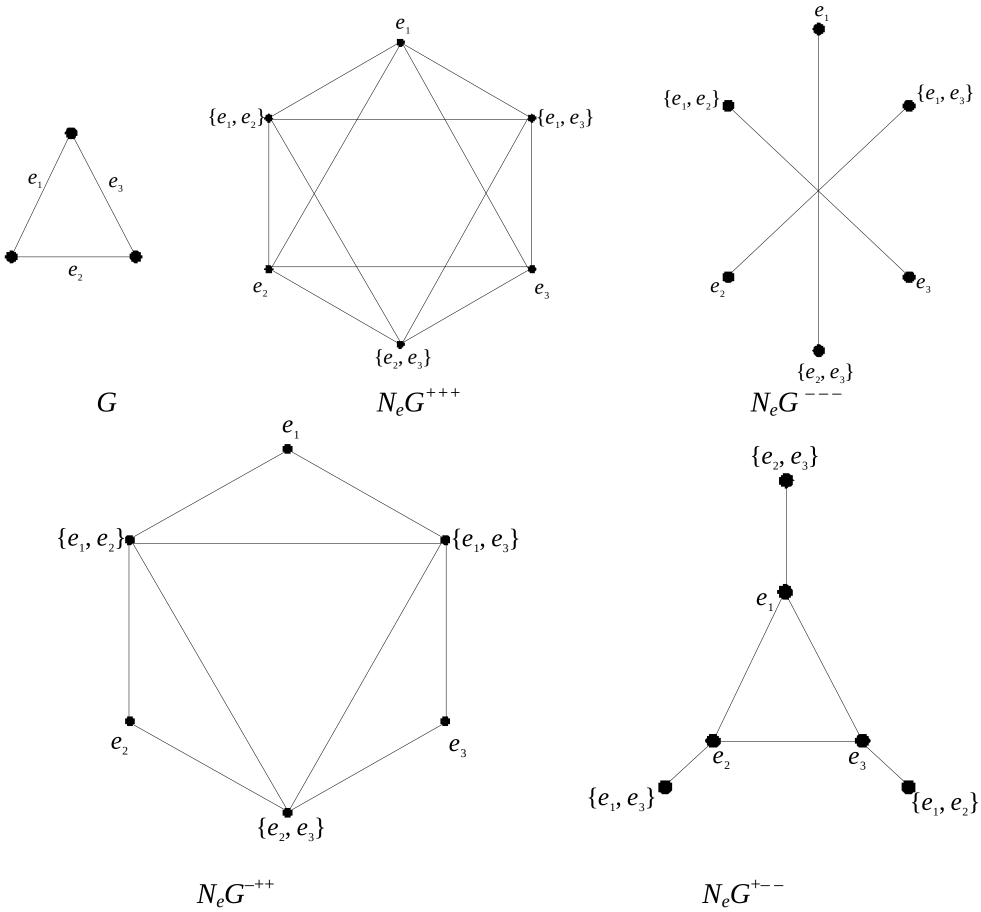


Figure 1:

3. The edge neighborhood transformation graph N_eG^{+++}

Among edge neighborhood transformation graphs one is the entire edge neighborhood graph $N_{ee}(G)$. It is easy to see that

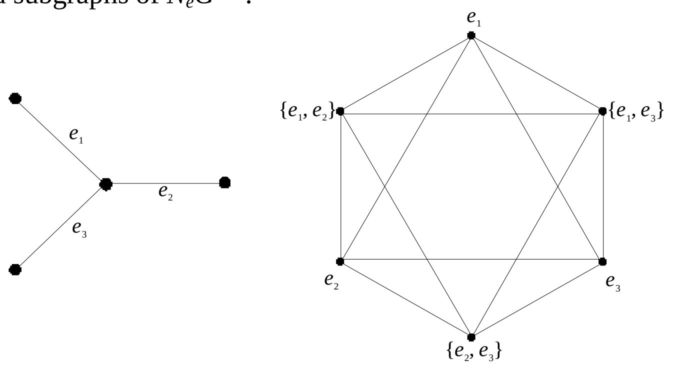
Proposition 4. For any graph G without isolated vertices, $N_{ee}(G)=N_eG^{+++}$.

Remark 5. For any graph G without isolated vertices, the edge neighborhood graph $N_e(G)$ of G is a spanning subgraph of N_eG^{+++}.

Remark 6. For any graph G without isolated vertices, the middle edge neighborhood graph $M_{en}(G)$ of G is a spanning subgraph of N_eG^{+++}.

Remark 7. For any graph G without isolated vertices, the semientire edge neighborhood graph $N_{se}(G)$ of G is a spanning subgraph of N_eG^{+++}.

Remark 8. For any graph G without isolated vertices, the open edge neighborhood graph $N_{oe}(G)$ and the common edge neighborhood graph $N_{ce}(G)$ are vertex and also edge disjoint induced subgraphs of N_eG^{+++}.



$K_{1,3}$ $N_eK_{1,3}^{+++}$

Figure 2:

Remark 9. Form Figure 1 and Figure 2, we have $N_eC_3^{+++} = N_eK_{1,3}^{+++}$. But $C_3 \neq K_{1,3}$.

Theorem 10. $N_eG^{+++} = \overline{K}_n$ if and only if $G = nK_2$, $n\geq 1$.
Proof: Suppose $G = nK_2$, $n\geq 1$. Then each component of G is an edge. Then $N(e)$ is a null set. Clearly $N_eG^{+++} = \overline{K}_n$.

Conversely suppose $N_eG^{+++}=\overline{K}_n$. We now prove that $G = nK_2$. On the contrary, assume $G \neq nK_2$. Then there exists a component of G which has at least 2 edges, say $uv = e_1$ and $vw = e_2$. Then $N(e_1)$ and $N(e_2)$ are nonempty open edge neighborhood sets of edges e_1 and e_2 respectively. Thus N_eG^{+++} contains an edge. Thus $N_eG^{+++} \neq \overline{K}_n$, which is a contradiction. Hence each component of G is an edge. Thus $G = nK_2$.

Theorem 10 may be written as
Theorem 11. N_eG^{+++} is totally disconnected if and only if each component of G is K_2.

Theorem 12. $N_eG^{+++} = 2mP_2$ if and only if $G = mP_3$, $m \geq 1$.
Proof: Suppose $G = mP_3$. Then each open edge neighborhood set of an edge of G contains exactly one edge. Thus the corresponding vertex of open edge neighborhood set is adjacent with exactly one vertex in N_eG^{+++}. Since G has $2m$ edges, it implies that G has $2m$ open edge neighborhood sets. Thus N_eG^{+++} has $4m$ vertices and the degree of each vertex is one. Thus $N_eG^{+++} = 2mP_2$.

Conversely suppose $N_eG^{+++} = 2mP_2$. We prove that $G = mP_3$, $m\geq 1$. On the contrary, assume $G \neq mP_3$. We consider the following two cases.
Case 1. Suppose $G = mP_2$. By Theorem 10, $N_eG^{+++} = mP_1$, which is a contradiction.
Case 2. Suppose $G = mG_1$, where G_1 is a component of G with at least 4 vertices. Then there exists at least one open edge neighborhood set N containing two or more edges of

G. Then v together with two edges will form a subgraph P_3 in N_eG^{+++}, which is a contradiction.

From the above two cases, we conclude that $G = mP_3$.

We characterize graphs G for which $N_eG^{+++} = N_e(G)$.

Theorem 13. For any graph G without isolated vertices and without isolated edges,

$$N_e(G) \subseteq N_eG^{+++} \qquad (1)$$

Furthermore, equality in (1) holds if and only if every open edge neighborhood set contains exactly one edge.

Proof: By Remark 5, $N_e(G) \subseteq N_eG^{+++}$.

We now prove the second part.

Suppose equality in (1) holds. Assume an open edge neighborhood set of an edge of G contains at least two edges, say $e_1, e_2, \ldots, e_n$. $n \geq 2$. Then the corresponding vertices of $e_1, e_2, \ldots, e_n$ are not mutually adjacent in $N_e(G)$, but they are mutually adjacent in N_eG^{+++}. Hence $N_e(G) \neq N_eG^{+++}$, which is a contradiction. Thus two or more edges of G are not in the same open edge neighborhood set. This proves that every open edge neighborhood set contains exactly one edge.

Conversely, suppose every open edge neighborhood set contains exactly one edge. Then every pair of open edge neighborhood sets of G are disjoint. Hence the corresponding vertices of open edge neighborhood sets in N_eG^{+++} are not adjacent. Thus $N_eG^{+++} \subseteq N_e(G)$ and since $N_e(G) \subseteq N_eG^{+++}$, it implies that equality in (1) holds.

Proposition 14. If $G = mP_3$, $m \geq 1$, then $N_e(G) = N_eG^{+++}$.

Proof: This follows from Theorem 12 and Theorem 13.

Next we characterize graphs G for which $N_eG^{+++} = N_{se}(G)$.

Theorem 15: For any graph G without isolated vertices and without isolated edges,

$$N_{se}(G) \subseteq N_eG^{+++} \qquad (2)$$

Furthermore, equality in (2) holds if and only if every pair of open edge neighborhood sets of edges of G is disjoint.

Proof: By Remark 7, $N_{se}(G) \subseteq N_eG^{+++}$.

We now prove the second part.

Suppose equality in (2) holds. We prove that every pair of open edge neighborhood sets of edges of G is disjoint. On the contrary, assume $N_1, N_2, \ldots, N_k$, $k \geq 2$ are open edge neighborhood sets of edges of G such that $N_i \cap N_j \neq \phi$, $1 \leq i, j \leq k$, $i \neq j$. Then the corresponding vertices of N_i and N_j are not adjacent in $N_{Se}(G)$ and they are adjacent in N_eG^{+++}. Hence $N_{se}(G) \neq N_eG^{+++}$, which is a contradiction. Thus every pair of open edge neighborhood sets of G is adjacent.

Conversely suppose every pair of open edge neighborhood sets of G is disjoint. Then two vertices corresponding to open edge neighborhood sets cannot be adjacent in N_eG^{+++}. Hence $N_eG^{+++} \subseteq N_{se}(G)$ and since $N_{se}(G) \subseteq N_eG^{+++}$, it implies that equality in (2) holds.

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, On edge neighborhood graphs, submitted.
3. V.R.Kulli, On common edge graphs, *J. Karnatak University Sci.*, 18 (1973) 321-324.
4. V.R.Kulli, The block-line forest of a graph, *Journal of Computer and Mathematical Sciences*, 6(4) (2015) 200-205.
5. V.R.Kulli, On line block graphs, *International Research Journal of Pure Algebra*, 5(4) 40-44 (2015).
6. V.R.Kulli, On full graphs, *Journal of Computer and Mathematical Sciences*, 6(5) 261-267 (2015).
7. V.R.Kulli, The semifull graph of a graph, *Annals of Pure and Applied Mathematics*, 10(1) (2015) 99-104.
8. V.R.Kulli, On block line graphs, middle line graphs and middle block graphs, *International Research Mathematical Archive*, 6(5) (2015) 80-86.
9. V.R.Kulli, On semifull line graphs and semifull block graphs, *Journal of Computer and Mathematical Sciences*, 6(7) (2015) 388-394.
10. V.R.Kulli and M.S.Biradar, The middle blict graph of graph, *International Research Journal of Pure Algebra,* 5(7) (2015) 111-117.
11. V.R.Kulli, On full line graph and the full block graph of a graph, *International Journal of Mathematical Archive,* 6(8) (2015) 91-95.
12. V.R.Kulli and N.S.Annigeri, The ctree and total ctree of a graph *Vijnana Ganga*, 2 (1981) 10-24.
13. V.R.Kulli and B.Basavanagoud, On the quasivertex total graph of a graph, *J. Karnatak University Sci.*, 42 (1998) 1-7.
14. V.R.Kulli and M.S. Biradar, The point block graph of a graph, *Journal of Computer and Mathematical Sciences*, 5(5) (2014) 476-481.
15. V.R.Kulli and K.M.Niranjan, The semi-splitting block graph of a graph, *Journal of Scientific Research*, 2(3) (2010) 485-488.
16. V.R.Kulli and N.S. Warad, On the total closed neighbourhood graph of a graph, *J. Discrete Mathematical Sciences and Cryptography*, 4 (2001) 109-114.
17. V.R.Kulli, *The edge dominating graph of a graph.* In Advances in Domination Theory I, V.R.Kulli, ed., Vishwa International Publications Gulbarga, India (2012) 127-131.
18. V.R.Kulli, The middle edge dominating graph, *Journal of Computer and Mathematical Sciences*, 4(5) (2013) 372-375.
19. V.R.Kulli, The entire edge dominating graph, *Acta Ciencia Indica*, 40(4) to appear.
20. V.R.Kulli, The common minimal total dominating graph, *J. Discrete Mathematical Sciences and Cryptography*, 17 (2014) 49-54.
21. V.R.Kulli, The total dominating graph, *Annals of Pure and Applied Mathematics*, 10(1) (2015) 123-128.
22. V.R.Kulli and R.R.Iyer, *The total minimal dominating graph*. In Advances in Domination Theory I, V.R.Kulli., ed., Vishwa International Publications, Gulbarga, India, (2012) 121-126.

23. V.R.Kulli, and B.Janakiram, The common minimal dominating graph, *Indian J.Pure Appl. Math*, 27(2) (1996) 193-196.
24. V.R.Kulli, B.Janakiram and K.M.Niranjan, The vertex minimal dominating graph *Acta Ciencia Indica*, 28 (2002) 435-440.
25. V.R.Kulli, B Janakiram and K.M. Niranjan, The dominating graph, *Graph Theory Notes of New York, New York Academy of Sciences*, 46 (2004) 5-8.
26. B.Basavanagoud, V.R.Kulli and V.V.Teli, Equitable total minimal dominating graph, *International Research Journal of Pure Algebra*, 3(10) (2013) 307-310.
27. B.Basavanagoud, V.R.Kulli and V.V. Teli, Semientire equitable dominating graphs, *International Journal of Mathematical Combinatorics*, 3 (2014) 49-54.
28. B.Basavanagoud, V.R.Kulli and V.V.Teli, Equitable dominating graph, *International Research Journal of Math. Sci. and Engg. Appls*, 9(2) (2015) 109-114.
29. V.R. Kulli, The open edge neighborhood graph and the common edge neighborhood graph of a graph, International Journal of Mathematical Archive,
30. V.R.Kulli, On middle edge neighborhood graphs, preprint.
31. V.R.Kulli, On semientire edge neighborhood graph of a graph, submitted.
32. V.R.Kulli, On entire edge neighborhood graph of a graph, preprint.
33. V.R.Kulli, On qlick transformation graphs, *International Journal of Fuzzy Mathematical Archive*, 8(1) (2015) 29-35.
34. V.R.Kulli, On entire dominating transformation graphs and fuzzy transformation graphs, *International Journal of Fuzzy Mathematical Archive*, 8(1) (2015) 43-49.
35. V.R.Kulli, Entire total dominating transformation graphs, *International Research Journal of Pure Algebra*, 5(5) (2015) 50-53.
36. V.R.Kulli, Entire edge dominating transformation graphs, *Inter. Journal of Advanced Research in Computer Science and Technology*, 3(2) (2015) 104-106.
37. V.R.Kulli, On neighborhood transformation graphs, *Annals of Pure and Applied Mathematics*, 10(2) (2015) 239-245.
38. V.R.Kulli, On entire equitable dominating transformation graphs and fuzzy transformation graphs, *Journal of Computer and Mathematical Sciences*, 6(8) (2015) 449-454.