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Application of Intuitionistic Fuzzy Sets in Electoral System

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Abstract. In this paper, we gave a concise note on intuitionistic fuzzy sets and presented an application of intuitionistic fuzzy sets in election process using an assumed data. The application was conducted with the aid of a new distance measure of intuitionistic fuzzy sets.

Keywords: fuzzy sets, intuitionistic fuzzy sets, electoral system, distance measures

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1. Introduction

The concept of intuitionistic fuzzy sets (IFSs) was proposed by Atanassov [1] as an extension of fuzzy sets introduced by Zadeh [19]. Several publications on IFSs theory and applications have been carried out, recent of such works can be found in [2,3,5-18]. In this paper, we give a brief note on IFSs and extend the works in [4, 8] with an explicit illustration.

2. Concept of intuitionistic fuzzy sets

Definition 2.1. [19] Fuzzy set A of a set X is defined by the membership function of the set A such that $\mu_A(x): X \to [0,1]$,

$$\text{where, } \mu_A(x) = \begin{cases} 1, \; if \; x \; is \; totally \; in \; A \\ 0, if \; x \; is \; not \; in \; A \\ (0,1), if \; x \; is \; partly \; in \; A \end{cases}$$

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, whereas the grades 1 and 0 represents full membership and full non-membership.

Definition 2.2. [2] Let X be nonempty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x): X \longrightarrow [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$ to the set A. For every $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the intuitionistic fuzzy set index or hesitation margin which is the degree of indeterminacy concerning the membership of x in A, then $0 \leq \mu_A(x) + \nu_A(x) + \pi_A(x) = 1$. Whenever $\pi_A(x) = 0$, an IFS reduces automatically to fuzzy set. Without ambiguity, we denote an IFS A as $A = (\mu_A, \nu_A)$.

For example, let A be an IFS with membership function μ_A and non-membership function ν_A respectively. If $\mu_A = 0.5$ and $\nu_A = 0.4$, then $\pi_A = 1 - 0.5 - 0.4 = 0.1$, (since $\mu_A + \nu_A + \pi_A = 1$) which could be interpreted as the degree to which the object x belongs to A is 0.5, the degree to which the object x does not belong to A is 0.4 and the hesitation margin is 0.1. Thus, A in X can be expressed as $\{\langle x, 0.5, 0.4, 0.1\rangle : x \in X\}$. If A is a crisp fuzzy set, it implies that $\pi_A = 0$ for each $x \in X$ i.e. $\pi_A = 1 - \mu_A - \nu_A = 1 - \mu_A - (1 - \mu_A) = 1 - \mu_A - 1 + \mu_A = 0$. Likewise, $\pi_A = 1 - \mu_A - \nu_A = \nu_A - \nu_A = 0$. In turn, it means that the third parameter π_A cannot be omitted or discarded since it determines the intuitionistic aspect of A.

Definition 2.3. [2] Let X be nonempty. If A is an IFS drawn from X , then
$A = \{\langle x, \mu_A\rangle : x \in X\} = \{\langle x, \mu_A, 1 - \mu_A\rangle : x \in X\}$,
$\Diamond A = \{\langle x, 1 - \nu_A\rangle : x \in X\} = \{\langle x, 1 - \nu_A, \nu_A\rangle : x \in X\}$.

Definition 2.4. Let X be nonempty. If $A, B \in IFS(X)$ then;
Complement: $A^c = (\nu_A, \mu_A)$
Union: $\mu_{A\cup B} = \vee(\mu_A, \mu_B)$ and $\nu_{A\cup B} = \wedge(\nu_A, \nu_B)$
Intersection: $\mu_{A\cap B} = \wedge(\mu_A, \mu_B)$ and $\nu_{A\cap B} = \vee(\nu_A, \nu_B)$
Addition: $\mu_{A\oplus B} = (\mu_A + \mu_B - \mu_A\mu_B)$ and $\nu_{A\oplus B} = (\nu_A\nu_B)$
Multiplication: $\mu_{A\otimes B} = (\mu_A\mu_B)$ and $\nu_{A\otimes B} = (\nu_A + \nu_B - \nu_A\nu_B)$
Difference: $A - B = \wedge(\mu_A, \nu_B), \vee(\nu_A, \mu_B)$
We also have the following derive operations;
(i) $A@B = \left(\frac{1}{2}(\mu_A + \mu_B), \frac{1}{2}(\nu_A + \nu_B)\right)$ (ii) $A\$B = \left((\mu_A\mu_B)^{\frac{1}{2}}, (\nu_A\nu_B)^{\frac{1}{2}}\right)$
(iii) $A\#B = \left(\frac{2\mu_A\mu_B}{\mu_A+\mu_B}, \frac{2\nu_A\nu_B}{\nu_A+\nu_B}\right)$ (iv) $A * B = \left(\frac{\mu_A+\mu_B}{2(\mu_A\mu_B+1)}, \frac{\nu_A+\nu_B}{2(\nu_A\nu_B+1)}\right)$.

Theorem 2.5. Let A and B be two IFSs in a nonempty set X, then; (i) $A - B = A \cap B^c$
(ii) $A - B = B - A$ iff $A = B$ (iii) $A - B = B^c - A^c$.

Theorem 2.6. For IFSs A, B, C in X and $A \subseteq B \subseteq C$, then $B - A \subseteq C - A$.

Theorem 2.7. Let A and B be two IFSs in a nonempty set X, then;(i)$A - A = \Phi$ (ii) $A - \Phi = A$
(iii) $A - B \subseteq A$ (iv) $A - B = \Phi$ iff $A = B$ (v) $A - B = A$ iff $B = \Phi$.

Theorem 2.8. Let A, B and C be IFSs in X, then; (i) $A@B = B@A$ (ii) $A\$B = B\$A$ (iii) $A\#B = B\#A$ (iv) $A * B = B * A$ (v) $\overline{A@B} = A@B$ (vi) $\overline{A\$B} = A\$B$ (vii) $\overline{A\#B} = A\#B$ (viii) $\overline{A * B} = A * B$

Theorem 2.9. Let A, B and C be IFSs in X, then;
(i) $A@(B \cap C) = (A@B) \cap (A@C)$ (ii) $A@(B \cup C) = (A@B) \cup (A@C)$
(iii) $A\#(B \cap C) = (A\#B) \cap (A\#C)$ (iv) $A\#(B \cup C) = (A\#B) \cup (A\#C)$
(v) $A\$(B \cap C) = (A\$B) \cap (A\$C)$ (vi) $A\$(B \cup C) = (A\$B) \cup (A\$C)$

Theorem 2.10. Let A, B and C be IFSs in X, then;

(i) $\Box(A@B) = \Box A@\Box B$ (ii) $\Box(A\$B) = \Box A\$\Box B$
(iii) $\Box(A\#B) = \Box A\#\Box B$ (iv) $\Diamond(A@B) = \Diamond A@\Diamond B$
(v) $\Diamond(A\$B) = \Diamond A\$\Diamond B$(vi) $\Diamond(A\#B) = \Diamond A\#\Diamond B$.
The proofs are straightforward.

Definition 2.11. [17] Let X be nonempty such that IFSs $A, B, C \in X$. Then the distance measure d between IFSs A and B is a mapping $d: X \times X \to [O, 1]$ satisfying the following axioms:
(i) $0 \leq d(A, B) \leq 1$(boundedness)
(ii) $d(A, B) = 0$ if and only if $A = B$
(iii) $d(A, B) = d(B, A)$ (symmetric)
(iv) $d(A, C) + d(B, C) \geq d(A, B)$
(v) if $A \subseteq B \subseteq C$, then $d(A, C) \geq d(A, B)$ and $d(A, C) \geq d(B, C)$.
Distance measure is a term that describes the difference between intuitionistic fuzzy sets and can be considered as a dual concept of similarity measure. We make use of the four distance measures in [17] between intuitionistic fuzzy sets, which were partly based on the geometric interpretation of intuitionistic fuzzy sets, and have some good geometric properties.

Let $A = \{\langle x, \mu_A(x_i), \nu_A(x_i), \pi_A(x_i)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x_i), \nu_B(x_i), \pi_B(x_i)\rangle : x \in X\}$ be two IFSs in $X = \{x_1, x_2, \ldots, x_n\}$, for $i = 1, 2, \ldots, n$. Based on the geometric interpretation of IFSs, Szmidt [17] proposed the following four distance measures between A and B:

The Hamming distance;

$$d_H(A, B) = \frac{1}{2}\sum_{i=1}^{n}\left(\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right| + \left|\pi_A(x_i) - \pi_B(x_i)\right| \right)$$

The Euclidean distance;

$$d_E(A, B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}$$

The normalized Hamming distance;

$$d_{n-H}(A, B) = \frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right| + \left|\pi_A(x_i) - \pi_B(x_i)\right| \right)$$

The normalized Euclidean distance ;

$$d_{n-E}(A, B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}$$

The new distance measure is given as thus [14];

$$d(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\Big[\,|\mu_A(x_i)-\mu_B(x_i)|+\big||\mu_A(x_i)-\nu_A(x_i)|-|\mu_B(x_i)-\nu_B(x_i)|\big| + \big||\mu_A(x_i)-\pi_A(x_i)|-|\mu_B(x_i)-\pi_B(x_i)|\big|\Big]$$

For example, let $A=\{\langle 0.6,0.2,0.2\rangle,\langle 0.5,0.3,0.2\rangle\}$ and $B=\{\langle 0.5,0.4,0.1\rangle,\langle 0.4,0.1,0.5\rangle\}$ be IFSs in X such that $X=\{x_1,x_2\}$. We use the above distance measures to calculate the distance from A to B and get the following results; $d_H(A,B)=0.25$, $d_E(A,B)=0.2189$, $d_{n-H}(A,B)=0.125$, $d_{n-E}(A,B)=0.1548$ and $d(A,B)=0.1$ i.e. the new distance measure of A and B. From these results, we observe that the new distance measure proposed in [14] is more accurate because it produces the shortest distance measure.

3. Intuitionistic fuzzy sets and electoral system

The concept of intuitionistic fuzzy sets is a veritable tool in decision making as reported in [2, 4, 6-8, 10-14, 16, 18]. Presently, we are in an era of democracy where the electorates exercise their franchise in the poll. Due to the existence of the fundamental human right, every electorate has the right to vote a preferable aspirant, and as such, decision on whom to elect preoccupies the electorates. In this scenario, some voters must of necessity vote a candidate, some against and some of course, will remain undecided or cast invalid vote. Interpreting into an intuitionistic fuzzy set, the electorates that voted for a candidate stand for the membership function μ, those that voted against stand for the non-membership function ν, and those that remain undecided or cast invalid ballot paper stand for the hesitation margin π [4].

Let $C=\{C_1,C_2,C_3,C_4,C_5\}$ be the set of all candidates, let X be the set of all electorates (i.e. total of ten million, equal proportion each from eight provinces), let $P=\{P_1,P_2,P_3,P_4,P_5,P_6,P_7,P_8\}$ be the set of provinces within the country, and let $O=\{O_1,O_2,O_3,O_4,O_5\}$ be the set of offices vying for. Let $F(x)$ be the number of electorates that voted for, $A(x)$ be the number of electorates that voted against, and $U(x)$ be the number of electorates that remained undecided or cast invalid votes for every $x\in X$ i.e. $X=F(x)+A(x)+U(x)$. From these, we get

$$\mu(x)=\frac{F(x)}{X} \quad (1)$$

$$\nu(x)=\frac{A(x)}{X} \quad (2)$$

Similarly, $\pi(x)=1-\frac{F(x)}{X}-\frac{A(x)}{X}\Longrightarrow$

$$\pi(x)=\frac{X-F(x)-A(x)}{X} \quad (3)$$

Adding (1) and (2), we get $\mu(x)+\nu(x)=\frac{F(x)}{X}+\frac{A(x)}{X}$, but from Def. 2.2, $\mu_A(x)+\nu_A(x)\le 1$ i.e. $\frac{F(x)}{X}+\frac{A(x)}{X}\le 1$, and yields

$$F(x)+A(x)\le X \quad (4)$$

and

$$U(x)=X-F(x)-A(x) \quad (5)$$

Equation (3) becomes

$$\pi(x)=\frac{U(x)}{X} \quad (6)$$

For any elected offices, the candidates to be returned elected must satisfied stipulated percentage votes in each of the provinces as showed in the Table below.

Offices and Provinces' Required Votes

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
O_1	(0.6,0.3)	(0.5,0.4)	(0.7,0.2)	(0.6,0.3)	(0.5,0.3)	(0.5,0.4)	(0.6,0.2)	(0.7,0.3)
O_2	(0.7,0.2)	(0.8,0.1)	(0.7,0.1)	(0.6,0.3)	(0.8,0.1)	(0.8,0.0)	(0.7,0.1)	(0.9,0.0)
O_3	(0.8,0.1)	(0.7,0.2)	(0.8,0.1)	(0.7,0.1)	(0.8,0.1)	(0.8,0.1)	(0.6,0.2)	(0.8,0.2)
O_4	(0.6,0.3)	(0.5,0.3)	(0.6,0.2)	(0.4,0.4)	(0.6,0.1)	(0.5,0.4)	(0.5,0.3)	(0.6,0.3)
O_5	(0.4,0.5)	(0.5,0.2)	(0.5,0.3)	(0.6,0.4)	(0.6,0.3)	(0.5,0.3)	(0.6,0.3)	(0.6,0.1)

Each of the entries is described by three numbers, the first entry is membership function μ, the second entry is non-membership function ν and the third entry is hesitation margin π i.e. $\pi = 1 - \mu - \nu$.
We assumed hypothetically that, the results below are the collated results after the election.

Candidates and Provinces' Votes

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
C_1	(0.5,0.4)	(0.8,0.2)	(0.4,0.3)	(0.4,0.5)	(0.6,0.2)	(0.7,0.1)	(0.8,0.1)	(0.7,0.2)
C_2	(0.8,0.2)	(0.6,0.3)	(0.6,0.1)	(0.3,0.5)	(0.8,0.1)	(0.8,0.1)	(0.9,0.0)	(0.8,0.1)
C_3	(0.6,0.3)	(0.7,0.3)	(0.7,0.2)	(0.2,0.6)	(0.9,0.1)	(0.8,0.2)	(0.9,0.1)	(0.7,0.2)
C_4	(0.7,0.1)	(0.8,0.1)	(0.9,0.0)	(0.5,0.4)	(0.5,0.3)	(0.8,0.1)	(0.9,0.0)	(0.7,0.1)
C_5	(0.8,0.1)	(0.5,0.4)	(0.8,0.1)	(0.8,0.2)	(0.6,0.3)	(0.5,0.3)	(0.6,0.3)	(0.6,0.2)

Using the new distance measure to calculate the distance between the offices and the candidates with respect to the provinces, we get the results below.

Final Collation

	O_1	O_2	O_3	O_4	O_5
C_1	0.0438	0.0406	0.0516	0.0406	0.0359
C_2	0.0219	0.0297	0.0313	0.0484	0.0547
C_3	0.0469	0.0344	0.0375	0.0484	0.0578
C_4	0.0438	0.0344	0.0375	0.0531	0.0578
C_5	0.0250	0.0500	0.0320	0.0359	0.0313

From the collation Table above; candidate C_1 is not elected for any position, candidate C_2 is returned elected for office O_1, candidate C_3 and candidate C_4 will go for a rerun election and candidate C_5 is returned elected for office O_5. The declaration is based on the distance of each candidate to the offices. The candidate with the shortest distance to any of the offices wins that particular office and no candidate can be elected into two offices.

4. Conclusion

We conclude from this paper that, the concept of intuitionistic fuzzy sets is a reliable technique for decision problems. This method placed each candidates on the right offices based on the stipulated office votes requirements. Intuitionistic fuzzy sets theory is of great advantage in fuzzy logic, fuzzy control, pattern recognition, decision science etc.

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