

## **Solving Multi Objective Inventory Model of Deteriorating Items with Two Constraints using Fuzzy Optimization Technique**

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**Abstract.** In this paper a multi objective and multi item EOQ model with stock dependent demand for deteriorating items under the space and investment constraints is considered in fuzzy environment. Various inventory costs, the storage area and the amount of investment are taken as triangular fuzzy numbers. Linear membership function is considered for fuzzy objective. The aim of this paper is present a method in which a fuzzy inventory model is reduced to crisp using ranking function and then the crisp EOQ is solved by fuzzy programming technique. The method is illustrated with a numerical example.

**Keywords:** Fuzzy inventory, Deteriorating items, Triangular fuzzy number, Fuzzy optimization, Fuzzy ware house capacity, Fuzzy maximum investment, Fuzzy ranking

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### **1. Introduction**

Inventory problems are common in manufacturing, maintenance service and business operations in general. Often uncertainties may be associated with demand, various relevant costs. Generally demand rate is considered to be constant, time dependent, ramp type and selling price dependent. However in present competitive market stock-dependent demand plays an important role in increase its demand. Deterioration is one of the factors in inventory system. Some items like food grains, vegetables, milk, eggs etc. deteriorating during their storage time and retailer suffers loss. In an inventory system available storage space, budget, number of orders etc. are always limited hence multi-item classical inventory models under these constraints have great importance.

In conventional inventory models, uncertainties are treated as randomness and are handled by probability theory. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. However in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory introduced by Zadeh [13] is applicable. There are several studies on fuzzy EOQ model.

Lin et al. have developed a fuzzy model for production inventory problem. Katagiri and Ishii [4] have proposed an inventory problem with shortage cost as fuzzy quantity.

In fuzzy optimization the degree of acceptance of objectives and constraints are considered here. In this paper, under limited storage area and investment, a multi-item multi-objective inventory model of deteriorating item with stock-dependent demand is formulated in crisp and fuzzy environment. We introduced a method in which a fuzzy inventory model is first reduced to crisp inventory model using ranking function suggested by Robust's and the resulting one is solved by fuzzy programming technique of Zimmermann [14]. The model is illustrated numerically and the results are obtained from different methods.

### 1.1. Review of literature

In recent years inventory problems in fuzzy environment have received much attention Lin et al. have developed a fuzzy model for production inventory. Katagiri and Ishii [4] have proposed an inventory problem with shortage cost as fuzzy quantity. Roy and Maiti (1995) considered a fuzzy inventory model with constraint. Srinivasan and Dhanam (2006) have considered cost analysis on a deterministic single item fuzzy inventory model with shortage. Also they have considered a multi-item EOQ inventory model with three constraints in a fuzzy environment. The model is solved by fuzzy non-linear programming method using Lagrange multipliers. Different kinds of inventory problems are solved in the papers [9,10,15] Zimmermann [14] has introduced fuzzy programming approach to solve crisp multi objective linear programming problem.

## 2. Preliminaries

**Definition 1.** A fuzzy set is characterized by a membership function mapping elements of a domain, space or universe of discourse  $X$  to the unit interval  $[0,1]$ . (i.e)  $A = \{X, \mu_A(x) : x \in X\}$ , here  $\mu_A : X \rightarrow [0,1]$  is a mapping called the degree of membership function of the fuzzy set  $A$  and  $\mu_A(x)$  is called the membership value of  $x \in X$  in the fuzzy set  $A$ . These membership grades are often represented by real numbers ranking from  $[0,1]$ .

### Definition 2. (Triangular fuzzy number)

For a triangular fuzzy number  $A(x)$ , it can be represented by  $A(a,b,c;1)$  with membership function  $\mu_A(x)$  given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)}; & a \leq x \leq b \\ 1 & ; x = b \\ \frac{(c-x)}{(c-b)}; & b \leq x \leq c \\ 0 & ; otherwise \end{cases}$$

### Definition 3. ( $\alpha$ -cut of a fuzzy number)

The  $\alpha$ -cut of a fuzzy number  $A(x)$  is defined as  $A(\alpha) = \{x : \mu(x) \geq \alpha, \alpha \in [0,1]\}$

## 3. Ranking function for fuzzy number

Assume that  $R: F(\mathcal{R}) \rightarrow \mathcal{R}$  be linear ordered function that maps each fuzzy number in to the real number, in which  $F(\mathcal{R})$  denotes the whole fuzzy numbers. Accordingly for any two fuzzy numbers  $\tilde{a}$  and  $\tilde{b}$  we have

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$$\tilde{a} \underset{R}{\geq} \tilde{b} \text{ iff } R(\tilde{a}) \geq R(\tilde{b})$$

$$\tilde{a} \underset{R}{>} \tilde{b} \text{ iff } R(\tilde{a}) > R(\tilde{b})$$

$$\tilde{a} \underset{R}{=} \tilde{b} \text{ iff } R(\tilde{a}) = R(\tilde{b})$$

We restrict our attention to linear ranking function, that is a ranking function  $R$  such that  $R(k\tilde{a} + \tilde{b}) = kR(\tilde{a}) + R(\tilde{b})$  for any  $\tilde{a}$  and  $\tilde{b}$  in  $F(\mathbb{R})$  and any  $k \in \mathbb{R}$ .

Robust's ranking function:

The ranking function proposed by Robust's is defined by

$$R(\tilde{a}) = \frac{1}{2} \int_0^1 (a_\alpha^L, a_\alpha^U) d\alpha.$$

#### 4. Assumptions and notations

- (i) The scheduling period is constant and no lead time.
- (ii) Replenishment rate is infinite.
- (iii) Selling price is known and constant.
- (iv) Demand rate is stock dependent.
- (v) Shortages are not allowed.
- (vi) Deteriorating rate is age specific failure rate.

- $T_i$  : Time period for each cycle for the  $i^{\text{th}}$  item.
  - $R_i$  : Demand rate per unit time of  $i^{\text{th}}$  item. [ $R_i = a_i + b_i q_i$ ]
  - $\theta_i$  : Deteriorating rate of  $i^{\text{th}}$  item.
  - $Q_i(t)$  : inventory level at time  $t$  of  $i^{\text{th}}$  item.
  - $C_H$  : Total Holding cost.
  - $C_{1i}$  : Holding cost per unit of  $i^{\text{th}}$  item.
  - $C_{3i}$  : Setup cost for  $i^{\text{th}}$  item.
  - $S_{di}$  : Total deteriorating units of  $i^{\text{th}}$  item.
  - $P_i$  : Selling price per unit of  $i^{\text{th}}$  item.
  - $Q_i$  : Initial stock level of  $i^{\text{th}}$  item.
  - $N$  : Number of items.
  - $TC(Q_i)$  : Sum of costs of the system
  - $PF(Q_i)$  : Sum of profits of the system
  - $WC(Q_i)$  : Sum of wastage costs of the system
- (wavy bar ( $\sim$ ) represents the fuzzification of the parameters)

#### 5. Mathematical formulation

##### 5.1. Crisp model

As  $Q_i(t)$  is the inventory level at time  $t$  of  $i^{\text{th}}$  item, then the differential equation describing the state of inventory is given by

$$\frac{d}{dt} Q_i(t) + \theta_i Q_i(t) = -(a_i + b_i Q_i(t)) \quad 0 \leq t \leq T_i$$

Solving the above differential equation using boundary condition  $Q_i(t) = Q_i$  at  $t=0$ , we get

$$Q_i(t) = -\frac{a_i}{(\theta_i + b_i)} + \left[ Q_i + \frac{a_i}{(\theta_i + b_i)} \right] e^{-(\theta_i + b_i)t} \quad (1)$$

And using boundary condition  $Q_i(t) = 0$  at  $t = T_i$

$$\therefore T_i = \frac{1}{(\theta_i + b_i)} \log \left\{ 1 + \frac{(\theta_i + b_i)Q_i}{a_i} \right\} \quad (2)$$

The holding cost of  $i^{\text{th}}$  item in each cycle is

$$C_H = C_{li} G_i(Q_i) \quad (3)$$

$$\begin{aligned} \text{where } G_i(Q_i) &= \int_0^{Q_i} \frac{q_i dq_i}{a_i + (\theta_i + b_i)q_i} \\ &= \frac{Q_i}{(\theta_i + b_i)} + \frac{a_i}{(\theta_i + b_i)^2} \log \left\{ 1 + \frac{(\theta_i + b_i)Q_i}{a_i} \right\} \end{aligned}$$

By neglecting the higher power terms, we get

$$G_i(Q_i) = \frac{Q_i^2}{2a_i} \left\{ 1 - 2 \frac{(\theta_i + b_i)Q_i}{3a_i} \right\}$$

The total number of deteriorating units of the  $i^{\text{th}}$  item is  $S_{di}(Q_i) = \theta_i G_i(Q_i)$

The net revenue for the  $i^{\text{th}}$  item is

$$\begin{aligned} N(Q_i) &= (P_i - C_i)Q_i - P_i S_{di}(Q_i) \\ N(Q_i) &= (P_i - C_i)Q_i - P_i \theta_i G_i(Q_i) \end{aligned} \quad (4)$$

The profit of  $i^{\text{th}}$  item is

$$\begin{aligned} PF_i(Q_i) &= N(Q_i) - C_{li} G_i(Q_i) - C_{3i}, \quad i=1,2,\dots,n. \\ PF_i(Q_i) &= (P_i - C_i)Q_i - P_i \theta_i G_i(Q_i) - C_{li} G_i(Q_i) - C_{3i} \quad i=1,2,\dots,n. \\ PF_i(Q_i) &= (P_i - C_i)Q_i - (C_{li} + P_i \theta_i) G_i(Q_i) - C_{3i}, \quad i=1,2,\dots,n. \end{aligned} \quad (5)$$

The cost of  $i^{\text{th}}$  item is

$$TC_i(Q_i) = C_{li} G_i(Q_i) + C_{3i} + C_i Q_i \quad i=1,2,\dots,n$$

The wastage cost of  $i^{\text{th}}$  item is

$$WC_i(Q_i) = C_i \theta_i G_i(Q_i) \quad i=1,2,\dots,n \quad (6)$$

Hence the problem is

$$\begin{aligned} \text{Maximize } PF(Q_i) &= \sum_{i=1}^n PF_i(Q_i) \\ \text{Minimize } TC(Q_i) &= \sum_{i=1}^n TC_i(Q_i) \\ \text{Minimize } WC(Q_i) &= \sum_{i=1}^n WC_i(Q_i) \end{aligned} \quad (7)$$

$$\begin{aligned} \text{Subject to: } & \sum_{i=1}^n f_i Q_i \leq F \\ & \sum_{i=1}^n C_i Q_i \leq B \end{aligned}$$

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**5.2. Fuzzy model**

When the inventory parameters such as setup cost, holding cost, rate of deterioration, total storage area and maximum amount of investment are fuzzy, the said crisp model (7) is transformed to a fuzzy model and represented as

$$\begin{aligned}
 \text{Max } PF &= \sum_{i=1}^n \left[ (P_i - C_i)Q_i - (\tilde{C}_{1i} + P_i\tilde{\theta}_i)G_i(Q_i) - \tilde{C}_{3i} \right] \\
 \text{Min } TC(Q_i) &= \sum_{i=1}^n \tilde{C}_{1i}G_i(Q_i) + \tilde{C}_{3i} + C_iQ_i \\
 \text{Min } WC(Q_i) &= \sum_{i=1}^n C_i\tilde{\theta}_iG_i(Q_i) \tag{8} \\
 \text{Subject to: } & \sum_{i=1}^n f_iQ_i \leq \tilde{F} \\
 & \sum_{i=1}^n C_iQ_i \leq \tilde{B} \\
 & Q_i \geq 0, \quad i=1,2,\dots,n
 \end{aligned}$$

Now the above fuzzy multi objective inventory problem (FMIP) can be easily transformed into a classical form of a multi objective inventory problem (MOIP) by considering R as a linear ranking function. By implementing the R on the above model (8). We obtain the classical form of MOIP :

$$\begin{aligned}
 \text{Max } PF' &= \sum_{i=1}^n \left[ (P_i - C_i)Q_i - (R(\tilde{C}_{1i}) + P_iR(\tilde{\theta}_i))G_i(Q_i) - R(\tilde{C}_{3i}) \right] \\
 \text{Min } TC' &= \sum_{i=1}^n \left[ R(\tilde{C}_{1i})G_i(Q_i) + R(\tilde{C}_{3i}) + C_iQ_i \right] \\
 \text{Min } WC' &= \sum_{i=1}^n \left[ C_iR(\tilde{\theta}_i)G_i(Q_i) \right] \\
 & \sum_{i=1}^n f_iQ_i \leq R(\tilde{F}) \\
 & \sum_{i=1}^n C_iQ_i \leq R(\tilde{B})
 \end{aligned}$$

**6. Mathematical analysis**

**Fuzzy optimization technique:**

To solve the MOIP (8) we have used the following fuzzy programming technique.

**Step 1:** Solve the multi-objective programming problem as a single objective problem using only one objective at a time and ignoring the rest objectives subject to constraints of storage space. Let X<sup>i</sup> be the optimal solution for the i<sup>th</sup> single objective problem.

**Step 2:** From the results of step 1, determine the corresponding values for every objective at each optimal solution derive. Using all the above optimal values of the objectives in step-1, construct a pay-off matrix (3x3) as follows:

$$\begin{array}{c} \text{PF}(X) \quad \text{TC}(X) \quad \text{WC}(X) \\ \begin{array}{c} X^1 \\ X^2 \\ X^3 \end{array} \left[ \begin{array}{ccc} PF(X^1) & TC(X^1) & WC(X^1) \\ PF(X^2) & TC(X^2) & WC(X^2) \\ PF(X^3) & TC(X^3) & WC(X^3) \end{array} \right] \end{array}$$

Here, the Diagonal elements represent the optimal values of the corresponding objectives. From the pay-off matrix we find lower bounds

$$L_{PF} = \text{Min} (PF(X^1), PF(X^2), PF(X^3)), \quad L_{TC} = \text{Min} (TC(X^1), TC(X^2), TC(X^3)), \quad L_{WC} = \text{Min} (WC(X^1), WC(X^2), WC(X^3)).$$

And the upper bounds,

$$U_{PF} = \text{Max} (PF(X^1), PF(X^2), PF(X^3)), \quad U_{TC} = \text{Max} (TC(X^1), TC(X^2), TC(X^3)), \quad U_{WC} = \text{Max} (WC(X^1), WC(X^2), WC(X^3)).$$

Then the objective summations are estimated as

$$L_{PF} \leq PF(X) \leq U_{PF} \quad L_{TC} \leq TC(X) \leq U_{TC} \quad \text{and} \quad L_{WC} \leq WC(X) \leq U_{WC}.$$

**Step 3:** From step 2, we may find for each objective the value  $L_K$  and  $U_K$  corresponding to the set of solutions. For the multi-objective problem (8), the membership functions  $\mu_{PF}(X)$ ,  $\mu_{TC}(X)$ ,  $\mu_{WC}(X)$ . We have considered linear membership functions which are defined below:

$$\mu_{PF}(X) = \begin{cases} 1 & ; PF(X) > U_{PF} \\ \frac{PF(X) - L_{PF}}{d_{PF}} & ; L_{PF} \leq PF(X) \leq U_{PF} \\ 0 & ; PF(X) < L_{PF} \end{cases}$$

$$\mu_{TC}(X) = \begin{cases} 1 & ; TC(X) < L_{TC} \\ \frac{U_{TC} - TC(X)}{d_{TC}} & ; L_{TC} \leq TC(X) \leq U_{TC} \\ 0 & ; TC(X) > U_{TC} \end{cases}$$

$$\mu_{WC}(X) = \begin{cases} 1 & ; WC(X) < L_{WC} \\ \frac{U_{WC} - WC(X)}{d_{WC}} & ; L_{WC} \leq WC(X) \leq U_{WC} \\ 0 & ; WC(X) > U_{WC} \end{cases}$$

**Step 4:** Use the above membership functions to formulated a crisp non-linear programming model following Zimmermann's approach as

$$\text{Max } \alpha,$$

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$$\begin{aligned} \text{Subject to: } & \mu_{PF}(Q) \geq \alpha \\ & \mu_{TC}(Q) \geq \alpha \\ & \mu_{WC}(Q) \geq \alpha \\ & \sum_{i=1}^n f_i Q_i \leq R(\tilde{F}) \\ & \sum_{i=1}^n C_i Q_i \leq R(\tilde{B}) \\ & 0 \leq \alpha \leq 1, Q_i > 0 \end{aligned}$$

**7. Numerical example**

For the model let us assume

For n=2,

$P_1=10, C_1=7, a_1=110, b_1=0.5, \theta_1=0.025, P_2= 10, C_2=6.75, a_2=100, b_2=0.5, \theta_2=0.03, C_{11}=2, C_{12}=2.2, C_{31}=65, C_{32}=50, B=1800.$

Taking  $\tilde{C}_{11}=(2.03,2.05,2.08); \tilde{C}_{12}=(2.15,2.20,2.3); \tilde{C}_{31}=(60,65,75); \tilde{C}_{32}=(45,50,55); \tilde{\theta}_1=(0.022,0.025,0.03) \tilde{\theta}_2=(0.025,0.028,0.030); \tilde{F}=(155,165,175), \tilde{B}=(1700,1800,1900)$

	PF	TC	WC
PF	497.56036	494.54190	495.579783
TC	1994.711443	1994.3144	1994.356398
WC	16.6919147	16.273999	16.399390

The optimum value of  $\alpha, Q_1^*$  and  $Q_2^*$  are

$\alpha$	$Q_1^*$	$Q_2^*$
0.56736213	40.593675	180.49202

Now the optimal values of the objective functions are

Max PF*	Min TC*	Min WC*
496.25445	1994.4861	16.454804

**8. Conclusion**

In this paper, a real life inventory problem under the investment and storage space constraints in fuzzy environment has been proposed. Various inventory costs, storage space and the maximum amount of investment are taken as triangular fuzzy numbers. Initially the model is defuzzified using Robust’s ranking technique. Then using fuzzy optimization technique the optimum results are obtained. Linear membership function is considered for the fuzzy objective functions.

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