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A Method Based on Intuitionistic Fuzzy Linear Programming for Investment Strategy

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Abstract. This article proposes a new ranking method for ordering Triangular Intuitionistic Fuzzy Number (TIFN) based on area of both membership and non-membership parts of the number. Firstly, the membership part is split into two regions. Then the area between the midpoints of the centroids of the membership part is calculated to order the TIFN. Later on, the same procedure is adopted to calculate the area of non-membership value also. This ranking procedure is applied to solve Intuitionistic Fuzzy Linear Programming Problem (IFLPP). The IFLPP, in which, the objective function may be maximization or minimization and/or the constraints may be equality or inequality is considered and demonstrated using Intuitionistic Fuzzy Dual Simplex Method. Numerical examples are provided to illustrate our new approach.

Keywords: Intuitionistic Fuzzy Linear Programming Problem, Triangular Intuitionistic Fuzzy Number, Intuitionistic Fuzzy Dual Simplex Method.

AMS Mathematics Subject Classification (2010): 90C05, 03E72, 03F55

1. Introduction

The first and most fruitful industrial applications of linear programming are in the petroleum industry, including oil extraction, refining, blending, and distribution. The computational task is then to devise an algorithm for these systems for choosing the best schedule of actions from among the possible alternatives. Since 1947, there have been so many algorithms for solving linear programming problems [11]. Zadeh's paper on Fuzzy sets [21] has influenced many researchers and has been applied to decision making, mathematical programming, fuzzy optimization, and fuzzy linear programming.

In 1986, Atanassov [1] launched the concept of intuitionistic fuzzy sets (IFS) which generalized the fuzzy set theory (FS). Out of several higher order fuzzy sets, IFS [1] have been found to be highly useful to deal with vagueness. Here, the degree of satisfaction and rejection are considered so that the sum of both values is always less than or equal to one. In 1997, Angelov [18] introduced intuitionistic fuzzy optimization (IFO) for linear programming problem in which the non-membership function is considered as the complement of membership function. The main advantages of the IFO problems are that they give the richest apparatus for the formulation of optimization problems and the solution of IFO problems satisfies the objective with a higher degree of determinacy than the fuzzy and crisp cases. In the research field of IFO, first comes ordering an Intuitionistic Fuzzy Number (IFN). Ranking Fuzzy numbers under different concepts are discussed in [10,20]. Also, ranking intuitionistic fuzzy numbers (IFN) are investigated in [13,14,15,17]. In recent years, many researchers have paid great attention to Intuitionistic fuzzy optimization methods and few works are here: Bharati et al. study on the application of IFO in multi objective linear programming [2, 3], IFO applies to medical diagnosis [4, 5], pattern recognition [6], multi objective optimization [12], Mahapatra and Roy have used triangular intuitionistic fuzzy number (TIFN) in reliability evaluation [16] and so on. Li defined a method for solving multi attribute decision making with interval valued intuitionistic fuzzy sets in [7]. Dubey et al. [8, 9] formulated the intuitionistic fuzzy linear programming problem (IFLPP). Being able to deal with vague and imprecise data, IFLPP is growing with branches in all areas.

The motivation of the current study is to give a unique ranking procedure for ranking triangular intuitionistic fuzzy number (TIFN). This proposed ranking procedure is applied to solve IFLPP. IFLPP is a useful tool for understanding complex problems. The IFLPP can be viewed as a resource allocation model in which the objective is to maximize revenue subject to the availability of limited resources. Looking at the problem from this point, the associated dual problem offers interesting economic interpretations of the IFLPP resource allocation model. From the literature survey, we understand that there is a wide opportunity to work in dual problem, as an extension of duality in FLP [19]. So an algorithm based on duality for solving IFLPP is proposed and it is applied to obtain solution for investment planning.

This paper is organized as follows: Section 2 introduces the necessary definitions and concepts of IFLPP and its related terminologies. Section 3 deals with the proposed ranking method and dual simplex algorithm for IFLPP. The effectiveness of the proposed algorithm is illustrated with examples in section 4. Finally, in Section 5, the paper is concluded.

2. Preliminaries

In this section some basic definitions and operations on TIFNs and IFLPP are reviewed.

2.1. Triangular fuzzy number

A fuzzy number $\tilde{A}$ (a_1, a_2, a_3) is a ***Triangular Fuzzy Number*** if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{x-a_3}{a_2-a_3}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

Where a_1, a_2, a_3 are real numbers

2.2. *Intuitionistic fuzzy number (IFN)*

An intuitionistic fuzzy number $\tilde{A}^I$ *is*

i) an intuitionistic fuzzy subset of the real line,

ii) normal, that is, there is some $x_0 \in R$ such that
$\mu_{\tilde{A}^I}(x_0) = 1, \vartheta_{\tilde{A}^I}(x_0) = 0$.

iii) convex for the membership function $\mu_{\tilde{A}^I}(x)$, that is,
$\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2)\right)$
$for\ every\ x_1, x_2 \in R, \lambda \in [0,1]$

iv) concave for the non membership function $\vartheta_{\tilde{A}^I}(x)$, that is,
$\vartheta_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq \max\left(\vartheta_{\tilde{A}^I}(x_1), \vartheta_{\tilde{A}^I}(x_2)\right)$,
$for\ every\ x_1, x_2 \in R, \lambda \in [0,1]$.

2.3. (α, β) – level intervals or (α, β) – cuts

A set of (α, β) – cut generated by IFS $\tilde{A}^I$, where α, β ϵ [0,1] are fixed numbers such that α + β is defined as

$\tilde{A}^I{}_{\alpha,\beta} = \{\langle x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x)\rangle : x \epsilon X, \mu_{\tilde{A}^I}(x) \leq \alpha, \vartheta_{\tilde{A}^I}(x) \leq \beta,$
$\alpha, \beta \epsilon [0,1]\}$

(α, β) – level interval or (α, β) – cut denoted by $\tilde{A}^I{}_{\alpha,\beta}$ is defined as the crisp set of elements of x which belong to $\tilde{A}^I$ at least to the degree α and which does belong to $\tilde{A}^I$ at most to the degree β.

2.4. *Triangular intuitionistic fuzzy number (TIFN)*

A ***Triangular intuitionistic fuzzy number (TIFN)*** $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\vartheta_{\tilde{A}^I}(x)$

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{x-a_3}{a_2-a_3}, & a_2 \leq x \leq a_3 \\ 0, & otherwise \end{cases} \text{ and } \vartheta_{\tilde{A}^I}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, & a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2}, & a_2 \leq x \leq a_3 \\ 1, & otherwise \end{cases}$$

where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and
$\mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \leq 1,\ or\ \mu_{\tilde{A}^I}(x) = \vartheta_{\tilde{A}^I}(x), for\ all\ x \epsilon R$. This TIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3; a_1', a_2, a_3') = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$

2.5. Arithmetic operations of triangular intuitionistic fuzzy number based on (α, β) – cut method:

i) If $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3); (b_1', b_2, b_3')\}$ are two TIFNs, then their sum
$\tilde{A}^I + \tilde{B}^I = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3); (a_1' + b_1', a_2 + b_2, a_3' + b_3')\}$
is also a TIFN.

ii) If $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3); (b_1', b_2, b_3')\}$ are two TIFNs, then their difference
$\tilde{A}^I - \tilde{B}^I = \{(a_1 - b_3, a_2 - b_2, a_3 - b_1); (a_1' - b_3', a_2 - b_2, a_3' - b_1')\}$ is also a TIFN.

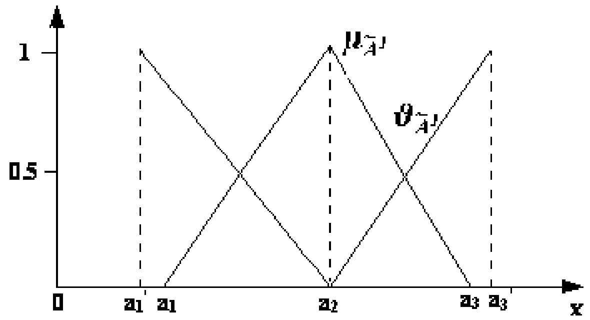

Figure 2.4.1. Membership and non-membership functions of TIFN

iii) If $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3); (b_1', b_2, b_3')\}$ are two TIFNs , then their product

$\tilde{A}^I \, X \, \tilde{B}^I = \{(a_1 b_1, a_2 b_2, a_3 b_3); (a_1' b_1', a_2 b_2, a_3' b_3')\}$ is also a TIFN.

iv) If TIFN $\tilde{A}^I = (a_1, a_2, a_3 ; a_1', a_2, a_3') = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and y = ka (with k > 0) then $\tilde{y}^I = k\tilde{A}^I = \{(ka_1, ka_2, ka_3); (ka_1', ka_2, ka_3')\}$ is also a TIFN.

v) If TIFN $\tilde{A}^I = (a_1, a_2, a_3 ; a_1', a_2, a_3') = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and y = ka (with k < 0) then $\tilde{y}^I = k\tilde{A}^I = \{(ka_3, ka_2, ka_1); (ka_3', ka_2, ka_1')\}$ is also a TIFN.

vi) If $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3); (b_1', b_2, b_3')\}$ are two positive TIFNs , then $\frac{\tilde{A}^I}{\tilde{B}^I}$ is also a TIFN ,

$\frac{\tilde{A}^I}{\tilde{B}^I} = \left\{\left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right), \left(\frac{a_1'}{b_3'}, \frac{a_2}{b_2}, \frac{a_3'}{b_1'}\right)\right\}$.

2.6. Intuitionistic fuzzy linear programming

Linear Programming with Triangular Intuitionistic Fuzzy Variables is defined as

$(IFLP) \max \ \tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^I \ \tilde{x}_j^I$ Subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij}^I \ \tilde{x}_j^I \lesssim \tilde{b}_i^I \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.1)$$

$$\tilde{x}_j^I \geq 0$$

for i = 1,2,....m , j = 1,2,......n , where $\tilde{A}^I = (\tilde{a}_{ij}^I)$, $\tilde{c}^I$, $\tilde{b}^I$, $\tilde{x}^I$ are (mxn), (1xn), (mx1), (nx1) intuitionistic fuzzy matrices consisting of Triangular Intuitionistic Fuzzy Numbers (TIFN).

2.7. *Accuracy function*

Let $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ be a TIFN, then

$H(\tilde{A}^I) = \frac{(a_1 + 2a_2 + a_3) + (a_1' + 2a_2 + a_3')}{8}$,

an accuracy function of $\tilde{A}^I$, is used to defuzzify the given number.

3. Proposed method

3.1. Proposed ranking method

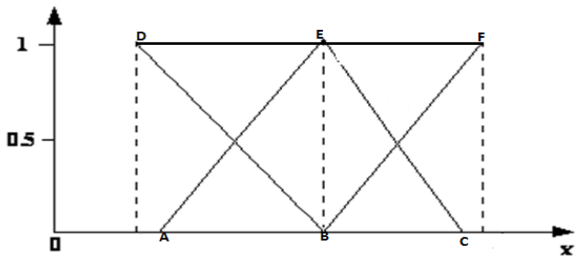

Figure 3.1.1.

Consider the TIFN $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$. Divide the triangle corresponding to the membership function $\mu_{\tilde{A}^I}(x)$ into two plane figures as given in fig. 3.1.1. These two planes are triangles AEB and BEC. Let the centroids of these two triangles be G_1 & G_2 respectively. The centre of these centroids G_1 & G_2 is taken as a point of reference to define the ranking of TIFN. The reason for selecting this point as a point of reference is that each centroid points G_1 & G_2 are balancing points of each individual triangles and the centre of these centroid points is a much more balancing point for a TIFN. The centroids of these plane figures are

$G_1 = \left(\frac{a_1 + 2a_2}{3}, \frac{1}{3}\right)$, $G_2 = \left(\frac{2a_2 + a_3}{3}, \frac{1}{3}\right)$ respectively. The centre of these centroid points are nothing but the midpoint of G_1 and G_2.

$$Therefore\ G = \left(\frac{\frac{a_1 + 2a_2}{3} + \frac{2a_2 + a_3}{3}}{2}, \frac{\frac{1}{3} + \frac{1}{3}}{2}\right) = \left(\frac{a_1 + 4a_2 + a_3}{6}, \frac{1}{3}\right)$$

Now define $S(\mu_{\tilde{A}^I}) = \left(\frac{a_1 + 4a_2 + a_3}{6}\right)\left(\frac{1}{3}\right)$.

Similarly we divide the triangle DBF corresponding to the non-membership function $\vartheta_{\tilde{A}^I}(x)$ into two plane figures. These plane figures are triangles DBE & EBF. Let these triangles be G_1' & G_2' respectively. Then the centre of these centroids of the non-membership part of TIFN is $G' = \left(\frac{a_1' + 4a_2 + a_3'}{6}, \frac{2}{3}\right)$

Now define $S(\vartheta_{\tilde{A}^I}) = \left(\frac{a_1' + 4a_2 + a_3'}{6}\right)\left(\frac{2}{3}\right)$.

Using the above definitions we define the rank of a TIFN. Let $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3); (b_1', b_2, b_3')\}$ be two TIFNs. The working procedure to compare A and B is as follows:

Step 1:

Compute $S(\mu_{\tilde{A}^I})$ & $S(\mu_{\tilde{B}^I})$

Case(i) $If\ S(\mu_{\tilde{A}^I}) > S(\mu_{\tilde{B}^I}), then\ \tilde{A}^I > \tilde{B}^I.$

Case(ii) $If\ S(\mu_{\tilde{A}^I}) < S(\mu_{\tilde{B}^I}), then\ \tilde{A}^I < \tilde{B}^I.$

Case(iii) $If\ S(\mu_{\tilde{A}^I}) = S(\mu_{\tilde{B}^I}), then\ go\ to\ step(2).$

Step 2:

Compute $S(\vartheta_{\tilde{A}^I})$ & $S(\vartheta_{\tilde{B}^I})$

Case(i) $If\ S(\vartheta_{\tilde{A}^I}) > S(\vartheta_{\tilde{B}^I}), then\ \tilde{A}^I > \tilde{B}^I.$

Case(ii) $If\ S(\vartheta_{\tilde{A}^I}) < S(\vartheta_{\tilde{B}^I}), then\ \tilde{A}^I < \tilde{B}^I.$

Case(iii) $If\ S(\vartheta_{\tilde{A}^I}) = S(\vartheta_{\tilde{B}^I}), then\ \tilde{A}^I = \tilde{B}^I.$

3.2. Duality

The dual of IFLPP given in equation (2.1) is

$$\min\ \tilde{Z}^{*I} = \sum_{i=1}^{m} \tilde{b}_i^I\ \tilde{y}_i^I$$

Subject to
$$\sum_{i=1}^{m} \tilde{a}_{ij}^{I\ T}\ \tilde{x}_j^I \gtrsim \tilde{c}_j^I \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$
$$\tilde{y}_i^I \geq 0$$

i = 1,2,....m , j = 1,2,......n , where $\tilde{A}^{I^T} = (\tilde{a}_{ij}^I)^T$, $\tilde{c}^I$, $\tilde{b}^I$, $\tilde{x}^I$ are (nxm), (1xn), (mx1), (nx1) intuitionistic fuzzy matrices consisting of triangular intuitionistic fuzzy numbers (TIFN).

3.3. Rules between primal and dual

The Table 3.3.1 gives the correspondence rules between Primal and Dual IFLPPs.

Table 3.3.1

S.No	**Primal**	**Dual**
1.	Objective : Minimize	Objective : Maximize
2.	Objective co-efficients	RHS of dual
3.	RHS of Primal	Objective co-efficients
4.	Co-efficient matrix	Transposed co-efficient matrix
5.	Primal relation i) (i^{th}) inequality : ≥ ii) (i^{th}) inequality : ≤ iii) (i^{th}) equation : =	Dual variable i) $\tilde{y}_i^I \geq 0$ ii) $\tilde{y}_i^I \leq 0$ iii) $\tilde{y}_i^I$ unrestricted in sign
6.	Primal Variable i) $\tilde{x}_j^I \geq 0$ ii) $\tilde{x}_j^I \leq 0$ iii) $\tilde{x}_j^I$ unrestricted in sign	Dual relation i) (j^{th}) inequality : ≤ ii) (j^{th}) inequality : ≥ iii) (j^{th}) equation : =

3.4. Theorem

The dual problem of a dual is a primal problem.

Proof: Consider IFLPP as stated in (2.1), which is the primal problem.

$$\max\ \tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^I\ \tilde{x}_j^I \quad \text{Subject to} \quad \sum_{j=1}^{n} \tilde{a}_{ij}^I\ \tilde{x}_j^I \lesssim \tilde{b}_i^I, \quad \tilde{x}_j^I \geq 0$$

The dual of this IFLPP is given in (3.1), as follows:

$$\min\ \tilde{Z}^{*I} = \sum_{i=1}^{m} \tilde{b}_i^I\ \tilde{y}_i^I \quad \text{Subject to} \quad \sum_{i=1}^{m} \tilde{a}_{ij}^{I\ T}\ \tilde{x}_j^I \gtrsim \tilde{c}_j^I, \quad \tilde{y}_i^I \geq 0 \quad \ldots\ldots\ldots (3.2)$$

The dual of the IFLPP (3.2), is as follows:

$$\max \ \tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^I \ \tilde{x}_j^I \quad \text{Subject to} \quad \begin{array}{r} \sum_{j=1}^{n} \tilde{a}_{ij}^I \ \tilde{x}_j^I \lesssim \tilde{b}_i^I \\ \tilde{x}_j^I \ \geq \ 0 \end{array}$$

Hence the proof.

3.5. Intuitionistic fuzzy dual simplex algorithm for solving IFLPP

In the dual simplex, the IFLPP starts at a better optimal solution but infeasible. Successive iterations remain infeasible but better than optimal, until feasibility is restored at the last iteration.

Step 1: Convert the problem into a maximization problem, if it is initially in the minimization form.

Step 2: Convert the equations, if any, into inequalities. (i.e., if $\tilde{a}_{ij}^I \tilde{x}_j^I = \tilde{b}_i^I$ for some i & for some j, then convert it into two inequalities as $\tilde{a}_{ij}^I \tilde{x}_j^I \lesssim \tilde{b}_i^I$& $\tilde{a}_{ij}^I \tilde{x}_j^I \gtrsim \tilde{b}_i^I$).

Step 3: Convert ≥ type constraints, if any, into ≤ type by multiplying both sides of such constraints by -1.

Step 4: Convert all the inequalities obtained after step (2) & step (3) into equations by adding of slack variables and obtain the initial intuitionistic fuzzy feasible solution. Express the above information in the form of a table known as the Intuitionistic Fuzzy Dual Simplex table.

Step 5: Compute $\tilde{c}_j^I - \tilde{z}_j^I$ for every column. Three cases arise:

Case(i) If all $\tilde{c}_j^I - \tilde{z}_j^I$ are either negative or zero and all $\tilde{b}_j^I$ are non-negative, the solution obtained above is the optimal intuitionistic feasible solution.

Case(ii) If all $\tilde{c}_j^I - \tilde{z}_j^I$ are either negative or zero and at least one $\tilde{b}_j^I$ is negative, then go to step(6).

Case(iii) If any $\tilde{c}_j^I - \tilde{z}_j^I$ is positive, the method fails.

Step 6: Intuitionistic fuzzy dual feasibility condition

Select the row that contains the most negative $\tilde{b}_j^I$. This row is called the key row or the pivot row. The corresponding basic variable leaves the basis.

Step 7: Intuitionistic fuzzy optimality condition

i) If all the elements of the key row are positive, the problem does not have a feasible solution.

ii) If at least one element is negative, find the ratio of the corresponding elements of $\tilde{c}_j^I - \tilde{z}_j^I$ row to these elements. Ignore the ratios associated with positive or zero elements of the key row. Choose the smallest of these ratios. i.e., compute $\theta = min\left\{\frac{\tilde{c}_j^I - \tilde{z}_j^I}{\tilde{a}_{ik}^I} \ , \ \tilde{a}_{ik}^I > 0\right\}$.

The corresponding column $\tilde{x}_k^I$ is the key column, and the associated variable is the entering variable.

Step 8: Make the key element unity. Perform the row operations as in the Simplex method for solving IFLPP and repeat the steps (5) to (7) until either an optimal feasible solution is obtained or it is indicated that there exists no feasible solution.

4. Numerical illustration

Example 4.1.

Let $\tilde{A}^I$ = {(1,2,3);(0.5,2,3.5)} and $\tilde{B}^I$ = {(2,3,4);(1,3,4.5)} be two TIFNs.

Here $S(\mu_{\tilde{A}^I}) = 0.67$, $S(\mu_{\tilde{B}^I}) = 1$

$since\ S(\mu_{\tilde{A}^I}) < S(\mu_{\tilde{B}^I}), we\ get\ \tilde{A}^I < \tilde{B}^I.$

Example 4.2.

Let $\tilde{A}^I = \{(0.8,1,1.2); (0.5, 1,1.5)\}$ and $\tilde{B}^I = \{(0.7,1,1.3); (0.2,1,1.3)\}$ be two TIFNS. Here $S(\mu_{\tilde{A}^I}) = 0.33$, $S(\mu_{\tilde{B}^I}) = 0.33$. Since both are equal, we compute for non-member $S(\vartheta_{\tilde{A}^I}) = 0.66$ & $S(\vartheta_{\tilde{B}^I}) = 0.61$.

i.e., $S(\vartheta_{\tilde{A}^I}) > S(\vartheta_{\tilde{B}^I})$. Hence $\tilde{A}^I > \tilde{B}^I$.

Example 4.3.

An individual wishes to invest his savings over the next year in two types of investments: Investment Plan A & Investment Plan B. Based on statistical surveys, he decided to invest twice the amount in Investment Plan A as in Investment Plan B, so as to yield at least a minimum profit without any loss. The following table provides the available policies in each investment plan and their respective claim(s), and the data are intuitionistic in nature

Policies	Investment Plan A (in lakhs)	Investment Plan B (in lakhs)
Risk cover	$\tilde{3}^I = \{(2.9,3,3.1); (3.8,3,3.2)\}$	$\tilde{1}^I = \{(0.9,1,1.2); (0.8,1,1.3)\}$
Home loan	$\tilde{4}^I = \{(3.8,4,4.1); (3.7,4,4.2)\}$	$\tilde{3}^I = \{(2.9,3,3.1); (2.7,3,3.2)\}$
Higher Education	$\tilde{1}^I = \{(0.9,1,1.1); (0.7,1,1.2)\}$	$\tilde{2}^I = \{(1.9,2,2.1); (1.7,2,2.2)\}$

The government has fixed some common criteria for each claim for every individual, it doesn't matter on the number of Plans he or she has. They are

i) Minimum Rs. $\tilde{3}^I = \{(2.9,3,3.2); (2.7,3,3.3)\}$ lakhs have to be given for risk coverage

ii) Each individual can claim Rs. $\tilde{6}^I = \{(5.9,6,6.2); (5.7,6,6.3)\}$ lakhs and above for housing loan

iii) At least $\tilde{3}^I = \{(2.8,3,3.1); (2.7,3,3.3)\}$ lakhs rupees have to be given for higher education

Suggest an optimum solution to the investor

Solution: Let the investor decide to invest in $\tilde{x}_1^I$ quantity in Plan A and $\tilde{x}_2^I$ quantity in Plan B.

Mathematical formulation of the given problem:

$$Min\ \tilde{z}^I = \tilde{2}^I\tilde{x}_1^I + \tilde{1}^I\tilde{x}_2^I$$

$Subject\ to\ \tilde{3}\tilde{x}_1^I + \tilde{1}^I\tilde{x}_2^I \underset{\sim}{\geq} \tilde{3}^I,\ \tilde{4}^I\tilde{x}_1^I + \tilde{3}^I\tilde{x}_2^I \underset{\sim}{\geq} \tilde{6}^I,\ \tilde{1}^I\tilde{x}_1^I + \tilde{2}^I\tilde{x}_2^I \underset{\sim}{\geq} \tilde{3}^I,$

$\tilde{x}_1^I,\ \tilde{x}_2^I \geq 0$ where

$\tilde{c}_1^I = \tilde{2}^I = \{(1.8,2,2.1);(1.7,2,2.2)\}$

$\tilde{c}_2^I = \tilde{1}^I = \{(0.9,1,1.1);(0.7,1,1.2)\}$

$\tilde{a}_{11}^I = \tilde{3}^I = \{(2.9,3,3.1);(3.8,3,3.2)\}$ $\quad \tilde{a}_{12}^I = \tilde{1}^I = \{(0.9,1,1.2);(0.8,1,1.3)\}$

$\tilde{a}_{21}^I = \tilde{4}^I = \{(3.8,4,4.1);(3.7,4,4.2)\}$ $\quad \tilde{a}_{22}^I = \tilde{3}^I = \{(2.9,3,3.1);(2.7,3,3.2)\}$

$\tilde{a}_{31}^I = \tilde{1}^I = \{(0.9,1,1.1);(0.7,1,1.2)\}$ $\quad \tilde{a}_{32}^I = \tilde{2}^I = \{(1.9,2,2.1);(1.7,2,2.2)\}$

$\tilde{b}_1^I = \tilde{3}^I = \{(2.9,3,3.2);(2.7,3,3.3)\}$ $\quad \tilde{b}_2^I = \tilde{6}^I = \{(5.9,6,6.2);(5.7,6,6.3)\}$

$\tilde{b}_3^I = \tilde{3}^I = \{(2.8,3,3.1);(2.7,3,3.3)\}$

Rewrite the problem in standard form:

$Maximize: \tilde{w}^I = -\tilde{2}^I\tilde{x}_1^I - \tilde{1}^I\tilde{x}_2^I + \tilde{0}^I\tilde{x}_3^I + \tilde{0}^I\tilde{x}_4^I + \tilde{0}^I\tilde{x}_5^I$

$Subject\ to - \tilde{3}\tilde{x}_1^I - \tilde{1}^I\tilde{x}_2^I + \tilde{1}^I\tilde{x}_3^I = -\tilde{3}^I, -\tilde{4}^I\tilde{x}_1^I - \tilde{3}^I\tilde{x}_2^I + \tilde{1}^I\tilde{x}_4^I = -\tilde{6}^I,$

$-\tilde{1}^I\tilde{x}_1^I - \tilde{2}^I\tilde{x}_2^I + \tilde{1}^I\tilde{x}_5^I = -\tilde{3}^I,\ \tilde{x}_1^I,\ \tilde{x}_2^I, \tilde{x}_3^I, \tilde{x}_4^I, \tilde{x}_5^I \geq 0$

Here the co-efficient of $\tilde{x}_3^I, \tilde{x}_4^I, \tilde{x}_5^I$ are given by

$\tilde{1}^I = \{(1,1,1);(1,1,1)\}\ and\ \tilde{0}^I = \{(0,0,0);(0,0,0\}$.

Using the arithmetic operations in the section (2.4)& (2.5) of this paper , and applying the proposed algorithm the following results are obtained :

Final Iteration:

	$\tilde{c}_j^I$	$-\tilde{2}^I$	$-\tilde{1}^I$	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$	
C_B	BV	$\tilde{x}_1^I$	$\tilde{x}_2^I$	$\tilde{x}_3^I$	$\tilde{x}_4^I$	$\tilde{x}_5^I$	$\widetilde{X}_B^{-I}$
$-\tilde{2}^I$	$\tilde{x}_1^I$	$\tilde{1}^I$	$\tilde{0}^I$	$-\widetilde{0.6}^I$	$\widetilde{0.19}^I$	$\tilde{0}^I$	$\widetilde{0.6}^I$
$-\tilde{1}^I$	$\tilde{x}_2^I$	$\tilde{0}^I$	$\tilde{1}^I$	$\widetilde{0.8}^I$	$-\widetilde{0.58}^I$	$\tilde{0}^I$	$\widetilde{1.2}^I$
$\tilde{0}^I$	$\tilde{x}_5^I$	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{1}^I$	$-\widetilde{0.97}^I$	$\tilde{1}^I$	$\tilde{1}^I$
	$\tilde{w}_j^I$	$-\tilde{2}^I$	$-\tilde{1}^I$	$\widetilde{0.4}^I$	$\widetilde{0.2}^I$	$\tilde{0}^I$	$-\widetilde{2.4}^I$
	$\tilde{c}_j^I - \tilde{w}_j^I$	$\tilde{0}^I$	$\tilde{0}^I$	$-\widetilde{0.4}^I$	$-\widetilde{0.2}^I$	$\tilde{0}^I$	-

The co-efficient of first row corresponding to $\tilde{x}_1^I$ are:

$\tilde{1}^I = \{(0.79,1,1.26);(0.61,1,1.64)\}$; $\quad \tilde{0}^I = \{(-0.11,0,0.14);(-0.3,0,0.22)\}$

$-\widetilde{0.6}^I = -\{(0.53,0.6,0.67);(0.48,0.6,0.8)\}$;

$\widetilde{0.19}^I = \{(0.17,0.19,0.22);(0.15,0.19,0.29)\}$;

$\tilde{0}^I = \{(0,0,0);(0,0,0)\}$; $\quad \widetilde{0.6}^I = \{(0.37,0.6,0.87);(0.18,0.6,1.21)\}$

The co-efficient of second row corresponding to $\tilde{x}_2^I$ are:

$\tilde{0}^I = \{(-0.45,0,0.36);(-1.03,0,0.74)\}$; $\quad \tilde{1}^I = \{(0.75,1,1.2);(0.55,1,2.2)\}$

$\widetilde{0.8}^I = \{(0.7,0.8,0.89);(0.63,0.8,1.06)\}$;

$-\widetilde{0.58}^I = -\{(0.54,0.58,0.63);(0.51,0.58,0.75)\}$

$\tilde{0}^I = \{(0,0,0);(0,0,0)\}$; $\quad \widetilde{1.2}^I = \{(0.75,1.2,1.64);(0.18,1.2,2.1)\}$

The co-efficient of third row corresponding to $\tilde{x}_5^I$ are:

$\tilde{0}^I = \{(-0.75,0,0.62);(-1.62,0,1.09)\}$;

$\tilde{0}^I = \{(-0.47,0,0.4);(-0.88,0,1.15)\}$; $\quad \tilde{1}^I = \{(0.88,1,1.11);(0.8,1,1.32)\}$

$-\widetilde{0.97}^I = -\{(0.92,0.97,1.04);(0.87,0.97,1.22)\}$; $\quad \tilde{1}^I = \{(1,1,1);(1,1,1)\}$

$\tilde{0}^I = \{(-0.74, 0, 0.85); (-1.74, 0, 1.67)\}$

The co-efficient of fourth row corresponding to $\tilde{w}_j^I$ are:

$-\tilde{2}^I = -\{(1.1, 2, 3.04); (0.31, 2, 4.48)\}$;

$-\tilde{1}^I = -\{(0.47, 1, 1.61); (-0.12, 1, 3.12)\}$;

$\widetilde{0.4}^I = \{(0.03, 0.4, 0.77); (-0.46, 0.4, 1.32)\}$

$\widetilde{0.2}^I = \{(0.02, 0.2, 0.37); (-0.28, 0.2, 0.65)\}$

$\tilde{0}^I = \{(0,0,0); (0,0,0)\}$; $\quad -\widetilde{2.4}^I = -\{(1.37, 2.4, 3.62); (0.42, 2.4, 5.16)\}$

The co-efficient of fifth row corresponding to $\tilde{c}_j^I - \tilde{w}_j^I$ are:

$\tilde{0}^I = \{(-1, 0, 1.14); (-1.89, 0, 2.78)\}$;

$\tilde{0}^I = \{(-0.63, 0, 0.08); (-1.32, 0, 2.42)\}$;

$-\widetilde{0.4}^I = -\{(0.3, 0.4, 0.7); (-0.46, 0.4, 1.32)\}$;

$-\widetilde{0.2}^I = -\{(0.02, 0.2, 0.37); (-0.28, 0.2, 0.65)\}$; $\quad \tilde{0}^I = \{(0,0,0); (0,0,0)\}$

The optimum solution is $\max \tilde{w}^I = -\widetilde{2.4}^I$, $\tilde{x}_1^I = \widetilde{0.6}^I$, $\tilde{x}_2^I = \widetilde{1.2}^I$

Therefore, $\min \tilde{z}^I = -\max \tilde{w}^I = \widetilde{2.4}^I$, $\tilde{x}_1^I = \widetilde{0.6}^I$, $\tilde{x}_2^I = \widetilde{1.2}^I$

The investor is suggested to invest 0.6 lakhs rupees in Plan A and 1.2 lakhs rupees in Plan B to get a guaranteed return of 2.4 lakhs rupees.

5. Conclusion

In this work, a new method that ranks Triangular Intuitionistic Fuzzy Number is proposed, which is simple and concrete. Though dual simplex methods are not recent, we have adopted them to solve Intuitionistic Fuzzy Linear Programming Problem because any kind of objectives and constraints can be solved without introducing any artificial variables and artificial constraints. Finally, numerical illustrations are carried out to describe the proposed study. In the future, we will continue working on the application of the proposed method to other domains.

REFERENCES

1. K.T.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986) 87-96.
2. S.K.Bharati and S.R.Singh, Solving multi-objective linear programming problems using intuitionistic fuzzy optimization method: A comparative study, *International Journal of Modeling and Optimization*, 4(1) (2014) 10-16.
3. S.K.Bharati, A.K.Nishad and S.R.Singh, Solution of multi-objective linear programming problems in intuitionistic fuzzy environment, *Advances in Intelligent Systems & Computing*, 236 (2014) 161-171.
4. T.Chaira, A novel intuitionistic C means clustering algorithm and its application to medical images, *Applied Soft Computing Journal*, 11(2) (2011) 1711-1717.
5. S.K.De, R.Biswas and A.R.Roy, An application of intuitionistic fuzzy systems in medical diagnosis, *Fuzzy Sets and Systems*, 117(2) (2001) 209-213.
6. D.F.Li and C.Chuntian, New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions, *Pattern Recognition Letters*, 23(1) (2002) 221-225.
7. D.F.Li, Linear Programming method for MADM with interval-valued intuitionistic fuzzy sets, *Expert Systems with Applications*, 37(8) (2010) 5939-5945.

8. D.Dubey and A.Mehra, Linear programming with triangular intuitionistic fuzzy number, *Advances in Intelligent Systems Research*, 1(1) (2011) 563-569.
9. D.Dubey, S.Chandra and A.Mehra, Fuzzy linear programming under uncertainty based on intuitionistic fuzzy representation, *Fuzzy Sets and Systems,* 188(1) (2012) 68-87.
10. D.Dubois and H.Prade, The mean value of a fuzzy number, *Fuzzy Sets and Systems*, 24 (1987) 279-300.
11. G.B.Dantzig and M.N.Thapa, Linear Programming, Springer Series in Operation Research, 1997.
12. H.Garg, M.Rani, S.P.Sharma and Y.Vishwakarma, Intuitionistic fuzzy optimization technique for solving multi-objective reliability optimization problems in interval environment, *Expert Systems with Applications*, 14(7) (2014) 3157-3167.
13. H.M.Nehi, A new ranking method for Intuitionistic Fuzzy Numbers, *International Journal of Fuzzy Systems*, 12(1) (2010) 80-86.
14. L.G.Nayagam.V, G.Venkateswari and G.Sivaraman, Modified ranking of intuitionistic fuzzy numbers, *Notes on IFS*, 17(1) (2010) 5-22.
15. L.G.Nayagam.V and G.Sivaraman, Ranking of interval-valued intuitionistic fuzzy sets, *Applied Soft Computing*, 11(4) (2011) 3368-3372.
16. G.S.Mahapatra and T.K.Roy, Reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations, *World Academy of Science, Engineering and Technology*, 50 (2009) 574-581.
17. H.B.Mitchall, Ranking–intuitionistic fuzzy numbers, *International Journal of Fuzzy Systems,* 12(1) (2004) 377-386.
18. P.P.Angelov, Optimization in an intuitionistic fuzzy environment, *Fuzzy Sets and Systems*, 80 (1997) 299-306.
19. W.Rodder and H.J.Zimmermann, Duality in fuzzy linear programming, *External Methods and System Analysis*, Berlin, New York, 1980.
20. J.S.Su, Fuzzy programming based on interval valued fuzzy numbers and ranking, *International Journal of Contemporary Mathematical Sciences*, 2 (2007) 393-410.
21. L.A. Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338-353.