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On K Indices of Graphs

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Abstract: In this paper, we introduce the first and second K indices of a graph. The first K index is defined as the sum of squares of the sum of the edge degrees of adjacent edges. The second K index is defined as the sum of the product of squares of the edge degrees of the adjacent edges. In this paper, some mathematical properties of the first K index of a graph are presented. Also some bounds for the first K index of a graph are established.

Keywords: reformulated Zagreb indices, K^* edge index, first K index, second K index.

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1. Introduction

Let G be a simple graph with n vertices and m edges with vertex set $V(G)$ and edge set $E(G)$. A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences.

In Chemical Science, the physico-chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [1].

The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e)= d_G(u)+ d_G(v) – 2$ with $e=uv$, $e\sim f$ means that the edges e and f are adjacent. The line graph $L(G)$ of a graph G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent.

The first and second Zagreb indices of a graph G are defined as

$$M_1(G)=\sum_{u\in V(G)} d_G(u)^2 \text{ and } M_2(G)=\sum_{uv\in E(G)} d_G(u)d_G(v).$$

These indices were introduced by Gutman *et al*. in [1].

In fact, one can rewrite the first Zagreb index as

$$M_1(G)=\sum_{uv\in E(G)} \left[d_G(u)+d_G(v)\right].$$

These indices in Chemical graph theory were studied, for example, in [2, 3, 4, 5, 6].

The reformulated first and second Zagreb indices of a graph G are defined as

$$EM_1(G)=\sum_{e\in E(G)} d_G(e)^2 \text{ and } EM_2(G)=\sum_{e\sim f\in E(G)} d_G(e)d_G(f).$$

These indices were introduced by Milićević *et al.* in [7]. The reformulated first and second Zagreb indices in chemical graph theory were studied, for example, in [8, 9, 10].

The K^*-edge index of a graph G is defined as

$$K_e^*(G)=\sum_{e\sim f\in E(G)}\left[d_G(e)^2+d_G(f)^2\right].$$

The K^*-edge index of G was introduced by Kulli in [11].

In this paper, we introduce the K indices of a graph and some properties of the K indices are obtained.

2. First and second *K* indices

We define K indices of a graph is terms of edge degrees where the degree of an edge e is defined as $d(e) = d(u) + d(v) - 2$ with $e = uv$.

Definition 1. The first and second K indices (called as Kulli indices) of a graph G are defined as

$$K^1(G)=\sum_{e\sim f\in E(G)}\left[d(e)+d(f)\right]^2$$

$$K^2(G)=\sum_{e\sim f\in E(G)}\left[d(e)d(f)\right]^2$$

where $e\sim f$ means that the edges e and f are adjacent .

3. Properties of first *K* index

Theorem 2. Let G be a simple graph. Then

$$K^1(G)=K_e^*(G)+2EM_2(G). \tag{1}$$

Proof: By the definition of the K^1 index, we have

$$\begin{aligned}K^1(G)&=\sum_{e\sim f\in E(G)}\left[d_G(e)+d_G(f)\right]^2\\&=\sum_{e\sim f\in E(G)}\left[d_G(e)^2+d_G(f)^2+2d_G(e)d_G(f)\right]\\&=\sum_{e\sim f\in E(G)}\left[d_G(e)^2+d_G(f)^2\right]+2\sum_{e\sim f\in E(G)}\left[d_G(e)d_G(f)\right].\end{aligned}$$

Therefore $K^1(G)=K_e^*(G)+2EM_2(G)$.

Corollary 2.1. Let G be a simple graph. Then

$$EM_2(G)=\frac{1}{2}\left(K^1(G)-K_e^*(G)\right).$$

4. Lower Bounds for *K*[1](G)

In this section, we give two lower bounds for $K^1(G)$.

Ilić and Zhou in [9] obtained the following inequality.

Theorem 3. Let G be a simple graph with n vertices and m edges. Then

$$EM_2(G) \ge EM_1(G) - \frac{1}{2}M_1(G) + m. \quad (2)$$

We establish a lower bound for $K^1(G)$.

Theorem 4. Let G be a simple graph with n vertices and m edges. Then

$$K^1(G) \ge K_e^*(G) + 2EM_1(G) - M_1(G) + 2m. \quad (3)$$

Furthermore, equality holds if and only if each component of G is P_3.

Proof: Using (2) in (1), we obtain the desired result.

In [9], Ilić and Zhou gave the following inequality.

Theorem 5. Let G be a simple graph with n vertices and m edges. Then

$$EM_2(G) \ge \frac{1}{2m^2}\left(M_1(G) - 2m\right)^3 \quad (4)$$

with equality if and only if L (G) is regular.

We now obtain another lower bound for $K^1(G)$.

Theorem 6. Let G be a simple graph with n vertices and m edges. Then

$$K^1(G) \ge K_e^*(G) + \frac{1}{m^2}\left(M_1(G) - 2m\right)^3. \quad (5)$$

Furthermore, equality holds if and only if *L(G)* is regular.

Proof: Using (4) in (1), we find the desired result. Obviously, equality holds in (5) if and only if *L(G)* is regular.

5. Upper bound for $K^1(G)$

In this section, we give an upper bound for $K^1(G)$.

In [8], De proved the following inequality.

Theorem 7. Let G be simple connected graph with n vertices and m edges. Then

$$EM_2(G) \le \frac{1}{2}\left(M_1(G) - 2m\right)^2 - (m-1)(\delta-1)\left(M_1(G) - 2m\right) + \frac{1}{2}(2\delta-3)EM_1(G). \quad (6)$$

with equality holds if and only if G is regular.

We now give an upper bound for $K^1(G)$.

Theorem 8. Let G be a simple connected graph with n vertices and $m\geq 2$ edges. Then

$$K^1(G) \le K_e^*(G) + \left(M_1(G) - 2m\right)^2 - 2(m-1)(\delta-1)\left(M_1(G) - 2m\right) + (2\delta-3)EM_1(G). \quad (7)$$

Furthermore, equality in (7) holds if and only if G is regular.

Proof: From (1) and (6), we obtain the inequality (7). Obviously equality in (7) holds if G is regular.

6. Other bounds for $K^1(G)$

In [8], De obtained the following result.

Theorem 9. Let *G* be a simple connected graph with *n* vertices and *m* edges. Then

$$2\left(M_1(G)-2m\right)(\delta-1)^2 \le EM_2(G) \le 2\left(M_1(G)-2m\right)(\Delta-1)^2. \tag{8}$$

We now give lower and upper bounds for $K^1(G)$.

Theorem 10. Let *G* be a simple connected graph with *n* vertices and *m* edges. Then

$$K_e^*(G)+4\left(M_1(G)-2m\right)(\delta-1)^2 \le K^1(G) \le K_e^*(G)+4\left(M_1(G)-2m\right)(\Delta-1)^2. \tag{9}$$

Furthermore, equality holds if and only if *G* is regular.

Proof: From (8), we have

$$2\left(M_1(G)-2m\right)(\delta-1)^2 \le EM_2(G) \le 2\left(M_1(G)-2m\right)(\Delta-1)^2.$$

Therefore

$$K_e^*(G)+4\left(M_1(G)-2m\right)(\delta-1)^2 \le K_e^*(G)+2EM_2(G) \le K_e^*(G)+4\left(M_1(G)-2m\right)+(\Delta-1)^2.$$

Thus $K_e^*(G)+4\left(M_1(G)-2m\right)(\delta-1)^2 \le K^1(G) \le K_e^*(G)+4\left(M_1(G)-2m\right)+(\Delta-1)^2$.

Obviously in (9), equality holds if *G* is regular.

In [8], De obtained the following result.

Theorem 11. Let *G* be a simple connected graph with *n* vertices and *m* edges. Then

$$(\delta-1)EM_1(G) \le EM_2(G) \le (\Delta-1)EM_1(G) \tag{10}$$

with equality holds if and only if *G* is regular.

We now obtain another lower and upper bounds for $K^1(G)$.

Theorem 12. Let *G* be a simple graph with *n* vertices and *m* vertices. Then

$$K_e^*(G)+2(\delta-1)EM_1(G) \le K^1(G) \le K_e^*(G)+2(\Delta-1)EM_1(G). \tag{11}$$

Furthermore, equality holds if and only if *G* is regular.

Proof: From (10), we have

$$(\delta-1)EM_1(G) \le EM_2(G) \le (\Delta-1)EM_1(G).$$

Therefore $K_e^*(G)+2(\delta-1)EM_1(G) \le K_e^*(G)+2EM_2(G) \le K_e^*(G)+2(\Delta-1)EM_1(G)$.

Thus $K_e^*(G)+2(\delta-1)EM_1(G) \le K^1(G) \le K_e^*(G)+2(\Delta-1)EM_1(G)$.

Obviously equality holds when *G* is regular.

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