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On K Edge Index and Coindex of Graphs

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Abstract: The *F*-index of a graph is defined as the sum of the cubes of the vertex degrees of a graph. It is known that this index can enhance the physico-chemical applicability of Zagreb index. In this paper, we introduce a new invariant the *K*-edge index which is defined as the sum of the cubes of the edge degrees of a graph. Also, we introduce another new invariant the *K*-edge coindex. We initiate a study of these new invariants.

Keywords: Zagreb indices, reformulated Zagreb indices, *K*-edge index, *K*-edge coindex

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

Let $G = (V, E)$ be a simple graph with $|V| = n$ vertices and $|E| = m$ edges.

The degree of a vertex v, denoted by $d_G(v)$, is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Let $\overline{G}$ be the complement of a graph G and $\left|E\left(\overline{G}\right)\right| = \overline{m} = \frac{n(n-1)}{2} - m$ and $d_{\overline{G}}(u) = n-1-d_G(u)$ for $u \in V$.

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences.

In Chemical Science, the physico-chemical properties of chemical compounds are often modeled by means of molecular-graph-based structure- descriptors, which are also referred to as topological indices, see [1].

The first and second Zagreb indices are defined as

$$M_1(G) = \sum_{u\in V(G)} d_G(u)^2 \text{ and } M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

These indices were introduced by Gutman *et al.* in [1]. Also the K indices and K coindices were introduced in [2] and [3] respectively.

In fact, one can rewrite the first Zagreb index as

$$M_1(G) = \sum_{uv\in E(G)} \left[d_G(u) + d_G(v)\right].$$

The forgotten topological index F_1 is defined [1, 4] as

$$F_1(G)=\sum_{u\in V(G)} d_G(u)^3=\sum_{uv\in E(G)}\left[d_G(u)^2+d_G(v)^2\right].$$

The forgotten topological index was studied, for example, in [5].

The first and second reformulated Zagreb indices are defined as

$$EM_1(G)=\sum_{e\in E(G)} d_G(e)^2,\ \ EM_2(G)=\sum_{e\sim f\in E(G)} d_G(e)d_G(f)$$

where $d_G(e)$ denotes the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$ and $e \sim f$ means that the edges e and f are adjacent.

The reformulated Zagreb indices were introduced by Milićević *et al*. in [6] and were studied, for example, in [7, 8, 9, 10].

The first *Ka*-index and the first *Ka*-coindex of a graph G are defined as

$$Ka(G)=\sum_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)^a$$

$$\overline{K}a(G)=\sum_{uv\notin E(G)}\left(d_G(u)+d_G(v)\right)^a.$$

These were introduced by Kulli in [11].

Terminology and notation not given here may be found in [12].

2. Some measures of irregularity

Definition 1. A graph whose all edge degrees are mutually equal is said to be edge regular.

We now define the irregular edge indices of a graph.

Definition 2. The first irregular edge index $IE_1(G)$ of a graph G is defined as

$$IE_1(G)=\sum_{e\sim f\in E(G)}\left|d_G(e)-d_G(f)\right|.$$

Definition 3. The second irregular edge index $IE_2(G)$ of a graph G is defined as

$$IE_2(G)=\sum_{e\sim f\in E(G)}\left[d_G(e)-d_G(f)\right]^2.$$

Also we define the irregular edge coindices of a graph.

Definition 4. The first irregular edge coindex $\overline{IE}_1(G)$ of a graph G is defined as

$$\overline{IE}_1(G)=\sum_{e\nsim f\in E(G)}\left|d_G(e)-d_G(f)\right|.$$

Definition 5. The second irregular edge coindex $\overline{IE}_2(G)$ of a graph G is defined as

$$\overline{IE}_2(G)=\sum_{e\nsim f\in E(G)}\left[d_G(e)-d_G(f)\right]^2.$$

3. The *K*-edge index of a graph

We define the *K*-edge index of a graph.

Definition 6. The *K*-edge index of a graph *G* is defined as

$$K_e(G) = \sum_{e \in E(G)} d_G(e)^3.$$

i.e. The *K*-edge index of a graph is the sum of the cubes of the edge degrees of a graph.

Definition 7. The K^*-edge index of a graph *G* is defined as

$$K_e^*(G) = \sum_{e \sim f \in E(G)} \left[d_G(e)^2 + d_G(f)^2 \right].$$

We now define the *K*-edge coindex of a graph.

Definition 8. The *K*-edge coindex of a graph *G* is defined as

$$\overline{K}_e(G) = \sum_{e \nsim f \in E(G)} \left[d_G(e)^2 + d_G(f)^2 \right].$$

Example 9. Consider methylcyclohexane and its molecular graph *G*, see Figure 1.

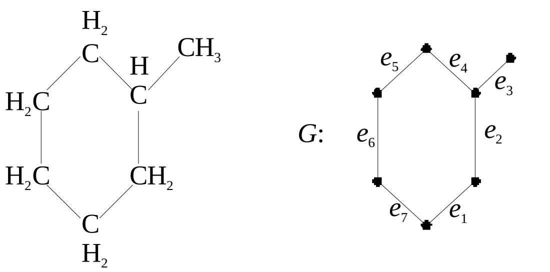

Figure 1: Methylcyclohexane

The values of the *K*-edge index of *G* and the K^*-edge index of *G* are $K_e(G) = 94$ and $K_e^*(G) = 94$.

Example 10. Consider ethylcyclopropane and its molecular graph *G*, see Figure 2.

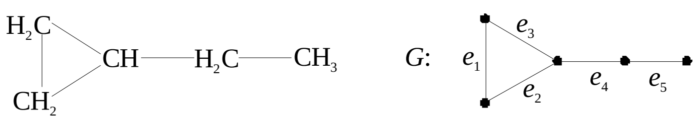

Figure 2:

The values of the *K*-edge index of *G* and the K^*-edge index of *G* are $K_e(G) = 90$ and $K_e^*(G) = 90$.

Now we have the following conjecture.

Conjecture 11. For any graph *G*, $K_e(G) = K_e^*(G)$.

Theorem 12. Let G be a graph with n vertices and m edges. Then

$$K_e(G) = K_3(G) - 6HM_1(G) + 12M_1(G) - 8m.$$

Proof: By definition K-edge index, we have

$$K_e(G) = \sum_{e\in E(G)} d_G(e)^3$$

$$= \sum_{e\in E(G)} \left[d_G(u) + d_G(v) - 2\right]^3$$

$$= \sum_{e\in E(G)} \left[\left\{d_G(u) + d_G(v)\right\}^3 - 6\left\{d_G(u) + d_G(v)\right\}^2 + 12\left\{d_G(u) + d_G(v)\right\} - 8\right]$$

$$= \sum_{e\in E(G)} \left[d_G(u) + d_G(v)\right]^3 - 6\sum_{e\in E(G)} \left[d_G(u) + d_G(v)\right] + 12\sum_{e\in E(G)} \left[d_G(u) + d_G(v)\right] - 8$$

$$= \sum_{e\in E(G)} \left[d_G(u) + d_G(v)\right]^3 - 6HM_1(G) + 12M_1(G) - 8m$$

$$= K_3(G) - 6HM_1(G) + 12M_1(G) - 8m.$$

Theorem 13. Let G be a graph with n vertices and m edges. Let $\overline{G}$ be the complement of G. Then

$$K_e(\overline{G}) = 8(n-2)^3\,\overline{m} - 12(n-2)^2\,\overline{M}_1(G) + 6(n-2)\overline{HM}_1(G) - \overline{K}_3(G).$$

Proof: If the degree of a vertex u in G is $d_G(u)$, then the degree of u in $\overline{G}$ is $n-1-d_G(u)$. If $e = uv$ is an edge of G, then the degree of an edge e in $\overline{G}$ is

$$n-1-d_G(u)+n-1-d_G(v)-2 \text{ or } 2(n-2)-\left[d_G(u)+d_G(v)\right].$$

Therefore

$$K_e(\overline{G}) = \sum_{e\in E(\overline{G})} d_{\overline{G}}(e)^3$$

$$= \sum_{e\in E(\overline{G})} \left[2(n-2) - \left\{d_G(u) + d_G(v)\right\}\right]^3$$

$$= \sum_{e\in E(\overline{G})} \left[2^3(n-2)^3 - 3.2^2(n-2)^2\left\{d_G(u)+d_G(v)\right\} + 3.2(n-2)\left\{d_G(u)+d_G(v)\right\}^2 - \left\{d_G(u)+d_G(v)\right\}^3\right]$$

$$= 8(n-2)^3\,\overline{m} - 12(n-2)^2\sum_{e\in E(\overline{G})}\left[d_G(u)+d_G(v)\right] + 6(n-2)\sum_{e\in E(\overline{G})}\left[d_G(u)+d_G(v)\right]^2 - \sum_{e\in E(\overline{G})}\left[d_G(u)+d_G(v)\right]^3$$

$$= 8(n-2)^3\,\overline{m} - 12(n-2)^2\,\overline{M}_1(G) + 6(n-2)\overline{HM}_1(G) - \overline{K}_3(G).$$

Theorem 14. Let G be a graph with n vertices m edges. Then

$$K_e(G) = IE_2(G) + 2EM_2(G).$$

Proof: Consider

$$K_e(G) = \sum_{e\sim f\in E(G)} \left[d_G(e)^2 + d_G(f)^2\right]$$

$$= \sum_{e\sim f\in E(G)} \left[d_G(e)-d_G(f)\right]^2 + 2\sum_{e\sim f\in E(G)} d_G(e).d_G(f)$$

$$= \sum_{e\sim f\in E(G)} \left[d_G(e)-d_G(f)\right]^2 + 2EM_2(G)$$

$= IE_2(G) + 2\,EM_2(G)$.

Corollary 15. Let G be a graph without isolated vertices. Then

$$IE_2(G) = K_e(G) - 2\,EM_2(G).$$

Theorem 16. Let G be a graph with n vertices and m edges. Then

$$\overline{K}_e(G) = 2(m-1)EM_1(G) - K_e(G).$$

Proof: Denote $\gamma_{e,f} = d_G(e)^2 + d_G(f)^2$. Then in view of

$$\sum_{e\in E}\sum_{f\in E} F(e,f) = \sum_{e\sim f\in E(G)} \gamma_{e,f} + \sum_{e\sim f\in E(\overline{G})} \gamma_{e,f} + \sum_{e\in E}\gamma_{e,e}$$

$$= K_e(G) + \overline{K}_e(G) + 2EM_1(G). \qquad (1)$$

Also

$$\sum_{e\in E}\sum_{f\in E}\left[d(e)^2 + d(f)^2\right] = m\sum_{e\in E} d(e)^2 + m\sum_{f\in E} d(f)^2$$

$$= 2m\,EM_1(G) \qquad (2)$$

From (1) and (2), we have

$$K_e(G) + \overline{K}_e(G) + 2EM_1(G) = 2mEM_1(G).$$

Thus $\overline{K}_e(G) = 2(m-1)EM_1(G) - K_e(G)$. (3)

Theorem 17. Let G be a graph with n vertices and m edges.
Let $\overline{G}$ be the complement of G. Then

i) $\overline{K}_e(\overline{G}) = K_e(G)$. (4)

ii) $\overline{K}_e(G) + \overline{K}_e(\overline{G}) = 2(m-1)EM_1(G)$.

Proof: (1) By definition $\overline{K}_e(G)$, we have

$$\overline{K}_e(G) = \sum_{e\sim f\in E(\overline{G})} \left[d_{\overline{G}}(e)^2 + d_{\overline{G}}(f)^2\right].$$

Therefore $$\overline{K}_e(\overline{G}) = \sum_{e\sim f\in E(G)} \left[d_G(e)^2 + d_G(f)^2\right] = K_e(G).$$

(ii) From (3) and (4) we have

$$\overline{K}_e(G) = 2(m-1)EM_1(G) - \overline{K}_e(\overline{G}).$$

Thus $$\overline{K}_e(G) + \overline{K}_e(\overline{G}) = 2(m-1)EM_1(G).$$

REFERENCES

1. I.Gutman and N.Trinajstić, Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.*, 17 (1972) 535-538.
2. V.R.Kulli, On *K* indices of graphs, *International Journal of Fuzzy Mathematical Archive*, 10(2) (2016) 105-109.
3. V.R.Kulli, On *K* coindices of graphs, *Journal of Computer and Mathematical Sciences*, 7(3) (2016) 107-112.
4. B.Furtula and I.Gutman, A forgotten topological index *J. Math. Comput. Chem.*, 53 (2015) 1184-1190.
5. S.M.Hosamani, A upper bound for the F_1-index and its applications to fullerences, submitted.
6. A.Milićević, S.Nikolić and N.Trinajstić, On reformulated Zagreb indices, *Molecular Diversity*, 8 (2004) 393-399.
7. B.Zhou and N.Trinajstić, Some properties of the reformulated Zagreb indices, *J. Math. Chem*., 48 (2010) 714-719.
8. A.Ilić and B.Zhou, On reformulated Zagreb indices, *Discrete Appl. Math.*, 160 (2012) 204-209.
9. S.Ji, X. Li and B. Huo, On reformulated Zagreb indices with respect to acyclic, unicyclic and bicyclic graphs, *MATCH Commun. Math. Comput. Chem.*, 72 (2014) 723-732.
10. N.De, Sk. Md. Abu Nayeem and A.Pal, Reformulated first Zagreb index of some graph operations, *Mathematics*, 3 (2015) 945-960.
11. V.R.Kulli, The first and second *Ka*-indices and coindices of graphs, submitted.
12. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).