# Fuzzy Optimal Solution for Fully Fuzzy Linear Programming Problems Using Hexagonal Fuzzy Numbers 

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#### Abstract

The fuzzy set theory has been applied in many fields such as management, engineering and almost in every business enterprise as well as day to day activities. In this paper fully fuzzy linear programming problems with hexagonal fuzzy numbers were discussed. A new approach for solving fully fuzzy linear programming problems (FFLPP) is proposed, based upon the new Ranking function, which is divided as two Trapezoidal and average values of the same were taken. This paper compares the three different ranking functions by solving some FFLLP problems which are tabulated.


Keywords: Fuzzy number, Trapezoidal fuzzy number, Hexagonal fuzzy number, Ranking function, Fuzzy variable linear programming, Fuzzy optimal solution.

## AMS Mathematics Subject Classification (2010): 90C05

## 1. Introduction

Fuzzy Logic was initiated in 1965 by Lotfi A. Zadeh, Professor at the University of California in Berkeley. Basically, Fuzzy Logic (FL) is a multivalued logic that allows intermediate values to be defined between conventional evaluations like true/false, yes/no, high/low, etc. Ranking fuzzy number is used mainly in decision-making, data analysis, artificial intelligence and various other fields of operation research. In fuzzy environment ranking fuzzy numbers is a very important decision making procedure. The idea of fuzzy set was first proposed by Bellman and Zadeh [5] as a mean of handling uncertainty that is due to imprecision rather than randomness. The concept of Fuzzy Linear Programming (FLP) was first introduced by Tanaka et al. [16, 17]. Zimmerman [19] introduced fuzzy linear programming in fuzzy environment. Chanas [6] proposed a fuzzy programming in multiobjective linear programming. Allahviranloo et al. [2] proposed a new method for solving fully fuzzy linear programming problems by the use of ranking function. Kumar et al. [11] proposed a new method for solving fully fuzzy linear programming problems with inequality constraints. Abbasbandy and Asady [1] suggested a sign distance method for ranking fuzzy numbers in 2006. Rajarajeswari et al. [15] presented a new operation on hexagonal fuzzy numbers. Liou and Wang [13] presented ranking fuzzy numbers with interval values. Verdegay [18] have developed

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three methods for solving three models of fuzzy integer linear programming based on the representation theorem and on fuzzy number ranking method. Nasseri et.al [14] proposed a new method for solving fuzzy linear programming problems in which he has used the fuzzy ranking method for converting the fuzzy objective function into crisp objective function. Lee and Li [12] discussed the comparison of fuzzy numbers. Amit Kumar et al. [3, 4] presented a new method for solving fuzzy linear programs with Trapezoidal fuzzy numbers. Cheng [8] used a centroid based distance method to rank fuzzy numbers in 1998.Kauffmann and Gupta [9] introduced to Fuzzy Arithmetic. Kolman and Hill [10] was introduced a FFLP problem. Chen [7] proposed the ranking trapezoidal fuzzy number using maximizing and minimizing set decomposition principle and sign distance. In this paper, some preliminaries are presented in section 2. Section 3 describes the proposed method with one numerical example and obtained results were discussed. Section 4 concludes the paper.

## 2. Preliminaries

Definition 2.1.The characteristic function $\mu_{\mathrm{A}}$ of a crisp set A C X assigns a value either 0 or 1 to each member in $X$. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set $X$ fall within a specified range i.e. $\mu_{\tilde{A}}$ : $\mathrm{X} \rightarrow[0,1]$. The assigned value indicate the membership function and the set $\widetilde{A}=\{(\mathrm{x}$, $\left.\left.\mu_{\tilde{A}}(x)\right) ; \mathrm{x} \in \mathrm{X}\right\}$ defined by $\mu_{\tilde{A}}(x)$ for $\mathrm{x} \in \mathrm{X}$ is called fuzzy set.

Definition 2.2. An effective approach for ordering the elements of $F(R)$ is also to define a ranking function $\mathfrak{R}: F(R) \rightarrow R$ which maps each fuzzy number into the real line, where a natural order exists. We define orders on $\mathrm{F}(\mathrm{R})$ by: $\tilde{a} \geq \tilde{b}$ if and only if $\mathrm{R}(\tilde{a}) \geq \mathrm{R}(\tilde{b})$, $\tilde{a} \leq \tilde{b}$ if and only if $\mathrm{R}(\tilde{a}) \leq \mathrm{R}(\tilde{b})$,
$\tilde{a}=\tilde{b} \quad$ if and only if $\mathrm{R}(\tilde{a})=\mathrm{R}(\tilde{b})$
Definition 2.3. A fuzzy number $\widetilde{A_{H}}$ is a hexagonal fuzzy number denoted by $\widetilde{A_{H}}=\left(a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}\right)$ where $\left(a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}\right)$ are real numbers and its membership function $\mu_{\widetilde{A_{H}}}(x)$ is given below.

$$
\mu_{\widetilde{A_{H}}}(x)= \begin{cases}0 & x<\mathrm{a}_{1} \\ \frac{1}{2}\left(\frac{x-a_{1}}{a_{2}-a_{1}}\right) & a_{1} \leq x \leq a_{2} \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_{2}}{a_{3}-a_{2}}\right) & a_{2} \leq x \leq a_{3} \\ 1 & a_{3} \leq x \leq a_{4} \\ 1-\frac{1}{2}\left(\frac{x-a_{4}}{a_{5}-a_{4}}\right) & a_{4} \leq x \leq a_{5} \\ \frac{1}{2}\left(\frac{a_{6}-x}{a_{6}-a_{5}}\right) & a_{5} \leq x \leq a_{6} \\ 0 & \mathrm{x}>a_{6}\end{cases}
$$

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Figure 1: Graphical representation of a hexagonal fuzzy number for $x \in[0,1]$

## 3. Proposed method



Figure 2


Figure 3:


Figure 4
In this paper, the hexagonal has been divided into two plane figures. These two plane figures are trapezoidal ABEF and BCDE (Fig.2).Then the ranking function were taken for ABEF and BCDE and the four lines were joined from x -axis to hexagonal (Fig.3). Average has been taken by using all six points (Fig.4). Let $\widetilde{A_{H}}=\left(a_{1}, a_{2}, a_{3}, a_{4}\right.$, $a_{5}, a_{6}$ ) be a hexagonal fuzzy number. The ranking functions are obtained as below:
$\mathfrak{R}(\widetilde{A})=\frac{a_{1}+2 a_{2}+a_{3}+a_{4}+2 a_{5}+a_{6}}{4}$

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$$
\begin{align*}
& \Re(\widetilde{A})=\frac{a_{1}+a_{2}+a_{3}+a_{4}+a_{5}+a_{6}}{4}  \tag{3.2}\\
& \Re(\widetilde{A})=\frac{a_{1}+a_{2}+a_{3}+a_{4}+a_{5}+a_{6}}{6} \tag{3.3}
\end{align*}
$$

## Algorithm

Step 1: Formulate the chosen problem into the following fully fuzzy linear programming Problem: $\operatorname{Max}($ or $) \min \tilde{z}=\tilde{c}_{j} \tilde{x}_{j}$
Subject to $\tilde{A}_{i j} \tilde{x}_{j} \leq \tilde{B}_{i}$,

$$
\tilde{x}_{j} \geq 0
$$

Step 2: Using the Ranking functions (3.1, 3.2, 3.3), the FFLPP transformed into FVLPP. Step 3: Solve the FVLPP by using simplex method / Big-M method. Let the solution be $\tilde{x}_{\mathrm{j}}$. Hence the solution of FFLPP is ${\widetilde{x_{j}}}^{*}$.
Step 4: Express the problem in standard form by introducing slack / surplus variables, to convert the inequality constraints into equations.
Step 5: Compute the value of $\tilde{Z}==C_{B} Y_{j}-C_{j} \quad j \neq B, \quad j=1 \ldots n$.
(i) If all $\tilde{Z} \geq 0 \forall j$ for maximization problem
(ii) If all $\tilde{Z}<0 \forall j$ for minimization problem. Then the current solution is optimal, otherwise go to step 6.
Step 6: Determine the basic variable $\tilde{x}_{\mathrm{k}}$, which will be replaced by the non-basic variable, Where $\mathrm{k}=\arg \min \left\{\Re\left(\widetilde{\widetilde{B}}_{l}\right)\right\} \mathrm{i}=1,2 \ldots . \mathrm{m}$, in maximization problem and $\mathrm{k}=\arg \max \left\{\Re\left(\widetilde{\widetilde{B}}_{l}\right)\right\} \mathrm{i}=1,2 \ldots \mathrm{~m}$, in minimization problem.
Step 7: Perform the pivot operation and return to step 5. Then repeat the procedure until a fuzzy optimal solution is obtained.

Example 3.1. Maximize $(11,13,15,17,19,21) \tilde{x}_{1}+(31,33,35,37,39,41) \tilde{x}_{2}$
Subject to
$(41,43,45,47,49,51) \tilde{x}_{1}+(61,63,65,67,69,71) \tilde{x}_{2} \leq(151,153,155,157,159,161)$
$(81,83,85,87,89,91) \tilde{x}_{1}+(101,103,105,107,109,111) \tilde{x}_{2} \leq(271,273,275,277$, 279, 281)
Solution:
Maximize $(11,13,15,17,19,21) \tilde{x}_{1}+(31,33,35,37,39,41) \tilde{x}_{2}$
Subject to
$(41,43,45,47,49,51) \tilde{x}_{1}+(61,63,65,67,69,71) \tilde{x}_{2}+(1,1,1,1,1,1) \tilde{x}_{3}=(151,153,155$, 157, 159, 161)
$(81,83,85,87,89,91) \tilde{x}_{1}+(101,103,105,107,109,111) \tilde{x}_{2}+(1,1,1,1,1,1) \widetilde{x}_{4}=(271,273$, 275, 277, 279, 281)
$\tilde{x}_{1}, \tilde{x}_{2}, \tilde{x}_{3}, \tilde{x}_{4}, \geq 0$
Ranking function (i)
Maximize $\tilde{Z}=32 \tilde{x}_{1}+72 \tilde{x}_{2}+0 \tilde{x}_{3}+0 \tilde{x}_{4}$
Subject to
$92 \tilde{x}_{1}+132 \tilde{x}_{2}+\tilde{x}_{3}=(151,153,155,157,159,161)$
$172 \tilde{x}_{1}+212 \tilde{x}_{2}+\tilde{x}_{4}=(271,273,275,277,279,281)$
$\tilde{x}_{1}, \tilde{x}_{2}, \tilde{x}_{3}, \tilde{x}_{4}, \geq 0$.

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Initial table

| Basis | $\tilde{x}_{1}$ | $\tilde{x}_{2}$ | $\tilde{x}_{3}$ | $\tilde{x}_{4}$ | RHS | $\mathfrak{R}\left(\widetilde{B}_{l}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\tilde{x}_{3}$ | 92 | 132 | 1 | 0 | $(151,153,155,157,159,161)$ | 312 |
| $\tilde{x}_{4}$ | 172 | 212 | 0 | 1 | $(271,273,275,277,279,281)$ | 552 |
| $\tilde{Z}$ | -32 | -72 | 0 | 0 | $(0,0,0,0,0,0)$ |  |

First iteration

| Basis | $\tilde{x}_{1}$ | $\tilde{x}_{2}$ | $\tilde{x}_{3}$ | $\tilde{x}_{4}$ | RHS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\tilde{x}_{2}$ | $\frac{92}{132}$ | 1 | $\frac{1}{132}$ | 0 | $\left(\frac{151}{132}, \frac{153}{132}, \frac{155}{132}, \frac{157}{132}, \frac{159}{132}, \frac{161}{132}\right)$ |
| $\tilde{x}_{4}$ | $\frac{3200}{132}$ | 0 | $-\frac{212}{132}$ | 1 | $\left(\frac{3760}{132}, \frac{3600}{132}, \frac{3440}{132}, \frac{3250}{132}, \frac{3120}{132}, \frac{2960}{132}\right)$ |
| $\widetilde{Z}$ | $\frac{2400}{132}$ | 0 | $\frac{72}{132}$ | 0 | $\left(\frac{10872}{132}, \frac{11016}{132}, \frac{11160}{132}, \frac{11304}{132}, \frac{11448}{132}, \frac{11592}{132}\right)$ |

Since $\tilde{Z} \geq 0$, the fuzzy optimal solution of the FVLPP and FFLPP is $\widetilde{x_{1}^{*}}=\quad(0,0,0,0,0,0), \quad \widetilde{x_{2}^{*}}=\left(\frac{151}{132}, \quad \frac{153}{132}, \frac{155}{132}, \frac{157}{132}, \frac{159}{132}, \frac{161}{132}\right) \quad$ and $\tilde{z} \quad=$ $\left(\frac{10872}{132}, \frac{11016}{132}, \frac{11160}{132}, \frac{11304}{132}, \frac{11448}{132}, \frac{11592}{132}\right) \cong 510.54$
Ranking function (ii)
Maximize $\tilde{Z}=24 \tilde{x}_{1}+54 \tilde{x}_{2}+0 \tilde{x}_{3}+0 \tilde{x}_{4}$
Subject to
$69 \tilde{x}_{1}+99 \tilde{x}_{2}+\tilde{x}_{3}=(151,153,155,157,159,161)$
$129 \tilde{x}_{1}+159 \tilde{x}_{2}+\tilde{x}_{4}=(271,273,275,277,279,281)$
$\tilde{x}_{1}, \tilde{x}_{2}, \tilde{x}_{3}, \tilde{x}_{4}, \geq 0$.
The fuzzy optimal solution of the FVLPP and FFLPP is $\widetilde{x_{1}^{*}}=(0,0,0,0,0,0)$,

$$
\widetilde{x_{2}^{*}}=\left(\frac{151}{99}, \frac{153}{99}, \frac{155}{99}, \frac{157}{99}, \frac{159}{99}, \frac{161}{99}\right) \text { and } \tilde{z}=\left(\frac{8154}{99}, \frac{8262}{99}, \frac{8370}{99}, \frac{8478}{99}, \frac{8586}{99}, \frac{8694}{99}\right) \cong 510.54
$$

Ranking function (iii)
Maximize $\tilde{Z}=16 \tilde{x}_{1}+36 \tilde{x}_{2}+0 \tilde{x}_{3}+0 \tilde{x}_{4}$
Subject to
$46 \tilde{x}_{1}+66 \tilde{x}_{2}+\tilde{x}_{3}=(151,153,155,157,159,161)$
$86 \tilde{x}_{1}+106 \tilde{x}_{2}+\tilde{x}_{4}=(271,273,275,277,279,281)$
$\tilde{x}_{1}, \tilde{x}_{2}, \tilde{x}_{3}, \tilde{x}_{4}, \geq 0$
The fuzzy optimal solution of the FVLPP and FFLPP is $\widetilde{x_{1}^{*}}=(0,0,0,0,0,0)$,
$\widetilde{x}_{2}^{*}=\left(\frac{151}{66}, \frac{153}{66}, \frac{155}{66}, \frac{157}{66}, \frac{159}{66}, \frac{161}{66}\right)$ and $\tilde{z}=\left(\frac{5436}{66}, \frac{5508}{66}, \frac{5580}{66}, \frac{5652}{66}, \frac{5724}{66}, \frac{5796}{66}\right) \cong 510.54$

| S 1. N o | No. of const raints | No. of varia bles | Ranking function I | Ranking function II | Ranking function III | Fuzzy optim al value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\widetilde{x_{j}^{*}}$ | $\widetilde{x_{j}^{*}}$ | $\widetilde{x_{j}^{*}}$ | $\tilde{Z}$ |

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| 1 | 2 | 2 | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0), \\ & \widetilde{x_{2}^{*}}=\left(\frac{151}{131}, \frac{153}{132}, \frac{155}{132},\right. \\ & \left.\frac{157}{132}, \frac{159}{132}, \frac{161}{132}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=\left(\frac{151}{99}, \frac{153}{99}, \frac{155}{99}, \frac{157}{99}, \frac{159}{99}, \frac{161}{99}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=\left(\frac{151}{66}, \frac{153}{66}, \frac{155}{66}\right. \\ & \left.\frac{157}{66}, \frac{159}{66}, \frac{161}{66}\right) \end{aligned}$ | 510.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 2 | 2 | $\begin{aligned} & \widetilde{x_{1}^{*}}=\left(\frac{202}{54}, \frac{204}{54}, \frac{206}{54},\right. \\ & \left.\frac{208}{\frac{54}{x_{2}^{*}}}, \frac{20}{54}, \frac{212}{54}\right) \\ & \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=\left(\frac{202}{40.5}, \frac{204}{40.5}, \frac{206}{40.5}, \frac{208}{40.5}, \frac{210}{40.5}, \frac{212}{40.5}\right) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=\left(\frac{202}{27}, \frac{204}{27}, \frac{206}{27},\right. \\ & \left.\frac{208}{\frac{27}{x_{2}^{*}}}, \frac{210}{27}, \frac{212}{27}\right) \\ & \end{aligned}$ | 966 |
| 3 | 2 | 2 | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=\left(\frac{171}{128}, \frac{173}{128}, \frac{175}{128},\right. \\ & \left.\frac{177}{128}, \frac{179}{128}, \frac{181}{128}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=\left(\frac{171}{96}, \frac{173}{96}, \frac{175}{96}, \frac{177}{96}, \frac{179}{96}, \frac{181}{96}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=\left(\frac{171}{64}, \frac{173}{64}, \frac{175}{64}\right. \\ & \left.\frac{177}{64}, \frac{179}{64}, \frac{181}{64}\right) \end{aligned}$ | 511.5 |
| 4 | 2 | 2 | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=\left(\frac{60}{15}, \frac{62}{15}, \frac{64}{15}, \frac{66}{15}, \frac{68}{15}, \frac{70}{15}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=\left(\frac{60}{11.25}, \frac{62}{11.25},\right. \\ & \left.\frac{64}{11.25}, \frac{66}{11.25}, \frac{68}{11.25}, \frac{70}{11.25}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=\left(\frac{60}{7.5}, \frac{62}{7.5}, \frac{64}{7.5},\right. \\ & \left.\frac{66}{7.5}, \frac{68}{7.5}, \frac{70}{7.5}\right) \end{aligned}$ | 364 |
| 5 | 3 | 3 | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=\left(\frac{60}{38}, \frac{62}{38}, \frac{64}{38}, \frac{66}{38}, \frac{68}{38}, \frac{70}{38}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=\left(\frac{60}{28.5}, \frac{62}{28.5}, \frac{64}{28.5}, \frac{66}{28.5}, \frac{68}{28.5}, \frac{70}{28.5}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \left.\widetilde{x_{2}^{*}}=0,0,0,0,0,0\right) \\ & \widetilde{\left(x_{3}^{*}\right.}=\left(\frac{60}{19}, \frac{62}{19}, \frac{64}{19}, \frac{66}{19}, \frac{68}{19}, \frac{70}{19}\right) \end{aligned}$ | 225.7 |
| 6 | 3 | 3 | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=\left(\frac{120}{98}, \frac{122}{98}, \frac{124}{98},\right. \\ & \left.\frac{126}{\frac{128}{98}}, \frac{128}{98}, \frac{130}{98}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=\left(\frac{120}{73.5}, \frac{122}{73.5}, \frac{124}{73.5}, \frac{126}{73.5}, \frac{128}{73.5}, \frac{130}{73.5}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=\left(\frac{120}{49}, \frac{122}{49}, \frac{124}{49},\right. \\ & \left.\frac{126}{49}, \frac{128}{49}, \frac{130}{49}\right) \end{aligned}$ | 290.8 |
| 7 | 4 | 4 | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & x_{2}^{*}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ & \frac{\widetilde{x_{4}^{*}}}{}=\left(\frac{302}{174}, \frac{304}{174}, \frac{306}{174},\right. \\ & \left.\frac{308}{174}, \frac{310}{174}, \frac{312}{174}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \frac{x_{3}^{*}}{=}=(0,0,0,0,0,0) \\ & \widetilde{x_{4}^{*}}=\left(\frac{302}{130.5}, \frac{304}{130.5}, \frac{306}{130.5}, \frac{308}{130.5}, \frac{310}{130.5}, \frac{312}{130.5}\right. \\ & ) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & x_{2}^{*}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ & \frac{x_{4}^{*}}{}=\left(\frac{302}{87}, \frac{304}{87}, \frac{306}{87},\right. \\ & \left.\frac{308}{87}, \frac{310}{87}, \frac{312}{87}\right) \end{aligned}$ | 783.3 |
| 8 | 4 | 4 | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{4}^{*}}=\left(\frac{103}{160}, \frac{105}{160}, \frac{107}{160},\right. \\ & \left.\frac{109}{160}, \frac{111}{160}, \frac{113}{160}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{4}^{*}}=\left(\frac{103}{120}, \frac{105}{120}, \frac{107}{120}, \frac{109}{120}, \frac{111}{120}, \frac{113}{120}\right) \end{aligned}$ | $\begin{aligned} & \frac{x_{1}^{*}}{*}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{4}^{*}}=\left(\frac{103}{80}, \frac{105}{80}, \frac{107}{80},\right. \\ & \left.\frac{109}{80}, \frac{111}{80}, \frac{, 13}{80}\right) \end{aligned}$ | 639.9 |
| 9 | 5 | 5 | $\begin{aligned} & \widetilde{x}_{1}^{*}=(0,0,0,0,0,0) \\ & \frac{x_{2}^{*}}{*}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{4}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{5}^{*}}=\left(\frac{200}{170}, \frac{202}{170}, \frac{204}{170},\right. \\ & \left.\frac{206}{170}, \frac{208}{170}, \frac{210}{170}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{4}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{5}^{*}}=\left(\frac{200}{127.5}, \frac{202}{127.5}, \frac{204}{127.5},\right. \\ & \left.\frac{206}{127.5}, \frac{208}{127.5}, \frac{210}{127.5}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x}_{1}^{*}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{4}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{5}^{*}}=\left(\frac{200}{85}, \frac{202}{85}, \frac{204}{85},\right. \\ & \left.\frac{206}{85}, \frac{208}{85}, \frac{210}{85}\right) \end{aligned}$ | $\begin{gathered} 332.8 \\ 2 \end{gathered}$ |
| $\begin{aligned} & 1 \\ & 0 \end{aligned}$ | 6 | 6 | $\begin{aligned} & \frac{x_{1}^{*}}{\widetilde{x}_{2}^{*}}=(0,0,0,0,0,0) \\ & \frac{x_{3}^{*}}{x}=(0,0,0,0,0,0) \\ & \frac{x_{4}^{*}}{x}=(0,0,0,0,0,0) \\ & \frac{x_{5}^{*}}{}=(0,0,0,0,0) \\ & \widetilde{x_{6}^{*}}=\left(\frac{200}{196}, \frac{202}{196}, \frac{204}{196},\right. \\ & \left.\frac{206}{196}, \frac{208}{196}, \frac{210}{196}\right) \end{aligned}$ | $\begin{aligned} & \widetilde{x_{1}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{2}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{4}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{5}^{*}}=(0,0,0,0,0,0) \\ & \widetilde{x_{6}^{*}}=\left(\frac{200}{147}, \frac{202}{147}, \frac{204}{147}, \frac{206}{147}, \frac{208}{147}, \frac{210}{147}\right) \end{aligned}$ | $\begin{aligned} & \frac{\widetilde{x_{1}^{*}}=(0,0,0,0,0,0)}{\widetilde{x_{2}^{*}}=(0,0,0,0,0,0)} \begin{array}{l} \widetilde{x_{3}^{*}}=(0,0,0,0,0,0) \\ \widetilde{x_{4}^{*}}=(0,0,0,0,0,0) \\ \widetilde{x_{5}^{*}}=(0,0,0,0,0,0) \\ \widetilde{x_{6}^{*}}=\left(\frac{200}{98}, \frac{202}{98}, \frac{204}{98},\right. \\ \left.\frac{206}{98}, \frac{208}{98}, \frac{210}{98}\right) \end{array}, \end{aligned}$ | $\begin{gathered} 476.9 \\ 3 \end{gathered}$ |

Table1: Comparison of fuzzy optimal solution and fuzzy optimal values using three ranking functions

# Fuzzy Optimal Solution For Fully Fuzzy Linear Programming Problems Using Hexagonal Fuzzy Numbers 

## 4. Conclusion

In this paper a new method is proposed for solving the fuzzy optimal solution of FFLP problem transform into FVLP problems. The FFLP problem is converted into FVLP problem using new Ranking function. We have obtained the same results by using the above three ranking functions (3.1), (3.2), (3.3). Ranking function is reasonable and effective for calculating the hexagonal weights of criteria. Therefore it is easier to solve fully fuzzy linear programming problem.

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