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Fuzzy Shortest Path by Type Reduction Method

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Abstract. Type-2 fuzzy sets are a generalization of the ordinary fuzzy sets in which each type-2 fuzzy set is characterized by a fuzzy membership function. A proposed algorithm gives the shortest path using centroid of type-2 fuzzy number from source node to destination node. Using the concept of centroid, type-2 fuzzy number reduced to crisp value. An illustrative example also included to demonstrate our proposed algorithm.

Keywords: Type-2 fuzzy number, centroid of a type-2 fuzzy set, centroid of type-1 fuzzy set, extension principle

AMS mathematics Subject Classification (2010): 94D05

1. Introduction

The shortest path problem concentrates on finding the path with minimum distance to find the shortest path from source node to destination node is a fundamental matter in graph theory. A directed acyclic network is a network that consists of a finite set of nodes and a set of direct acyclic arcs.

For a fuzzy graph the fuzzy shortest path problem has been solved by many authors, among them the work of [10] is very interesting.Type-2 fuzzy set was introduced by Zadeh [8] as an extension of the concept of an ordinary fuzzy set. The type-2 fuzzy logic has gained much attention recently due to its ability to handle uncertainty and many advances appeared in both theory and applications. Recently, type-2 fuzzy shortest path has been studied in [1] and shortest path fuzzy network in [3,9].

Type-reduction was proposed by Karnik and Mendel [5,6,7]. It is an 'extended version' [8] of type-1 defuzzification methods and is called type reduction because this operation takes us from the type-2 output sets of the fuzzy logic system to a type-1 set that is called the "type-reduced set". This set may then be defuzzified to obtain a single crisp number: however, in many applications, the type reduced set may be more important than a single crisp number since it conveys a measure of uncertainties that have flown through the type-2 fuzzy logic system.

There exist many kinds of type reduction such as centroid, centre-of-sets, heights, and modified height, the details of which are given in [2,4,5,6,7]. In this paper we focus on the centroid of Gaussian type-2 fuzzy sets for finding fuzzy shortest path.

The structure of paper is following: In Section 2, we have some basic concepts required for analysis. In section 3, an algorithm is proposed to find shortest path and shortest path length based on type reduction method. Section 4 gives the network terminology. To illustrate the proposed algorithm the numerical example is solved in Section 5.

2. Concepts

2.1. Type-2 fuzzy set

A type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$ where $x \in X$ and $u \in J_x \subseteq [0,1]$.

ie., $\tilde{A} = \{ ((x,u), \mu_{\tilde{A}}(x,u)) \,/\, \forall\, x \in X, \forall\, u \in J_x \subseteq [0,1] \}$ in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$.

$\tilde{A}$ can be expressed as $\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$ $J_x \subseteq [0,1]$, where $\int\int$ denotes union over all admissible x and u. For discrete universe of discourse $\int$ is replaced by $\sum$.

2.2. Type-2 fuzzy number

Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. $\tilde{A}$ is normal,
2. $\tilde{A}$ is a convex set,
3. The support of $\tilde{A}$ is closed and bounded, then $\tilde{A}$ is called a type-2 fuzzy number.

2.3. Discrete type-2 fuzzy number

The discrete type-2 fuzzy number $\tilde{A}$ can be defined as follows:

$\tilde{A} = \sum_{x\in X} \mu_{\tilde{A}}(\mathrm{x})/\mathrm{x}$ where $\mu_{\tilde{A}}(\mathrm{x}) = \sum_{u\in J_x} f_x(\mathrm{u})/\mathrm{u}$ where J_x is the primary membership.

2.4. Extension principle

Let $A_1, A_2, \ldots, A_r$ be type-1 fuzzy sets in $X_1, X_2, \ldots, X_r$, respectively. Then, Zadeh's Extension Principle allows us to induce from the type-1 fuzzy sets $A_1, A_2, \ldots, A_r$ a type-1 fuzzy set B on Y, through f, i.e, $B = f(A_1, \ldots, A_r)$, such that

$$\mu_B(y) = \begin{cases} \sup\limits_{x_1,x_2,\ldots x_n \in f^{-1}(y)} \min\{\mu_{A_1}(x_1),\ldots,\mu_{A_n}(x_n) & iff^{-1}(y) \neq \phi \\ 0, & f^{-1}(y) = \phi \end{cases}$$

2.5. Addition on type-2 fuzzy numbers

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number be $\tilde{A}=\sum\mu_{\tilde{A}}(x)/\mathrm{x}$ and $\tilde{B}=\sum\mu_{\tilde{B}}(\mathrm{y})/\mathrm{y}$ where $\mu_{\tilde{A}}(x)=\sum f_x(\mathrm{u})/\mathrm{u}$ and $\mu_{\tilde{B}}(x)=\sum g_y(\mathrm{w})/\mathrm{w}$. The addition of these two types-2 fuzzy numbers $\tilde{A}\oplus\tilde{B}$ is defined as

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z})=\bigcup_{z=x+y}(\mu_{\tilde{A}}(\mathrm{x})\bigcap\mu_{\tilde{B}}(\mathrm{y}))$$

$$=\bigcup_{z=x+y}((\sum_i \mathrm{f}_x(\mathrm{u}_i)/\mathrm{u}_i)\bigcap(\sum_j g_y(\mathrm{w}_j)/\mathrm{w}_j))$$

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z})==\bigcup_{z=x+y}((\sum_{i,j}(\mathrm{f}_x(\mathrm{u}_i)\wedge g_y(\mathrm{w}_j))/(\mathrm{u}_i\wedge\mathrm{w}_j))$$

2.6. Complement of type-2 fuzzy set

The complement of the type-2 fuzzy set having a fuzzy membership function given in the following formula.

$$\overline{\tilde{A}}=\sum_{x\in X}\left[\sum_{u\in J_x} f_x(u)/(1-u)\right]/x$$

2.7. Centroid of type-2 fuzzy sets

Suppose that $\tilde{A}$ is a type-2 fuzzy set in the discrete caase. The centroid of $\tilde{A}$ can be defined as follows:

$$C_{\tilde{A}}=\frac{\int_{\theta_1\in J_{x_1}}\cdot\int_{\theta_2\in J_{x_2}}\cdots\cdots\int_{\theta_R\in J_{x_R}}[f_{x_1}(\theta_1).f_{x_2}(\theta_2)\ldots\ldots f_{x_R}(\theta_R)}{\dfrac{\sum_{j=1}^{R}x_j\mu_A(x_j)}{\sum_{j=1}^{R}\mu_A(x_j)}}$$

where $\tilde{A}=\sum_{j=1}^{R}\left[\sum_{u\in J_{x_j}} f_{x_j}(\mathrm{u})/\mathrm{u}\right]/\mathrm{x}_j$.

2.8. Centroid of type-1 fuzzy sets

Suppose that A is a type-1 fuzzy set, $A\subseteq X$ in a discrete case, $x_1,x_2,\ldots.x_R\in X$. The centroid of A can be defined as follows:

$$C_A=\frac{\sum_{j=1}^{R}x_j\mu_A(x_j)}{\sum_{j=1}^{R}\mu_A(x_j)}$$

2.8. **Minimum of two discrete type-2 fuzzy number**

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then minimum of two type-2 fuzzy sets is denoted as Min($\tilde{A}$, $\tilde{B}$) is given by

$$Min(\tilde{A},\tilde{B})(z) = \underset{z=Min(x,y)}{Sup}\left[(f_x(u_i) \wedge g_y(w_j)) / (\mathrm{u}_i \wedge w_j) \right]$$

where $\tilde{A} = \sum f_x(\mathrm{u}) / \mathrm{u} / \mathrm{x}$ and $\tilde{B} = \sum g_y(\mathrm{w}) / \mathrm{w} / \mathrm{y}$.

3. Algorithm

Step 1: Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

Step2: Find $C_{\tilde{L}_i}$ using definition 2.7

Step 3: Find C_L using definition 2.8

Step 4: Choose the path which is having the highest C_{L_i} , that corresponding path is the shortest path.

Step 5: Form the complement type-2 fuzzy number using def 2.6 for each edge weight.

Step 6: Continue the procedure from step 1 to step 4.

4. Network technology

Consider a directed network G(V,E) consisting of a finite set of nodes V = {1,2, . . .n} and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i,j), where i,j∈ V and i$\neq$j. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence $P_{ij} = \{i = i_1, (i_1,i_2),i_2,. . . ., i_{l-1}, (i_{l-1},i_l), i_l = j\}$ of alternating nodes and edges. The existence of at least one path P_{si} in G(V,E) is assumed for every node i∈ V – {s}.

$\tilde{d}_{ij}$ denotes a Type-2 Fuzzy Number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i,j\in P)} \tilde{d}_{ij}$

5. Numerical example

The problem is to find the shortest path and shortest path length between source node and destination node in the network having 6 vertices and 8 edges with type-2 fuzzy number.

Solution:

The edge lengths are

$\tilde{P}$ = (0.3/0.2+0.2/0.3)/2 + (0.3/0.1)/4

$\tilde{Q}$ = (0.5/0.2 + 0.3/0.4)/3

$\tilde{R}$ = (0.7/0.4)/1 + (0.5/0.3)/2

$\tilde{S}$ = (0.4/0.6 + 0.5/0.5)/2 + (0.2/0.1)/3

$\tilde{T}$ = (0.5/0.3)/2 + (0.7/0.6)/3
$\tilde{U}$ = (0.2/0.4)/1 + (0.3/0.5 + 0.4/0.6)/3
$\tilde{V}$ = (0.6/0.4)/2 +(0.7/0.5 + 0.4/0.6)/3
$\tilde{W}$ = (0.6/0.2)/1 +(0.4/0.5)/3

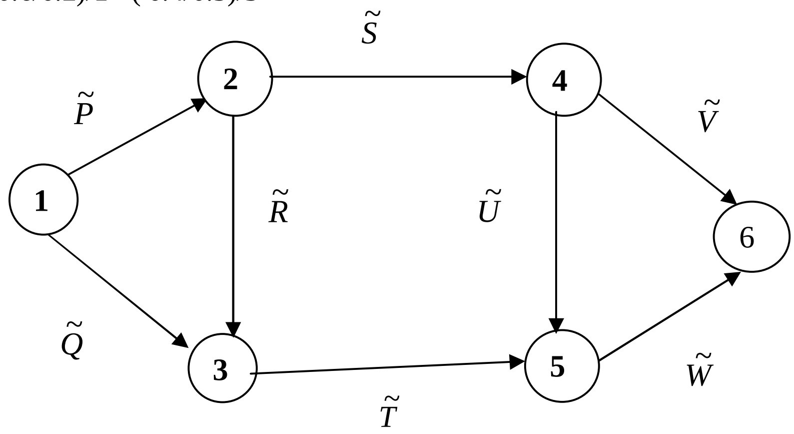

Figure 5.1:

Step 1: Computation of possible paths

Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

$\tilde{P}_1$: 1 – 2 – 4 – 6
$\tilde{P}_2$: 1 – 2 – 4 – 5 – 6
$\tilde{P}_3$: 1 – 2 – 3 – 5 – 6
$\tilde{P}_4$: 1 – 3 – 5 – 6

$\tilde{L}_1$ = (0.3/0.2 + 0.2/0.3)/6+ (0.2/0.2+0.2/0.3)/7 + (0.2/0.1)/8 + (0.2/0.1)/9 (0.2/0.1)/10

$\tilde{L}_2$ = (0.2/0.2)/6 + (0.2/0.1)/7 + (0.2/0.2+ 0.2/0.3)/8 + (0.2/0.1)/9 + (0.2/0.2 + 0.2/0.3)/10 + (0.2/0.1) /11 + (0.3/0.1)/12 + (0.2/0.1)/13

$\tilde{L}_3$ = (0.3/0.2)/6 + (0.3/0.2)/7 + (0.3/0.2 +0.2 /0.3)/8 + (0.3/0.2 + 0.2/0.3)/9 + (0.3/0.2 + 0.2/0.3)/10 + (0.3/0.1)/11 + (0.3/0.1)/12

$\tilde{L}_4$ = (0.5/0.2 + 0.3/0.2)/6 + (0.5/0.2)/7 + 0.4/0.2 +0.3/0.3)/8 + (0.4/0.2 + 0.3/0.4)/9

Step 2: Find $C_{\tilde{L}_i}$ using def 2.7

$C_{\tilde{L}_1}$ = 0.00048/7.6 + 0.00048/7.5 + 0.00032/7.4 + 0.00032/7.3

$C_{\tilde{L}_2}$ = 0.0000038/9.1 + 0.0000038/9.2 + 0.0000038/9 + 0.0000038/9.8

$C_{\tilde{L}_3}$ = 0.000097/8.9 + 0.000146/8.7 + 0.000218/8.6 + 0.000146/8.5

$C_{\tilde{L}_4}$ = 0.04/7.5 + 0.03/7.8 + 0.03/6.12

Step 3: Find C_L using def 2.8

$C_{L_1} = 7.4313$

$C_{L_2} = 9.38$

$C_{L_3} = 8.646$

$C_{L_4} = 7.176$

Step 4: Choose the path which is having the highest C_{L_i}, that corresponding path is the shortest path. $C_{L_2} = 9.38$

Shortest path is 1-2-4-5-6.

Step 5: Form the complement type-2 fuzzy number using def 2.6 for each edge weight.

Complement edge Lenght are,

$\bar{\tilde{P}}$ = (0.3/0.8+0.2/0.7)/2 + (0.3/0.9)/4

$\bar{\tilde{Q}}$ = (0.5/0.8 + 0.3/0.6)/3

$\bar{\tilde{R}}$ = (0.7/0.6)/1 + (0.5/0.7)/2

$\bar{\tilde{S}}$ = (0.4/0.4 + 0.5/0.5)/2 + (0.2/0.9)/3

$\bar{\tilde{T}}$ = (0.5/0.7)/2 + (0.7/0.4)/3

$\bar{\tilde{U}}$ = (0.2/0.6)/1 + (0.3/0.5 + 0.4/0.4)/3

$\bar{\tilde{V}}$ = (0.6/0.6)/2 +(0.7/0.5 + 0.4/0.4)/3

$\bar{\tilde{W}}$ = (0.6/0.8)/1 + (0.4/0.5)/3

Step 6: Continue the procedure from step 1 to step 4.

Step 1: Computation of possible paths for complement type-2 fuzzy number

Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\bar{\tilde{L}}_i$ i = 1,2,. . . .n for possible n paths.

$\bar{\tilde{P}}_1$: 1 – 2 – 4 – 6

$\bar{\tilde{P}}_2$: 1 – 2 – 4 – 5 – 6

$\bar{\tilde{P}}_3$: 1 – 2 – 3 – 5 – 6

$\bar{\tilde{P}}_4$: 1 – 3 – 5 – 6

$\bar{\tilde{L}}_1$ = (0.3/0.2 + 0.2/0.3)/6+ (0.2/0.2+0.2/0.3)/7 + (0.2/0.1)/8 + (0.2/0.1)/9 + (0.2/0.1)/10

$\bar{\tilde{L}}_2$ = (0.2/0.2)/6 + (0.2/0.1)/7 + (0.2/0.2+ 0.2/0.3)/8 + (0.2/0.1)/9 + (0.2/0.2 + 0.2/0.3)/10 + (0.2/0.1) /11 + (0.3/0.1)/12 + (0.2/0.1)/13

$\bar{\tilde{L}}_3$ = (0.3/0.2)/6 + (0.3/0.2)/7 + (0.3/0.2 +0.2 /0.3)/8 + (0.3/0.2 + 0.2/0.3)/9 + (0.3/0.2 +0.2/0.3)/10 + (0.3/0.1)/11 + (0.3/0.1)/12

$\bar{\tilde{L}}_4$ = (0.5/0.2 + 0.3/0.2)/6 + (0.5/0.2)/7 + 0.4/0.2 +0.3/0.3)/8 + (0.4/0.2 + 0.3/0.4)/9

Step 2: Find $C_{\bar{\tilde{L}}_i}$ using definition 2.7

$C_{\bar{\tilde{L}}_1}$ = 0.00048/8.4 + 0.00048/8.1 + 0.00048/8 + 0.00048/7.9 + 0.00048/7.8

$C_{\bar{\tilde{L}}_2}$ = 0.0000038/9.3 + 0.0000038/9.4 + 0.0000038/9.5 + 0.0000038/9.6

$C_{\bar{\tilde{L}}_3}$ = 0.00022/8.9

$C_{\bar{\tilde{L}}_4}$ = 0.04/7.3 + 0.024/7.4

Step 3: Find $\mathrm{C}_{\bar{\mathrm{L}}}$ using definition 2.8

$\mathrm{C}_{\bar{\mathrm{L}}_1}$ = 8.0375

$\mathrm{C}_{\bar{\mathrm{L}}_2}$ = 9.4474

$\mathrm{C}_{\bar{\mathrm{L}}_3}$ = 8.9

$\mathrm{C}_{\bar{\mathrm{L}}_4}$ = 7.3375

Step 4: Choose the path which is having the highest $C_{\bar{\tilde{L}}_i}$, that corresponding path is theshortest path for complement type-2 fuzzy number.

$\mathrm{C}_{\bar{\mathrm{L}}_2}$ = 9.4474

Shortest path is 1-2-4-5-6.

Table 5.1:

S.No.	**Possible Paths**	$C_{\tilde{L}_i}$	$C_{\bar{\tilde{L}}_i}$	**Rank**
1.	1-2-4-6	7.4314	8.0375	3
2.	1-2-4-5-6	9.38	9.4474	1
3.	1-2-3-5-6	8.646	8.9	2
4.	1-3-5-6	7.176	7.3375	4

Here the shortest path is same for both type-2 fuzzy number and its complement also.

6. Conclusion

In this paper, we have developed an algorithm for shortest path with type-2 fuzzy number and its complement. Here we have used the concept of centroid of type-2 fuzzy number for type reduction. Then we can easily find out the shortest path without using any measures. Type-2 fuzzy number is reduced to type-1 fuzzy number and then it is reduced to crisp number using this type reduction method. We are getting the same shortest path for both type-2 fuzzy number and its complement with small variations in values.

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