Intern. J. Fuzzy Mathematical Archive
Vol. 5, No. 1, 2014, 19-27
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 25 November 2014
www.researchmathsci.org

International Journal of
Fuzzy Mathematical
Archive

Complement of Type-2 Fuzzy Shortest Path Using Possibility Measure

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Received 2 November 2014; accepted 12 November 2014

Abstract. This paper presents a complement fuzzy shortest path problem on a network by possibility measure using extension principle. A proposed algorithm gives the complement shortest path and complement shortest path length. Complement type-2 fuzzy number have been taken for each edge which have been assigned by type-2 fuzzy number. An illustrative example also included to demonstrate our proposed algorithm.

Keywords: Type-2 fuzzy set, complement type-2 fuzzy set, possibility measure, type-2 fuzzy number, extension principle

AMS mathematics Subject Classification (2010): 94D05

1. Introduction

In graph theory, the shortest path problem is the problem of finding a path between source node to destination node on a network. It has applications in various fields like transportation, communication, routing and scheduling. In real world problem the arc length of the network may represent the time or cost which is not stable in the entire situation, hence it can be considered to be a fuzzy set.

The fuzzy shortest path problem was first analyzed by Dubois and Prade [4]. Okada and Soper [11] developed an algorithm based on the multiple labeling approach, by which a number of nondominated paths can be generated. Type-2 fuzzy set was introduced by Zadeh [15] as an extension of the concept of an ordinary fuzzy set. Many researchers have studied on shortest path in fuzzy network [10], and one may refer the Book by Klir et al. [5], operations on type-2 fuzzy sets [1,6,7,8,9].The type-2 fuzzy logic has gained much attention recently due to its ability to handle uncertainty, and many advances appeared in both theory and applications. A complement of type-2 fuzzy set is also considered as a type-2 fuzzy set having complement for fuzzy membership function only.

The fuzzy measures was introduced by sugeno [13], Good surveys of various types of measures subsumed under this broad concept were prepared by Dubois and Prade [3], Bacon [2] and Wierzchon [14]. There were many researches using different approaches, but we will mainly focus on an approach based on possibility theory proposed by Okada [12].

This paper is organized as follows: In Section 2, we have some basic concepts required for analysis. Section 3, gives an algorithm to find the complement shortest path and shortest path length with complement type-2 fuzzy number. Section 4 gives the network terminology. To illustrate the proposed algorithm the numerical example is solved in section 5. Finally in section 6, conclusion is included.

2. Preliminaries

2.1. Type-2 fuzzy set

A Type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$ where x ∈ X and u ∈ $J_x \subseteq [0,1]$.

ie., $\tilde{A}$ = { ((x,u), $\mu_{\tilde{A}}(x,u)$) / ∀ x ∈ X, ∀ u ∈ $J_x \subseteq [0,1]$ } in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$. $\tilde{A}$ can be expressed as $\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$ $J_x \subseteq [0,1]$, where $\int\int$ denotes union over all admissible x and u. For discrete universe of discourse $\int$ is replaced by $\sum$.

2.2. Complement of type-2 fuzzy set

The complement of the type-2 fuzzy set having a fuzzy membership function given in the following formula.

$$\bar{\tilde{A}} = \sum_{x\in X}\left[\sum_{u\in J_x} f_x(u)/(1-u)\right]/x$$

2.3. Type-2 fuzzy number

Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. $\tilde{A}$ is normal,
2. $\tilde{A}$ is a convex set,
3. The support of $\tilde{A}$ is closed and bounded, then $\tilde{A}$ is called a type-2 fuzzy number.

2.4. Discrete type-2 fuzzy number

The discrete type-2 fuzzy number $\tilde{A}$ can be defined as follows:

$\tilde{A} = \sum_{x \in X} \mu_{\tilde{A}}(x) / x$ where $\mu_{\tilde{A}}(x) = \sum_{u \in J_x} f_x(u) / u$ whereJ_x is the primary membership.

2.5. Extension principle

Let $A_1, A_2, \ldots, A_r$ be type-1 fuzzy sets in $X_1, X_2, \ldots, X_r$, respectively. Then, Zadeh's Extension Principle allows us to induce from the type-1 fuzzy sets $A_1, A_2, \ldots, A_r$ a type-1 fuzzy set B on Y, through f, i.e,$B = f(A_1, \ldots, A_r)$, such that

$$\mu_B(y) = \begin{cases} \sup\limits_{x_1, x_2, \ldots x_n \in f^{-1}(y)} \min\{\mu_{A_1}(x_1), \ldots, \mu_{A_n}(x_n) & iff^{-1}(y) \neq \phi \\ 0, & f^{-1}(y) = \phi \end{cases}$$

2.6. Addition on type-2 fuzzy numbers

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number be $\tilde{A} = \sum \mu_{\tilde{A}}(x) / x$ and $\tilde{B} = \sum \mu_{\tilde{B}}(y) / y$ where $\mu_{\tilde{A}}(x) = \sum f_x(u) / u$ and $\mu_{\tilde{B}}(x) = \sum g_y(w) / w$. The addition of these two types-2 fuzzy numbers $\tilde{A} \oplus \tilde{B}$ is defined as

$$\mu_{\tilde{A} \oplus \tilde{B}}(z) = \bigcup_{z=x+y} (\mu_{\tilde{A}}(x) \bigcap \mu_{\tilde{B}}(y))$$

$$= \bigcup_{z=x+y} ((\sum_i f_x(u_i) / u_i) \bigcap (\sum_j g_y(w_j) / w_j))$$

$$\mu_{\tilde{A} \oplus \tilde{B}}(z) == \bigcup_{z=x+y} ((\sum_{i,j} (f_x(u_i) \wedge g_y(w_j)) / (u_i \wedge w_j))$$

2.7. Relative possibility

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then the relative possibility for av($\tilde{A}$) = x and av($\tilde{B}$)= y can be defined as

$$s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y) = \sum \sum u.f(u).w.g(w)$$

where $av(\tilde{A})$ is the actual value of fuzzy set $\tilde{A}$ and $\tilde{A} = \sum f_x(u) / u / x$ and $\tilde{B} = \sum g_y(w) / w / y$.

2.8. Possibility measure

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then the possibility that $\tilde{A} < \tilde{B}$ is defined as

$$s(\tilde{A} < \tilde{B}) = \frac{\sum_{(x,y)\in\Omega(\tilde{A}<\tilde{B})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}{\sum_{(x,y)\in\Omega(\mathrm{T})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}$$

where $\Omega(\tilde{A} < \tilde{B})$ is the set of all possible situations where $\mathrm{av}(\tilde{A}) < av(\tilde{B})$.

$\Omega(\mathrm{T}) = \Omega(\tilde{\mathrm{A}} < \tilde{\mathrm{B}}) + \Omega(\tilde{\mathrm{A}} = \tilde{\mathrm{B}}) + \Omega(\tilde{\mathrm{A}} > \tilde{\mathrm{B}})$.

Similarly

$$s(\tilde{A} > \tilde{B}) = \frac{\sum_{(x,y)\in\Omega(\tilde{A}>\tilde{B})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}{\sum_{(x,y)\in\Omega(\mathrm{T})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}$$ and

$$s(\tilde{A} = \tilde{B}) = \frac{\sum_{(x,y)\in\Omega(\tilde{A}=\tilde{B})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}{\sum_{(x,y)\in\Omega(\mathrm{T})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}.$$

2.9. Degree of possibility

The Degree of possibility D_p for the path P is defined as

$$D_p = \min_{p^1 \in p}\{s(p_i \le p_j) + s(p_i = p_j)\}.$$

3. Algorithm

Step 1: Complement type-2 fuzzy number

Form the complement type-2 fuzzy number using definition 2.2 for each edge.

Step 2: Computation of possible paths

Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\overline{\tilde{L}_i}$ i = 1,2,. . . .n for possible n paths.

Step 3: Computation of relative possibilities

$\overline{\tilde{A}} = \sum \mu_{\overline{\tilde{A}}}(x)/\mathrm{x}$ and $\overline{\tilde{B}} = \sum \mu_{\overline{\tilde{B}}}(\mathrm{y})/\mathrm{y}$

Compute relative possibility $s\ (av(\overline{\tilde{p}}_i(x_k)) \wedge av(\overline{\tilde{p}}_j(\mathrm{x}_k)))$ for all situations where x_k is the actual value of the path $\overline{\tilde{p}_i}$ and j = i + 1.

Step 4: Computation of possibility measure

i) i = 1

ii) j = i+1

iii) Compute $s_{ij}(\overline{\tilde{p}_i} < \overline{\tilde{p}_j})$ using definition 2.7

iv) Write $s_{ij}(\overline{\tilde{p}_i} < \overline{\tilde{p}_j})$

v) Put j = j +1

vi) If j≤ n then go to (iii)

vii) Put i = i + 1

viii) If i<n then go to (ii)

ix) Compute the possibility of $s_{ij}(\overline{\tilde{p}_i} = \overline{\tilde{p}_j})$

Step 5: Computation of degree of possibility

$D(\overline{\tilde{p}_i}) = \min\{s_{ij}(\overline{\tilde{p}_i} < \overline{\tilde{p}_j}) + (1/2)s_{ij}(\overline{\tilde{p}_i} = \overline{\tilde{p}_j})\}$ where i = 1,2, . . .n and j = i+1.

Step 6: Shortest path and shortest path length

The path which is having highest degree of possibility is the Shortest path and the corresponding path length is the Shortest path Length for the complement type-2 fuzzy number.

4. Network technology

Consider a directed network G(V,E) consisting of a finite set of nodes V = {1,2, . . .n} and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i,j), where i,j$\in$ V and i$\neq$j. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence $P_{ij} = \{i = i_1, (i_1,i_2), i_2, \ldots, i_{l-1}, (i_{l-1},i_l), i_l = j\}$ of alternating nodes and edges. The existence of at least one path P_{si} in G(V,E) is assumed for every node i$\in$ V – {s}. $\tilde{d}_{ij}$ denotes a Type-2 Fuzzy Number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i,j\in P)} \tilde{d}_{ij}$

5. Numerical example

The problem is to find the shortest path and shortest path length between source node and destination node in the network having 6 vertices and 7edges with complement type-2 fuzzy number.

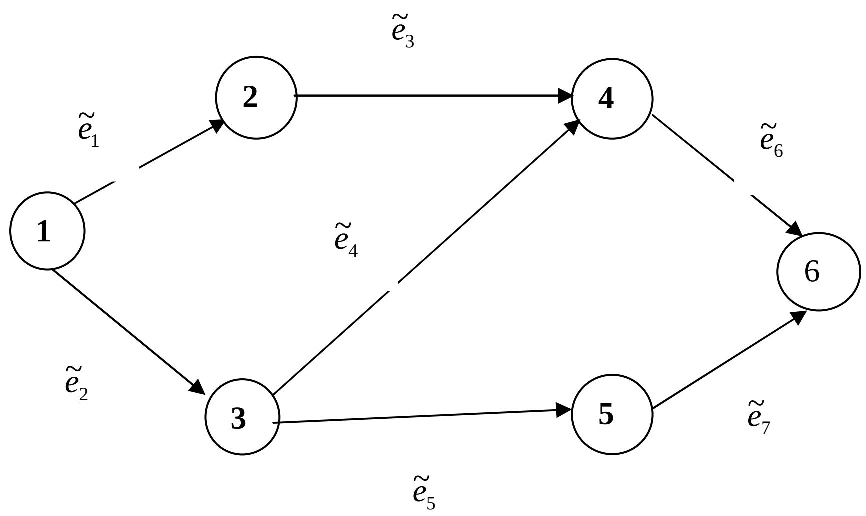

Figure 5.1:

Solution:

The edge Lengths are

$\tilde{e}_1$ = (0.5/0.2+0.4/0.3)/2 + (0.4/0.2)/3

$\tilde{e}_2$ = (0.3/0.2 + 0.8/0.3)/1 + (0.2/0.8)/3

$\tilde{e}_3$ = (0.7/0.2)/2 + (0.9/0.4 +0.7/0.5)/4

$\tilde{e}_4$ = (0.6/0.2)/4

$\tilde{e}_5$ = (0.9/0.4 +0.7/0.5)/3 + (0.4/0.7)/5

$\tilde{e}_6$ = (0.8/0.3 + 0.4/0.5)/2

$\tilde{e}_7$ = (0.6/0.4)/2 +(0.7/0.5+ 0.5/0.6)/4

Step 1: Complement type-2 fuzzy number

The complement of type-2 fuzzy numbers are

$\overline{\tilde{e}_1}$ = (0.5/0.8+0.4/0.7)/2 + (0.4/0.8)/3

$\overline{\tilde{e}_2}$ = (0.3/0.8 + 0.8/0.7)/1 + (0.2/0.2)/3

$\overline{\tilde{e}_3}$ = (0.7/0.8)/2 + (0.9/0.6 +0.7/0.5)/4

$\overline{\tilde{e}_4}$ = (0.6/0.8)/4

$\overline{\tilde{e}_5}$ = (0.9/0.6 +0.7/0.5)/3 + (0.4/0.3)/5

$\overline{\tilde{e}_6}$ = (0.8/0.7 + 0.4/0.5)/2

$\overline{\tilde{e}_7}$ = (0.6/0.6)/2 +(0.7/0.5+ 0.5/0.4)/4

Step 2: Computation of Possible Paths

Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\overline{\tilde{L}_i}$ i = 1,2,. . . .n for possible n paths.

$\tilde{p}_1$ = $\overline{\tilde{e}_1}$ + $\overline{\tilde{e}_3}$ + $\overline{\tilde{e}_6}$ = 1 – 2- 4- 6

$\tilde{p}_2$ = $\overline{\tilde{e}_2}$ + $\overline{\tilde{e}_4}$ + $\overline{\tilde{e}_6}$ =1 – 3- 4- 6

$\tilde{p}_3$ = $\overline{\tilde{e}_2}$ + $\overline{\tilde{e}_5}$ + $\overline{\tilde{e}_7}$ =1 – 3- 5- 6

$\overline{\tilde{L}_1}$ = (0.5/0.7+0.4/0.5)/6+(0.4/0.7+0.4/0.5)/7+ (0.5 /0.6 +0.5/0.5)/8 + (0.4/0.6+0.4/0.5)/9

$\overline{\tilde{L}_2}$ = (0.6/0.7+0.4/0.5)/7 + (0.2/0.2)/9

$\overline{\tilde{L}_3}$ = (0.6/0.6 + 0.6/0.5)/6 + (0.2/0.5+0.2/0.4)/8 + (0.2/0.3)/10 + (0.2/0.2)/12

Step 3: Computation of Relative Possibilities

$\overline{\tilde{A}} = \sum \mu_{\tilde{A}}(x)$ /x and

$$\bar{\tilde{B}} = \sum \mu_{\tilde{B}}(\mathrm{y})/\mathrm{y}$$

Compute relative possibility $s\ (av(\overline{\tilde{p}}_i(x_k)) \wedge av(\overline{\tilde{p}}_j(\mathrm{x}_k)))$ for all situations where x_k is the actual value of the path $\overline{\tilde{p}_i}$ and j = i + 1.

$s(\overline{\tilde{p}_1} = 6 \wedge \overline{\tilde{p}_2} = 7) = 0.341$	$s(\overline{\tilde{p}_1} = 6 \wedge \overline{\tilde{p}_2} = 9) = 0.022$
$s(\overline{\tilde{p}_1} = 6 \wedge \overline{\tilde{p}_3} = 6) = 0.363$	$s(\overline{\tilde{p}_1} = 6 \wedge \overline{\tilde{p}_3} = 8) = 0.099$
$s(\overline{\tilde{p}_1} = 6 \wedge \overline{\tilde{p}_3} = 10) = 0.033$	$s(\overline{\tilde{p}_1} = 6 \wedge \overline{\tilde{p}_3} = 12) = 0.022$
$s(\overline{\tilde{p}_1} = 7 \wedge \overline{\tilde{p}_2} = 7) = 0.2976$	$s(\overline{\tilde{p}_1} = 7 \wedge \overline{\tilde{p}_2} = 9) = 0.0192$
$s(\overline{\tilde{p}_1} = 7 \wedge \overline{\tilde{p}_3} = 6) = 0.3168$	$s(\overline{\tilde{p}_1} = 7 \wedge \overline{\tilde{p}_3} = 8) = 0.0864$
$s(\overline{\tilde{p}_1} = 7 \wedge \overline{\tilde{p}_3} = 10) = 0.0288$	$s(\overline{\tilde{p}_1} = 7 \wedge \overline{\tilde{p}_3} = 12) = 0.0192$
$s(\overline{\tilde{p}_1} = 8 \wedge \overline{\tilde{p}_2} = 7) = 0.341$	$s(\overline{\tilde{p}_1} = 8 \wedge \overline{\tilde{p}_2} = 9) = 0.022$
$s(\overline{\tilde{p}_1} = 8 \wedge \overline{\tilde{p}_3} = 6) = 0.33$	$s(\overline{\tilde{p}_1} = 8 \wedge \overline{\tilde{p}_3} = 8) = 0.099$
$s(\overline{\tilde{p}_1} = 8 \wedge \overline{\tilde{p}_3} = 10) = 0.028$	$s(\overline{\tilde{p}_1} = 8 \wedge \overline{\tilde{p}_3} = 12) = 0.022$
$s(\overline{\tilde{p}_1} = 9 \wedge \overline{\tilde{p}_2} = 7) = 0.2728$	$s(\overline{\tilde{p}_1} = 9 \wedge \overline{\tilde{p}_2} = 9) = 0.0176$
$s(\overline{\tilde{p}_1} = 9 \wedge \overline{\tilde{p}_3} = 6) = 0.2904$	$s(\overline{\tilde{p}_1} = 9 \wedge \overline{\tilde{p}_3} = 8) = 0.0792$
$s(\overline{\tilde{p}_1} = 9 \wedge \overline{\tilde{p}_3} = 10) = 0.0264$	$s(\overline{\tilde{p}_1} = 9 \wedge \overline{\tilde{p}_3} = 12) = 0.0176$
$s(\overline{\tilde{p}_2} = 7 \wedge \overline{\tilde{p}_3} = 6) = 0.4092$	$s(\overline{\tilde{p}_2} = 7 \wedge \overline{\tilde{p}_3} = 8) = 0.1116$
$s(\overline{\tilde{p}_2} = 7 \wedge \overline{\tilde{p}_3} = 10) = 0.0372$	$s(\overline{\tilde{p}_2} = 7 \wedge \overline{\tilde{p}_3} = 12) = 0.0248$
$s(\overline{\tilde{p}_2} = 9 \wedge \overline{\tilde{p}_3} = 6) = 0.0264$	$s(\overline{\tilde{p}_2} = 9 \wedge \overline{\tilde{p}_3} = 8) = 0.0072$
$s(\overline{\tilde{p}_2} = 9 \wedge \overline{\tilde{p}_3} = 10) = 0.0024$	$s(\overline{\tilde{p}_2} = 9 \wedge \overline{\tilde{p}_3} = 12) = 0.0016$

Step 4: Computation of Possibility Measure

i) i = 1
ii) j = i+1
iii) Compute $s_{ij}(\overline{\tilde{p}_i} < \overline{\tilde{p}_j})$ using definition 2.7
iv) Write $s_{ij}(\overline{\tilde{p}_i} < \overline{\tilde{p}_j})$
v) Put j = j +1
vi) If j≤ n then go to (iii)
vii) Put i = i + 1
viii) If i<n then go to (ii)
ix) Compute the possibility of $s_{ij}(\overline{\tilde{p}_i} = \overline{\tilde{p}_j})$

$s_{12}(\overline{\tilde{p}_1} < \overline{\tilde{p}_2})$ = 0.1059 $s_{13}(\overline{\tilde{p}_1} < \overline{\tilde{p}_3})$ = 0.1003

$s_{12}(\overline{\tilde{p}_1} = \overline{\tilde{p}_2})$ = 0.0826 $s_{13}(\overline{\tilde{p}_1} = \overline{\tilde{p}_3})$ = 0.12112

$s_{21}(\overline{\tilde{p}_2} < \overline{\tilde{p}_1})$ = 0.1609 $s_{21}(\overline{\tilde{p}_2} = \overline{\tilde{p}_1})$ = 0.0826

$s_{23}(\overline{\tilde{p}_2} < \overline{\tilde{p}_3})$ = 0.0466 $s_{23}(\overline{\tilde{p}_2} = \overline{\tilde{p}_3})$ = 0

$s_{31}(\overline{\tilde{p}_3} < \overline{\tilde{p}_1})$ =0.2665 $s_{32}(\overline{\tilde{p}_3} < \overline{\tilde{p}_2})$ = 0.11609

Step 5: Computation of Degree of Possibility

$D(\overline{\tilde{p}_i}) = \min\{s_{ij}(\overline{\tilde{p}_i} < \overline{\tilde{p}_j}) + (1/2)s_{ij}(\overline{\tilde{p}_i} = \overline{\tilde{p}_j})\}$ where i = 1,2, . . .n and j = i+1.

$D(\overline{\tilde{p}_1})$ = min{0.1472, 0.1609} = 0.1472

$D(\overline{\tilde{p}_2})$ = min{ 0.2022, 0.0466} = 0.0466

$D(\overline{\tilde{p}_3})$ = min{0.32706, 0.11609} = 0.11609

Step 6: Shortest Path and Shortest Path Length

The path which is having highest degree of possibility is the Shortest path and the corresponding path length is the Shortest path Length for the complement type-2 fuzzy number.

Since highest degree is $\overline{\tilde{p}_1}$ ie.,0.1472.

We conclude that the Shortest Path is 1 – 2 – 4 – 6 and the shortest length is

$\overline{\tilde{L}_1}$ =(0.5/0.7+0.4/0.5)/6+(0.4/0.7+0.4/0.5)/7+(0.5/0.6+0.5/0.5)/8+(0.4/0.6+0.4/0.5)/9

6. Conclusion

In this paper we have developed an algorithm for finding complement shortest path and shortest path length using possibility measure with complement type-2 fuzzy number. The order of getting the degree of possibility for possible paths for the given edge weights in a network is getting reversed while we use the complement of the edge weights in a same network. We conclude that the complement shortest path is not at all a shortest path for the given edge weight but it is the shortest path for complement of given type-2 fuzzy number.

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