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Implication Operator on Intuitionistic Fuzzy Tautological Matrix

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Abstract. In this paper, we study some properties of intuitionistic fuzzy tautological matrix using intuitionistic fuzzy implication operator.

Keywords: Intuitionistic fuzzy matrix, intuitionistic fuzzy set, intuitionistic fuzzy implication operator and intuitionistic fuzzy tautological matrix

AMS Mathematics Subject Classification (2010): 03E72,15B15

1. Introduction

A phenomenal set theory named and introduced by Zadeh[20] as fuzzy set theory is the most developed one in the last five decades. An interesting extension is Intuitionistic Fuzzy set theory by Atanassov [2]; Atanassov interpret the Zadeh's fuzzy set as, if X is a non empty set, an Intuitionistic Fuzzy Set(IFS) A in X(universal set) is defined as an object of the following form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, where the functions: $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ define the membership function and non membership function of the element $x \in X$ respectively and for every $x \in X: 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Xu, Yager[19] represents $\langle \mu_A(x), \nu_A(x)\rangle$ as Intuitionistic Fuzzy Values with $\mu_A(x) + \nu_A(x) \leq 1$. An IFS A is an Intuitionistic fuzzy tautological set[3] if and only if for every $x \in X: \mu_A(x) = 1, \nu_A(x) = 0$ holds. For simplicity we consider the pair $\langle x, x'\rangle$ as membership and non membership function of an IFS with $x + x' \leq 1$. Also we can interpret an element in IFS in classical way as (1,0), one being the membership degree and zero being the non membership degree; and an element does not belong to the IFS when the membership degree is zero and the non membership degree is one, (0,1). A fuzzy implication **I**, is a function of the form **I**: $[0,1]^2 \to [0,1]$, which for any possible truth values x,y of the given fuzzy proposition p,q respectively, defines the truth value, **I**(x,y) of the conditional proposition "if q then p". This function should be an extension of the classical implication, p $\to$ q , from the restricted domain (0,1) to the full domain [0,1] of truth values in fuzzy logic. This can be extended in Intuitionistic Fuzzy sense, when the propositions p,q of the conditional if p then q are intuitionistic fuzzy. That is, when each one of them is defined by two values, where the first indicates the degree of the truth of the proposition and the second the degree of non truth. The Intuitionistic Fuzzy truth value is $\langle 1,0\rangle$ and non truth value is $\langle 0,1\rangle$. This implication operator must be an extension of fuzzy implication in the sense of Fodor and Roubens [7]. Atanassov and

Gargov [4] and later Cornlis and Deschrijver [5,6] gave the following definition of intuitionistic fuzzy implication operator.

An Intuitionistic Fuzzy Implication is any $I_I: D^2 \to D$ mapping satisfying the border conditions: $I_I\big((0,1),(0,1)\big) = (1,0), I_I\big((0,1),(1,0)\big) = (1,0), I_I\big((1,0),(1,0)\big) = (1,0)$ $I_I\big((1,0),(0,1)\big) = (0,1)$ and the two following conditions: 1. If $(x,y) \leq (x',y')$ then $I_I\big((x,y),(z,t)\big) \geq I_I\Big(\big(x',y'\big),(z,t)\Big)$ for all $(z,t) \in D$. 2. If $(z,t) \leq (z'.t')$ then $I_I\big((x,y),(z,t)\big) \leq I_I\Big(\big(x',y'\big),(z,t)\Big)$ for all $(z,t) \in D$. In classical logic theory a formula φ(for variables $p,q: p \wedge q, p \vee q, p \to q, p \leftrightarrow q$ are formulae) is said to be a tautology if φ has the truth value true. In this paper, we study some properties which gives intuitionistic fuzzy truth values for intuitionistic fuzzy formula φ (if we treat p,q as Intuitionistic fuzzy proposition). Hasimoto [9] used Godel implication operator in fuzzy matrix theory and obtained results in sub-inverse of fuzzy matrix using fuzzy relational equation. After the generalization of fuzzy theory by Atanassov [2] as intuitionistic fuzzy set theory, Im et al. [10] extended it to intuitionistic fuzzy matrix. Meenakshi and Gandhimathi [12], Sriram and Murugadas [16,17,18], Pal, Khan and Shyamal [11] and Shyamal and Pal [21-25] developed this intuitionistic fuzzy matrix in all fields such as g-inverse, intuitionistic fuzzy linear equation, intuitionistic fuzzy linear transformation etc. Sriram and Murugadas [16] extended the concepts of Implication operator to IFM and discussed several properties like sub-inverse, semi-inverse and necessary and sufficient condition for the existence of g-inverse using the implication operator. Hashimoto [8] traced the fuzzy relation under dual operations. The authors in [13] introduced hook implication operator $\hookleftarrow$ for IFS as well as IFM, discussed the relation with $\leftarrow$ implication operator and obtained maximum solution (minimum solution) for the inequality A$\times X \times B \leq C (A \circ B \circ C \geq C)$using max-min (min-max) product. Further the authors in [14,15] defined bi-implication operator for IFS, extended it to IFM, its relation with IFIO and obtained sub inverses and g-inverses of an IFM.

2. Preliminaries

Throughout this section $\langle x,x'\rangle \vee \langle y,y'\rangle$ or $\langle x,x'\rangle + \langle y,y'\rangle$ means maximum of $\langle x,x'\rangle$ and $\langle y,y'\rangle$ (component wise addition) and $\langle x,x'\rangle \wedge \langle y,y'\rangle$ or $\langle x,x'\rangle\langle y,y'\rangle$ means minimum of $\langle x,x'\rangle$ and $\langle y,y'\rangle$ (component wise multiplication).

Definition 2.1. [17] For $(x,x'),(y,y') \in$ IFS, define
$\langle x,x'\rangle \vee \langle y,y'\rangle = \langle \max\{x,y\}, \min\{x',y'\}\rangle$
$\langle x,x'\rangle \wedge \langle y,y'\rangle = \langle \min\{x,y\}, \max\{x',y'\}\rangle, \langle x,x'\rangle^c = \langle x',x\rangle$. We can also use +(component wise addition) for $\vee$ and $\langle x,x'\rangle \wedge \langle y,y'\rangle = \langle x,x'\rangle\langle y,y'\rangle$ (component wise multiplication).

Definition 2.2. [17] Let $X = \{x_1, x_2, \ldots. x_m\}$ be an Universal set and $Y = \{y_1, y_2 \ldots. y_n\}$ be the attribute set of each element of X. An Intuitionistic Fuzzy Matrix (IFM) is defined by $A = \big(\langle (x_i,y_j), \mu_A(x_i,y_j), \nu_A(x_i,y_j)\rangle\big)$ for I = 1,2,…m and j= 1,2…n, where $\mu_A: X \times Y \to [0,1]$ satisfy the condition $0 \leq \mu_A(x_i,y_j) + \nu_A(x_i,y_j) \leq 1$. For simplicity we denote an Intuitionistic Fuzzy Matrix(IFM) is a matrix of pairs $A = \big(\langle a_{ij}, a'_{ij}\rangle\big)$ of non negative real numbers satisfying $a_{ij} + a_{ij}' \leq 1$ for all I,j. We denote the set of all IFMs of order $m \times n$ by $\mathcal{F}_{mn}$. For any two elements $A = \left(\langle a_{ij}, a_{ij}^{'}\rangle\right), B = \left(\langle b_{ij}, b_{ij}^{'}\rangle\right) \in \mathcal{F}_{mn}$, define $A \vee B = \left(\langle a_{ij} \vee b_{ij}, a_{ij}^{'} \wedge b_{ij}^{'}\rangle\right) = A \oplus B$, (Component wise addition), $A \wedge B = \left(\langle a_{ij} \wedge b_{ij}, a_{ij}^{'} \vee b_{ij}^{'}\rangle\right) = A \odot B$,(component wise multiplication) for all $1 \leq i \leq m$ and $1 \leq j \leq n$. Here $a_{ij} \vee b_{ij} = a_{ij} + b_{ij}$(maximum of a_{ij} and b_{ij}) and $a_{ij} \wedge b_{ij} = a_{ij} b_{ij}$ (minimum of a_{ij} and b_{ij}). Denote $J = (\langle 1,0\rangle)$ the Universal matrix (matrix in which all entries are $\langle 1,0\rangle$) and $O = \left(\langle \delta_{ij}, \delta_{ij}^{'}\rangle\right)$ where $\langle \delta_{ij}, \delta_{ij}^{'}\rangle = \langle 1,0\rangle$ if $i = j$ and $\langle \delta_{ij}, \delta_{ij}^{'}\rangle = \langle 0,1\rangle$ if $i \neq j$, the Zero matrix. $\bar{A} = \left(\langle a_{ij}^{'}, a_{ij}\rangle\right)$. Adak et al. [1] proved that generalized IFM forms a distributive lattice using this component wise addition and component wise multiplication.

Definition 2.3. [16] Let A and B are two IFMs of same order and for $\langle x, x'\rangle, \langle y, y'\rangle \in$ IFS define $\langle x, x'\rangle \leftarrow \langle y, y'\rangle = \begin{cases} \langle 1,0\rangle \; if \; \langle x, x'\rangle \geq \langle y, y'\rangle \\ \langle x, x'\rangle \; if \; \langle x, x'\rangle < \langle y, y'\rangle \end{cases}$
$A \leftarrow B = \left(\langle a_{ij}, a_{ij}^{'}\rangle \leftarrow \langle b_{ij}, b_{ij}^{'}\rangle\right)$.

Definition 2.4. An Intuitionistic fuzzy matrix is called an Intuitionistic fuzzy tautological matrix (IFTM) if and only if $a_{ij} \geq a_{ij}^{'}$ for all i, j.

Definition 2.5. An Intuitionistic fuzzy matrix is called an Intuitionistic fuzzy co-tautological matrix(IFCTM) if and only if $a_{ij} \leq a_{ij}^{'}$ for all i, j.

3. Results on IFTMs

Throughout this section the matrices used are of compatible order for the use of implication operator.

Theorem 3.1. Let A and B be two IFMs then the following expressions are IFTMs
$(i) A \rightarrow A$
$(ii) \bar{A} \rightarrow A (if\ A$ is an IFTM)
$(iii) A \rightarrow (B \rightarrow A)$

Proof:

(i) Let $A = \left(\langle a_{ij}, a_{ij}^{'}\rangle\right)$.
As $\langle a_{ij}, a_{ij}^{'}\rangle = \langle a_{ij}, a_{ij}^{'}\rangle$ for all i, j, $\left(\langle a_{ij}, a_{ij}^{'}\rangle\right) \rightarrow \left(\langle a_{ij}, a_{ij}^{'}\rangle\right) = (\langle 1,0\rangle)$
So, $A \rightarrow A$ is an IFTM.

(ii) Let $A = \left(\langle a_{ij}, a_{ij}^{'}\rangle\right)$. $\bar{A} = \left(\langle a_{ij}^{'}, a_{ij}\rangle\right)$ then $\bar{A} \rightarrow A = \left(\langle a_{ij}^{'}, a_{ij}\rangle\right) \rightarrow \left(\langle a_{ij}, a_{ij}^{'}\rangle\right) = (\langle 1,0\rangle)$. Since A is a IFTM $a_{ij} \geq a_{ij}^{'}, \langle a_{ij}, a_{ij}^{'}\rangle \geq \langle a_{ij}^{'}, a_{ij}\rangle$.

(iii) Let $A = \left(\langle a_{ij}, a_{ij}^{'}\rangle\right), B = \left(\langle b_{ij}, b_{ij}^{'}\rangle\right)$, $B \rightarrow A = (\langle b_{ij}, b_{ij}^{'}\rangle) \rightarrow (\langle a_{ij}, a_{ij}^{'}\rangle)$.
Case(i): If $\langle b_{ij}, b_{ij}^{'}\rangle \leq \langle a_{ij}, a_{ij}^{'}\rangle$ for all i, j, then $\langle b_{ij}, b_{ij}^{'}\rangle \rightarrow \langle a_{ij}, a_{ij}^{'}\rangle = \langle 1,0\rangle$
$B \rightarrow A = (\langle 1,0\rangle)$
$A \rightarrow (B \rightarrow A) = A \rightarrow (\langle 1,0\rangle) = \left(\langle a_{ij}, a_{ij}^{'}\rangle\right) \rightarrow (\langle 1,0\rangle) = (\langle 1,0\rangle)$
Case(ii): $\langle b_{ij}, b_{ij}^{'}\rangle > \langle a_{ij}, a_{ij}^{'}\rangle$ for all i, j, then $\langle b_{ij}, b_{ij}^{'}\rangle \rightarrow \langle a_{ij}, a_{ij}^{'}\rangle = \langle a_{ij}, a_{ij}^{'}\rangle$.

$B \to A = \left(\langle a_{ij}, a'_{ij} \rangle \right)$
$A \to (B \to A) = \left(\langle a_{ij}, a'_{ij} \rangle \right) \to \left(\langle a_{ij}, a'_{ij} \rangle \right) = (\langle 1,0 \rangle)$.
$Thus\ A \to (B \to A)$ is an IFTM.

Theorem 3.2. If A and B be two IFMS then
$(i) A \to (A \vee B)$
$(ii) B \to (A \vee B)$
$(iii) A \to (B \to (A \wedge B))$ are IFTMS.
Proof:

(i) $A \vee B = \left(\langle a_{ij}, a'_{ij} \rangle \right) \vee \left(\langle b_{ij}, b'_{ij} \rangle \right)$
Case (i): $\langle a_{ij}, a'_{ij} \rangle \leq \langle b_{ij}, b'_{ij} \rangle$ for all i, j, $A \to (A \vee B) = A \to \left(\langle b_{ij}, b'_{ij} \rangle \right) = \left(\langle a_{ij}, a'_{ij} \rangle \right) \to \left(\langle b_{ij}, b'_{ij} \rangle \right) = (\langle 1,0 \rangle)$.
Case (ii): $\langle a_{ij}, a'_{ij} \rangle > \langle b_{ij}, b'_{ij} \rangle$ for all i, j, $A \to (A \vee B) = A \to \left(\langle a_{ij}, a'_{ij} \rangle \right) = \left(\langle a_{ij}, a'_{ij} \rangle \right) \to \left(\langle a_{ij}, a'_{ij} \rangle \right) = (\langle 1,0 \rangle)$.
So $A \to (A \vee B)$ is an IFTM.

(ii) $B \to (A \vee B)$
Case (i): If $\langle a_{ij}, a'_{ij} \rangle \leq \langle b_{ij}, b'_{ij} \rangle$ for all i, j, $B \to (A \vee B) = B \to \left(\langle b_{ij}, b'_{ij} \rangle \right) = \left(\langle b_{ij}, b'_{ij} \rangle \right) \to \left(\langle b_{ij}, b'_{ij} \rangle \right) = (\langle 1,0 \rangle)$.
Case (ii): $\langle a_{ij}, a'_{ij} \rangle > \langle b_{ij}, b'_{ij} \rangle$ for all i, j, $B \to (A \vee B) = \left(\langle b_{ij}, b'_{ij} \rangle \right) \to \left(\langle a_{ij}, a'_{ij} \rangle \right) = (\langle 1,0 \rangle)$.
So, $B \to (A \vee B)$ is an IFTM.

(iii) $A \to (B \to (A \wedge B))$
$A \wedge B = \left(\langle a_{ij}, a'_{ij} \rangle \right) \wedge \left(\langle b_{ij}, b'_{ij} \rangle \right)$.
Case (i): If $\langle a_{ij}, a'_{ij} \rangle \leq \langle b_{ij}, b'_{ij} \rangle$ for all i, j, $\left(B \to (A \wedge B) \right) = B \to \left(\langle a_{ij}, a'_{ij} \rangle \right) = \left(\langle b_{ij}, b'_{ij} \rangle \right) \to \left(\langle a_{ij}, a'_{ij} \rangle \right) = \left(\langle a_{ij}, a'_{ij} \rangle \right)$.
$A \to \left(B \to (A \wedge B) \right) = A \to \left(\langle a_{ij}, a'_{ij} \rangle \right) = (\langle 1,0 \rangle)$
Case (ii): $\langle a_{ij}, a'_{ij} \rangle > \langle b_{ij}, b'_{ij} \rangle$ for all i, j, $\left(B \to (A \wedge B) \right) = B \to \left(\langle b_{ij}, b'_{ij} \rangle \right) = (\langle 1,0 \rangle)$
$A \to \left(B \to (A \wedge B) \right) = A \to (\langle 1,0 \rangle) = (\langle 1,0 \rangle)$.
$so, A \to \left(B \to (A \wedge B) \right)$ is an IFTM.

Theorem 3.3. If A and B are IFMs then

(i) $A \wedge B \to A$
(ii) $A \wedge B \to B$ are IFTMs

Proof:

(i) $A \wedge B = \left(\langle a_{ij}, a'_{ij} \rangle \right) \wedge \left(\langle b_{ij}, b'_{ij} \rangle \right)$
Case (i): If $\langle a_{ij}, a'_{ij} \rangle \leq \langle b_{ij}, b'_{ij} \rangle$ for all i, j, $A \wedge B \to A = \left(\langle a_{ij}, a'_{ij} \rangle \right) \to \left(\langle a_{ij}, a'_{ij} \rangle \right) = (\langle 1,0 \rangle)$.
Case (ii): $\langle a_{ij}, a'_{ij} \rangle > \langle b_{ij}, b'_{ij} \rangle$ for all i, j, $A \wedge B \to A = \left(\langle b_{ij}, b'_{ij} \rangle \right) \to \left(\langle a_{ij}, a'_{ij} \rangle \right) = (\langle 1,0 \rangle)$.

Hence $A \wedge B \to A$ is an IFTM.

(ii) Case(i): If $\langle a_{ij}, a_{ij}^{'} \rangle \leq \langle b_{ij}, b_{ij}^{'} \rangle$ for all i, j, $A \wedge B \to B = \left(\langle a_{ij}, a_{ij}^{'} \rangle\right) \to \left(\langle b_{ij}, b_{ij}^{'} \rangle\right) = (\langle 1,0 \rangle)$.

Case (ii): $\langle a_{ij}, a_{ij}^{'} \rangle > \langle b_{ij}, b_{ij}^{'} \rangle$ for all i, j, $A \wedge B \to B = \left(\langle b_{ij}, b_{ij}^{'} \rangle\right) \to \left(\langle b_{ij}, b_{ij}^{'} \rangle\right) = (\langle 1,0 \rangle)$

Hence $A \wedge B \to B$ is an IFTM.

Theorem 3.4: For any three IFMS A, B, C the following expressions are IFTMS.

(i) $(A \to C) \to ((B \to C) \to ((A \vee B) \to C))$

(ii) $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$

(iii) $(\bar{A} \to B) \to ((\bar{A} \to B) \to A)$ if A is a tautological matrix.

Proof:

(i) Case(i) $A \leq B$.

Sub case (i) $A \leq B \leq C$

$A \to C = \left(\langle a_{ij}, a'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = (\langle 1,0 \rangle)$

$B \to C = \left(\langle b_{ij}, b'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = (\langle 1,0 \rangle)$

$(A \vee B) \to C = \left(\langle b_{ij}, b'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = (\langle 1,0 \rangle)$

$(A \to C) \to \Big((B \to C) \to \big((A \vee B) \to C\big)\Big)$

$= (\langle 1,0 \rangle) \to \big((\langle 1,0 \rangle) \to (\langle 1,0 \rangle)\big) = (\langle 1,0 \rangle)$.

Sub case (ii) $A \leq C < B$

$A \to C = \left(\langle a_{ij}, a'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = (\langle 1,0 \rangle), B \to C = \left(\langle c_{ij}, c'_{ij} \rangle\right)$

$(A \vee B) \to C = \left(\langle b_{ij}, b'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = \left(\langle c_{ij}, c'_{ij} \rangle\right)$

$(A \to C) \to \Big((B \to C) \to \big((A \vee B) \to C\big)\Big)$

$= (\langle 1,0 \rangle) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = (\langle 1,0 \rangle)$

Sub case (iii): $C < A \leq B$

$A \to C = \left(\langle a_{ij}, a'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = \left(\langle c_{ij}, c'_{ij} \rangle\right), B \to C = \left(\langle c_{ij}, c'_{ij} \rangle\right)$

$(A \vee B) \to C = \left(\langle b_{ij}, b'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = \left(\langle c_{ij}, c'_{ij} \rangle\right)$

$(A \to C) \to \Big((B \to C) \to \big((A \vee B) \to C\big)\Big)$

$= \left(\langle c_{ij}, c'_{ij} \rangle\right) \to \Big(\left(\langle c_{ij}, c'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right)\Big) = (\langle 1,0 \rangle)$

In all the above three sub cases the expression (i) is a tautology.

Similarly we can prove when $A > B > C, A > C > B$ and $C > A > B$.

(ii) Case (i): $A \leq B$

Sub case (i): $A \leq B \leq C$

$(B \to C) = (\langle 1,0 \rangle), A \to B = (\langle 1,0 \rangle), A \to C = (\langle 1,0 \rangle)$

$A \to (B \to C) = \left(\langle a_{ij}, a'_{ij} \rangle\right) \to (\langle 1,0 \rangle) = (\langle 1,0 \rangle)$

$(A \to B) \to (A \to C) = (\langle 1,0 \rangle)$

$\left(A \to (B \to C)\right) \to \left((A \to B) \to (A \to C)\right) = (\langle 1,0 \rangle) \to (\langle 1,0 \rangle) = (\langle 1,0 \rangle)$.

Sub case (ii) $A \leq C \leq B$

$(B \to C) = \left(\langle c_{ij}, c'_{ij} \rangle\right), \ A \to B = (\langle 1,0 \rangle), A \to C = (\langle 1,0 \rangle)$

$A \to (B \to C) = \left(\langle a_{ij}, a'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = (\langle 1,0 \rangle)$

$(A \to B) \to (A \to C) = (\langle 1,0 \rangle) = (\langle 1,0 \rangle)$

Therefore $\left(A \to (B \to C)\right) \to \left((A \to B) \to (A \to C)\right)$

$= (\langle 1,0 \rangle) \to (\langle 1,0 \rangle) = (\langle 1,0 \rangle)$.

Sub case (iii): $C < A \leq B$

$(B \to C) = \left(\langle c_{ij}, c'_{ij} \rangle\right), \ A \to B = (\langle 1,0 \rangle), A \to C = \left(\langle c_{ij}, c'_{ij} \rangle\right)$.

$A \to (B \to C) = \left(\langle a_{ij}, a'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = \left(\langle c_{ij}, c'_{ij} \rangle\right)$.

$(A \to B) \to (A \to C) = (\langle 1,0 \rangle) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = \left(\langle c_{ij}, c'_{ij} \rangle\right)$.

$\left(A \to (B \to C)\right) \to \left((A \to B) \to (A \to C)\right)$

$= \left(\langle c_{ij}, c'_{ij} \rangle\right) \to \left(\langle c_{ij}, c'_{ij} \rangle\right) = (\langle 1,0 \rangle)$

Similarly we can prove when $A > B > C, A > C > B$ and $C > A > B$.

(iii) Let $\bar{A} = \left(\langle a'_{ij}, a_{ij} \rangle\right)$. Since A is tautological matrix, $a_{ij} \geq a'_{ij}$.

Case (i): $A \leq B$, that is $\langle a_{ij}, a'_{ij} \rangle \leq \langle b_{ij}, b'_{ij} \rangle$ for all i, j, also, $\bar{A} \leq B$.

$(\bar{A} \to B) = \left(\langle a'_{ij}, a_{ij} \rangle\right) \to \left(\langle b_{ij}, b'_{ij} \rangle\right) = (\langle 1,0 \rangle)$

$((\bar{A} \to B) \to A) = (\langle 1,0 \rangle) \to \left(\langle a'_{ij}, a_{ij} \rangle\right) = \left(\langle a'_{ij}, a_{ij} \rangle\right)$

$Hence\ (\bar{A} \to B) \to ((\bar{A} \to B) \to A) = (\langle 1,0 \rangle) \to \left(\langle a'_{ij}, a_{ij} \rangle\right) = \left(\langle a'_{ij}, a_{ij} \rangle\right)$

Since A is a tautological matrix, the above expression is a tautology.

$Case\ (ii)$: $If\ A > B, \langle a_{ij}, a'_{ij} \rangle > \langle b_{ij}, b'_{ij} \rangle$ for all i, j, $(\bar{A} \to B) = \left(\langle b_{ij}, b'_{ij} \rangle\right)$

$((\bar{A} \to B) \to A) = \left(\langle b_{ij}, b'_{ij} \rangle\right) \to \left(\langle a'_{ij}, a_{ij} \rangle\right) = (\langle 1,0 \rangle)$

$(\bar{A} \to B) \to ((\bar{A} \to B) \to A) = \left(\langle b_{ij}, b'_{ij} \rangle\right) \to (\langle 1,0 \rangle) = (\langle 1,0 \rangle)$.

$So, (\bar{A} \to B) \to ((\bar{A} \to B) \to A)$ is an IFTM.

Theorem 3.5: If A, B are IFMs and $A < B$ then $(A \to B)$ is an IFTM.

Proof:

Given $A < B$, then $(A \to B) = (\langle 1,0 \rangle)$. So $(A \to B)$ is an IFTM.

Theorem 3.6: If A, B are IFMs then the following expressions are IFTMs:

(i) $(A\wedge(A\to B))\to B$.

(ii) $A\to\big((A\leftarrow B)\vee B\big)$.

(iii) $B\to\big((A\to B)\vee A\big)$.

(iv) $(A\to C)\to\big((A\to B)\vee(B\to C)\big)$.

(v) $B\to\big((A\vee B)\to B\big)$.

(vi) $(A\to C)\to\Big((B\to C)\to\big((A\wedge B)\to C\big)\Big)$.

Proof:

(i) Case (i): If $A\le B$

$$A\to B=(\langle 1,0\rangle),(A\wedge(A\to B))\to B=$$
$$\Big(\big(\langle a'_{ij},a_{ij}\rangle\big)\wedge(\langle 1,0\rangle)\Big)\to\big(\langle b_{ij},b'_{ij}\rangle\big)$$
$$=\big(\langle a'_{ij},a_{ij}\rangle\big)\to\big(\langle b_{ij},b'_{ij}\rangle\big)=(\langle 1,0\rangle).$$

Case (ii): If $A>B$

$(A\wedge(A\to B))\to B=(A\wedge B)\to B=B\to B=(\langle 1,0\rangle)$.

So $(A\wedge(A\to B))\to B$ is an IFTM.

(ii) Case (i): If $A<B$

$A\to\big((A\leftarrow B)\vee B\big)$.$=$ A $\to(A\vee B)=A\to B=(\langle 1,0\rangle)$.

Case (ii): If $A\ge B$

$A\to\big((A\leftarrow B)\vee B\big)$.$=A\to\big((\langle 1,0\rangle)\vee B\big)=A\to(\langle 1,0\rangle)=(\langle 1,0\rangle)$.

(iii) Case (i): If $A\le B$,

$B\to\big((A\to B)\vee A\big)=B\to\big((\langle 1,0\rangle)\vee B\big)=B\to(\langle 1,0\rangle)=(\langle 1,0\rangle)$.

$Case(ii)$: If $A>B$, $B\to\big((A\to B)\vee A\big)=B\to(B\vee B)=B\to B=(\langle 1,0\rangle)$

(iv) Case (i): If $A\le B\le C$,

$(A\to C)\to\big((A\to B)\vee(B\to C)\big)=(\langle 1,0\rangle)\to\big((\langle 1,0\rangle)\vee(\langle 1,0\rangle)\big)=(\langle 1,0\rangle)$.

$Case\ (ii)$: If $A\le C\le B$,

$(A\to C)\to\big((A\to B)\vee(B\to C)\big)=(\langle 1,0\rangle)\to\big((\langle 1,0\rangle)\vee C\big)=(\langle 1,0\rangle)$.

$Case(iii)$: If $C<A\le B$,

$(A\to C)\to\big((A\to B)\vee(B\to C)\big)=C\to\big((\langle 1,0\rangle)\vee C\big)=(\langle 1,0\rangle)$.

$Similarly$ we can prove when $A>B>C, A>C>B$ and $C>A>B$.

(v) Case (i): If $A\ge B, B\to\big((A\vee B)\to B\big).=B\to(A\to B)=B\to B=(\langle 1,0\rangle)$.

Case(ii): If $A<B, B\to\big((A\vee B)\to B\big).=B\to(\langle 1,0\rangle)=(\langle 1,0\rangle)$.

(vi) Case (i): If $A\le B\le C$,

$(A\to C)\to\Big((B\to C)\to\big((A\wedge B)\to C\big)\Big)=(\langle 1,0\rangle)\to((\langle 1,0\rangle)\to$
$(A\to C))=(\langle 1,0\rangle)\to((\langle 1,0\rangle)\to\langle 1,0\rangle)=(\langle 1,0\rangle)$

Case (ii): If $A \leq C \leq B$

$$(A \to C) \to \Big((B \to C) \to \big((A \wedge B) \to C\big)\Big) = (\langle 1,0 \rangle) \to \big(C \to (A \to C)\big)$$

$$= (\langle 1,0 \rangle) \to \big(C \to (\langle 1,0 \rangle)\big) = (\langle 1,0 \rangle) \to (\langle 1,0 \rangle) = (\langle 1,0 \rangle)$$

Case (iii): If $C < A \leq B$

$(A \to C) \to \Big((B \to C) \to \big((A \wedge B) \to C\big)\Big) = C \to (C \to C) = (\langle 1,0 \rangle) \to ((\langle 1,0 \rangle) \to (\langle 1,0 \rangle)) = (\langle 1,0 \rangle)$.

$Similarly$ we can prove when $A > B > C, A > C > B$ and $C > A > B$.

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