Intern. J. Fuzzy Mathematical Archive
Vol. 5, No. 2, 2014, 103-111
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 26 December 2014
www.researchmathsci.org

International Journal of
Fuzzy Mathematical
Archive

New Class of rg*b-Continuous Functions in Topological Spaces

***G.Sindhu*[2] and *K.Indirani*[2]**

[1]Department of Mathematics with CA, Nirmala College for Women, Coimbatore
Tamil Nadu, India
[2]Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu
India. E-mail: sindhukannan23@gmail.com

Received 1 November 2014; accepted 20 December 2014

Abstract. In this paper, a new class of functions called rg*b-totally continuous functions and totally rg*b-continuous functions are introduced and their properties are studied. Further the relationship between this new class with other classes of existing functions are established.

Keywords: rg*b-totally continuous functions and totally rg*b-continuous functions

AMS Mathematics Subject Classification (2010): 14P25

1. Introduction

Levine [7] and Jain [6] introduced strongly continuous and totally continuous functions respectively. Nour [9] developed the notion of totally semi continuous functions. Further Caldas et al [3] introduced the concept of Totally b-continuous functions. The purpose of the present paper is to introduce a new class of continuous functions called rg*b-totally continuous functions and totally rg*b-continuous functions and investigate some of their fundamental properties. Relationship between this new class and other classes of functions are also established.

2. Preliminaries

Throughout this paper (X,τ) and (Y,σ) represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. (X,τ) will be replaced by X if there is no chance of confusion. We denote the family of all rg*b-closed sets in X by RG*BC(X,τ).

Let us recall the following definitions which we shall require later.

Definition 2.1. A subset A of a space (X, τ) is called

1) a regular open set[11] if A= int (cl(A)) and a regular closed set if A= cl(int (A))
2) a b-open set [2] if $A\subset$ cl(int(A))$\cup$int (cl(A)).
3) a regular generalized closed set (briefly, rg-closed)[10] if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.

4) a generalized b-closed (briefly gb-closed)[1] if $bcl(A) \subset U$ whenever $A \subset U$ and U is open.
5) a regular generalized b-closed set (briefly rgb-closed) [8] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
6) a regular generalized star b- closed set (briefly rg*b-closed set)[4] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is rg-open in X.

Definition 2.2. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called
1) b-irresolute: [8] if for each b-open set V in Y,$f^{-1}(V)$ is b-open in X;
2) b-continuous: [8] if for each open set V in Y,$f^{-1}(V)$ is b-open in X.
3) totally continuous [6] if $f^{-1}(V)$ is clopen set in X for each open set V of Y .
4) totally b-continuous [3] at each point of X if for each open subset V in Y containing f(x), there exists a b-clopen subset U in X containing x such that $f(U) \subset V$.
5) rg*b-continuous [5] if $f^{-1}(V)$ is rg*b Closed in X for every closed set V in Y.
6) rg*b-irresolute [5] if the inverse image of each rg*b Closed set in Y is a rg*b Closed set in X .
7) rg*b-closed [5], if the image of each closed set in X is a rg*b Closed set in Y.
7) rg*b-open [5], if the image of each open set in X is a rg*b open in Y.

Definition 2.3: A space (X,τ) is called
1) an rg*b-space[4] if every rg*b-closed set is closed.
2) a T_{rg*b}-space[4] if every rg*b-closed set is b-closed.

3. rg*b-totally continuous functions

In this section, a new generalization of strong continuity called rg*b-totally continuity which is stronger than totally continuity is defined. Furthermore, the basic properties of these functions are discussed.

Definition 3.1. A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is said to be a rg*b-totally continuous function if the inverse image of every rg*b-open set of Y is clopen in X.

Theorem 3.2. A bijective function $f : (X,\tau) \rightarrow (Y,\sigma)$ is a rg*b-totally continuous function if and only if the inverse image of every rg*b-closed subset of Y is clopen in X.
Proof: Let F be any rg*b-closed set in Y. Then $Y \setminus F$ is a rg*b-open set in Y. By definition $f^{-1}(Y \setminus F)$ is clopen in X. That is, $X \setminus f^{-1}(F)$ is clopen in X. This implies $f^{-1}(F)$ is clopen in X. Conversely if V is rg*b-open in Y, then $Y \setminus V$ is rg*b-closed in Y. By assumption, $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is clopen in X ,which implies $f^{-1}(V)$ is clopen in X. Therefore f is rg*b-totally continuous function.

Theorem 3.3. (i) Every rg*b-totally continuous function is totally continuous.
(ii) Every rg*b-totally continuous function is rg*b-continuous.
(iii) Every totally continuous function is rg*b-continuous.
Proof: (i) Let U be any open subset of Y. Since every open set is rg*b-open, U is rg*b-open in Y and $f : (X,\tau) \rightarrow (Y,\sigma)$ is rg*b-totally continuous, it follows that $f^{-1}(U)$ is clopen in X.

Proof is obvious for (ii) and (iii).

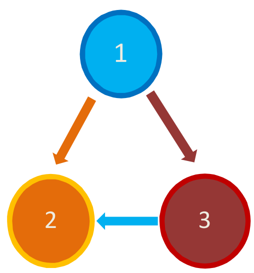
Remark 3.4. The converse of Theorem 3.3 is not true, which can be verified from the following examples.

Example 3.5. (i) Let X= Y= {a,b,c},τ = {ϕ, {a},{b},{a,b},{b,c},X} and σ = {ϕ, {a},Y}. Let f: (X,τ) $\rightarrow$ (Y,σ) be an identity map. Then τ-closed sets are {ϕ, {a},{c},{a,c},{b,c}, X} and RG*BO(Y, σ) ={ϕ,{a},{a,b},{a,c},Y}. Then f is totally continuous but not rg*b-totally continuous, since f^{-1}({a,b}) ={a,b} is not closed in (X,τ).

(ii) Let X = Y = {a,b,c}, τ = {ϕ, {a},{b},{a,b}, X} and σ = {ϕ, {a},Y}. Let f: (X,τ) $\rightarrow$ (Y,σ) be an identity map. Then τ-closed sets are {ϕ,{c},{a,c},{b,c}, X} and σ-closed sets are {ϕ,{b,c}, Y}. Also RG*BC(X,τ) = {ϕ,{a},{b},{c},{a,c},{b,c}, X} and RG*BO(Y, σ) = {ϕ,{a},{a,b}, {a,c},Y}. Then f is rg*b-continuous but not totally continuous and rg*b-totally continuous, since f^{-1}({a}) = {a} is not closed in (X,τ).

Remark 3.6. From the above discussions the following implication diagram is obtained. The numbers 1-3 represents the following continuities.

1. rg^*b-totally continuous 2. rg^*b-continuous 3. totally continuous.



Theorem 3.7. Let f : (X,τ) $\rightarrow$ (Y,σ) be a function, where X and Y are topological spaces. Then the following are equivalent:

1. f is rg*b-totally continuous.
2. For each x $\in$ X and each rg*b-open set V in Y with f(x) $\in$ V, there is a clopen set U in X such that x $\in$ U and f(U) $\subset$ V.

Proof: (1) $\Rightarrow$ (2): Suppose f is rg*b-totally continuous and V be any rg*b-open set in Y containing f(x) such that x $\in$ f^{-1}(V). Since f is rg*b-totally continuous, f^{-1}(V) is clopen in X. Let U = f^{-1}(V) then U is a clopen set in X and x $\in$ U. Also f(U) = f(f^{-1}(V)) $\subset$ V. This implies f(U) $\subset$ V.

(2) $\Rightarrow$ (1) : Let V be a rg*b-open set in Y. Let x$\in$ f^{-1}(V) be any arbitrary point. This implies f(x) $\in$ V. Therefore by (2) there is a clopen set G_x containing x such that f(G_x) $\subset$ V , which implies $G_x \subset f^{-1}$(V) is a clopen neighbourhood of x. Since x is arbitrary, it

implies $f^{-1}(V)$ is a clopen neighbourhood of each of its points. Hence it is a clopen set in X. Therefore f is rg*b-totally continuous.

Theorem 3.8. A function $f : (X,\tau) \to (Y,\sigma)$ is rg*b-totally continuous, if its graph function is rg*b-totally continuous.

Proof: Let $g: X \to X\times Y$ be a graph function of $f : X \to Y$ .Suppose g is rg*b–totally continuous and F be rg*b-open in Y, then $X\times F$ is a rg*b-open set of $X\times Y$. Since g is rg*b-totally continuous, $g^{-1}(X\times F) = f^{-1}(F)$ is clopen in X. Thus the inverse image of every rg*b-open set in Y is clopen in X. Therefore f is rg*b-totally continuous.

Theorem 3.9. If $f : (X,\tau) \to (Y,\sigma)$ is rg*b-totally continuous surjection and X is connected then Y is rg*b-connected.

Proof: Suppose Y is not rg*b-connected, let A and B form a disconnection of Y. Then A and B are rg*b-open sets in Y and $Y = A \cup B$ where $A \cap B = \phi$. Also $f^{-1}(Y) = X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non empty clopen sets in X, because f is rg*b-totally continuous. Further, $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(\phi) = \phi$. This implies X is not connected, which is a contradiction. Hence Y is rg*b-connected.

Theorem 3.10. If $f : (X,\tau) \to (Y,\sigma)$ is rg*b-totally continuous (totally continuous), injective, rg*b-open function from clopen regular space X on a space Y, then Y is rg*bc-regular (rg*b-regular).

Proof: Let F be a rg*b-closed (closed) set in Y and $y \notin F$. Take $y = f(x)$. Since f is rg*b-totally continuous (totally continuous) $f^{-1}(F)$ is clopen in X. Let $G = f^{-1}(F)$, then we have $x \notin G$. Since X is a clopen regular space, there exists disjoint open sets U and V such that $G \subset U$ and $x \in V$. This implies $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$. Further, since f is injective and rg*b-open, $f(U \cap V) = f(\phi) = \phi$, where f(U) and f(V) are rg*b-open in Y. Therefore Y is rg*bc-regular (rg*b-regular).

Theorem 3.11. If $f : (X,\tau) \to (Y,\sigma)$ is rg*b-totally continuous, rg*b-closed (closed) injection, and if Y is rg*bc-regular (rg*b-regular) then X is ultra regular.

Proof: Let F be a closed set not containing x. Since f is rg*b-closed (closed) f(F) is rg*b-closed (closed) in Y not containing f(x). Since Y is rg*bc-regular (rg*b-regular), there exists disjoint rg*b-open sets A and B such that $f(X) \in A$ and $f(F) \subset B$ which implies $x \in f^{-1}(A)$ and $F \subset f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are clopen sets because f is rg*b-totally continuous. Moreover, since f is injective, $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(\phi) = \phi$. Thus for a pair of points and a closed set not containing the points, they can be separated by clopen sets. Therefore X is ultra regular.

Theorem 3.12. If a function $f : (X,\tau) \to (Y,\sigma)$ is totally continuous and Y is a rg*b-space then f is rg*b-totally continuous.

Proof: Let V be rg*b-open in Y. Since Y is a rg*b-space, V is open in Y. Also as f is totally continuous, $f^{-1}(V)$ is open and closed in X. Hence $f^{-1}(V)$ is clopen in X. Therefore f is rg*b-totally continuous.

Theorem 3.13. (i) If $f : X \to Y$ and $g: Y \to Z$ are rg*b-totally continuous, then $g\circ f : X \to Z$ is also rg*b-totally continuous.

(ii) If $f : X \rightarrow Y$ is rg*b-totally continuous and $g: Y \rightarrow Z$ is rg*b-continuous, then $g \circ f : X \rightarrow Z$ is totally continuous.

Proof: Straightforward.

Theorem 3.14. Let $f: X \rightarrow Y$ be a rg*b-open map and $g: Y \rightarrow Z$ be any function. If $g \circ f: X \rightarrow Z$ is rg*b-totally continuous, then g is rg*b-irresolute.

Proof: Let $g \circ f : X \rightarrow Z$ be rg*b-totally continuous. Let V be rg*b-open set in Z. Since $g \circ f$ is rg*b-totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in X. Since f is rg*b-open, $f(f^{-1}(g^{-1}(V)))$ is rg*b-open in Y. Then $g^{-1}(V)$ is rg*b-open in Y. Hence g is rg*b-irresolute.

Theorem 3.15. Let $f : X \rightarrow Y$ be rg*b-totally continuous and $g: Y \rightarrow Z$ be any function, then $g \circ f : X \rightarrow Z$ is rg*b-totally continuous if and only if g is rg*b-irresolute.

Proof: Let V be a rg*b-open subset of Z. Then $g^{-1}(V)$ is rg*b-open in Y as g is rg*b-irresolute. Then $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is clopen in X. Hence $g \circ f : X \rightarrow Z$ is rg*b-totally continuous. Conversely, let $g \circ f : X \rightarrow Z$ be rg*b-totally continuous. Let V be a rg*b-open set in Z, then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in X. Since f is rg*b-totally continuous, $g^{-1}(V)$ is rg*b-open in Y. Hence g is rg*b-irresolute.

4 Totally rg*b-continuous functions

In this section, the concepts of totally rg*b-continuity is introduced and characterized. Also some of the properties of the separation axioms, by utilizing totally rg*b-continuity are studied.

Definition 4.1. A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is called

(i) totally rg*b-continuous at a point $x \in X$ if for each open subset V in Y containing $f(x)$, there exists a rg*b-clopen subset U in X containing x such that $f(U) \subset V$.

(ii) totally rg*b-continuous if it has this property at each point of X.

Theorem 4.2. The following statements are equivalent for a function $f : (X,\tau) \rightarrow (Y,\sigma)$, whenever the class of rg*b-closed sets in (X,τ) are closed under finite union:

(i) f is totally rg*b-continuous.

(ii) For every open set V of Y, $f^{-1}(V)$ is rg*b-clopen in X.

Proof: (i) $\Rightarrow$ (ii): Let V be an open subset of Y and let $x \in f^{-1}(V)$. Since $f(x) \in V$, by (i), there exists a rg*b-clopen set U_x in X containing x such that $U_x \subset f^{-1}(V)$. We obtain $f^{-1}(V) = \cup \{U_x : x \in f^{-1}(V)\}$. Thus $f^{-1}(V)$ is rg*b-clopen in X.

(ii) $\Rightarrow$ (i): Straightforward.

Definition 4.3. A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is said to be strongly $(rg*b)^*$-continuous if the inverse image of every rg*b-open set of (Y,σ) is rg*b-clopen in (X,τ).

Theorem 4.4. (i) Every strongly $(rg*b)^*$-continuous function is totally rg*b-continuous.

(ii) Every totally rg*b-continuous function is rg*b-continuous.

(iii) Every totally continuous function is totally rg*b-continuous.

(iv) Every rg*b-totally continuous function is totally rg*b-continuous.

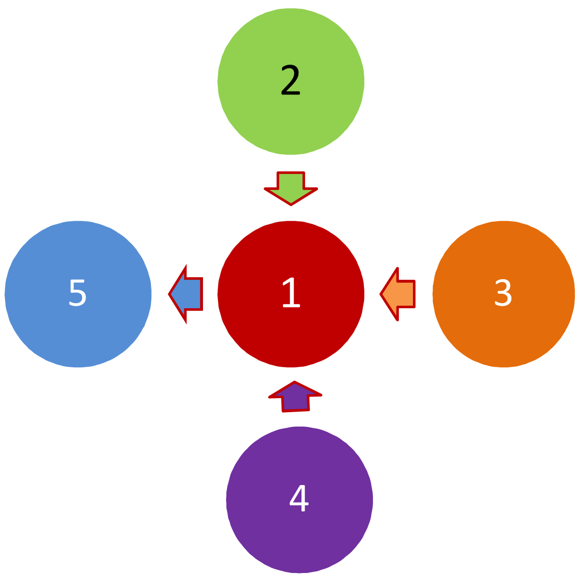
Proof: (i) Let V be an open set in Y. Then V is rg*b-open in Y. Then $f^{-1}(V)$ is rg*b-clopen in X as f is a strongly $(rg*b)^*$-continuous function. Hence f is totally rg*b-continuous.
Proof is obvious for (ii) to (iv).

Remark 4.5. The converse of Theorem 4.4 is not true, which can be verified from the following examples.

Example 4.6. (i) In Example 3.5 (ii), f is a totally rg*b-continuous and a rg*b-continuous function. But it is not a strongly $(rg*b)^*$-continuous, totally continuous and rg*b-totally continuous function.

(ii) Let X = Y = {a,b,c}, τ = {ϕ, {a},{a,b},{a,c}, X} and σ = {ϕ,{a},Y}. Let f: $(X,\tau) \to (Y,\sigma)$ be an identity map. Then τ-closed sets are {ϕ,{b},{c},{b,c}, X} and σ-closed sets are {ϕ,{b,c}, Y}. Also RG*BC(X,τ) ={ϕ,{b},{c},{b,c},X} and RG*BO(X,τ) ={ϕ,{a},{a,b}, {a,c},Y}. Then f is rg*b-continuous but not totally rg*b-continuous, since $f^{-1}(\{a\})$ = {a} is not rg*b-closed in (X,τ).

Remark 4.7. From the above discussions the following implication diagram is obtained. The numbers 1-5 represents the following continuities.
1. totally rg^*b-continuous 2. strongly $(rg^*b)^*$-continuous 3. totally continuous 4. rg^*b-totally continuous 5. rg^*b-continuous.



Theorem 4.8. If f : $(X,\tau) \to (Y,\sigma)$ is a totally rg*b-continuous map from a rg*b-connected space (X,τ) onto a space (Y,σ), then (Y,σ) is an indiscrete space.
Proof: On the contrary, suppose that (Y,σ) is not an indiscrete space. Let A be a proper non-empty open subset of (Y,σ). Since f is totally rg*b-continuous map, then $f^{-1}(A)$ is a

proper non-empty rg*b-clopen subset of X. Then $X = f^{-1}(A) \cup (X \setminus f^{-1}(A))$ which is a contradiction to the fact that X is rg*b-connected. Therefore Y must be an indiscrete space.

Theorem 4.9. Let $f : (X,\tau) \to (Y,\sigma)$ be a totally rg*b-continuous map and Y be a T_1 space. If A is a non-empty subset of a rg*b-connected space X, then f(A) is singleton.

Proof: Suppose if possible f(A) is not singleton, let $f(x_1) = y_1 \in A$ and $f(x_2) = y_2 \in A$. Since $y_1, y_2 \in Y$ and Y is a T_1 space, then there exists an open set G in (Y,σ) containing y_1 but not y_2. Since f is totally rg*b-continuous, $f^{-1}(G)$ is a rg*b-clopen set containing x_1 but not x_2. Now $X = f^{-1}(G) \cup (X \setminus f^{-1}(G))$. Thus X is a union of two non-empty rg*b-open sets which is a contradiction.

Definition 4.10. Let X be a topological space and $x \in X$. Then the set of all points y in X such that x and y cannot be separated by rg*b-separation of X is said to be the quasi rg*b-component of X.

Theorem 4.11. Let $f : (X,\tau) \to (Y,\sigma)$ be a totally rg*b-continuous function from a topological space (X,τ) into a T_1 space (Y,σ) .Then f is constant on each quasi rg*b-component of X.

Proof: Let x and y be two points of X that lie in the same quasi rg*b-component of X. Assume that $f(x) = \alpha \neq \beta = f(y)$. Since Y is a T_1 space, $\{\alpha\}$ is closed in Y and so $Y \setminus \{\alpha\}$is an open set. Since f is totally rg*b-continuous, $f^{-1}\{\alpha\}$ and $f^{-1}\{Y \setminus \{\alpha\}\}$ are disjoint rg*b-clopen subsets of X. Further $x \in f^{-1}\{\alpha\}$ and $y \in f^{-1}\{Y \setminus \{\alpha\}\}$ which is a contradiction to the fact that y belongs to the quasi rg*b-component of X and hence y must belong to every rg*b-open set containing x.

Definition 4.12. A space (X,τ) is said to be

(i) rg*b-co-T_1 if for each pair of disjoint points x and y of X, there exists rg*b-clopen sets U and V containing x and y, respectively such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.

(ii) rg*b-co-T_2 if for each pair of disjoint points x and y of X, there exists rg*b-clopen sets U and V in X, respectively such that $x \in U$ and $y \in V$.

(iii) rg*b-co-regular if for each rg*b-clopen set F and each point $x \notin F$, there exists disjoint open sets U and V such that $F \subset U$ and $x \in V$.

(iv) rg*b-co-normal if for any pair of disjoint rg*b-clopen subsets F_1 and F_2 of X, there exists disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

(v) rg*b-co-Hausdorff if every two distinct points of X can be separated by disjoint rg*b-clopen sets.

Theorem 4.13. If $f : (X,\tau) \to (Y,\sigma)$ is totally rg*b-continuous injective function and Y is a T_1 space, then X is rg*b-co-T_1.

Proof: Since Y is T_1, for any distinct points x and y in X, there exists open sets V,W in Y such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is totally rg*b-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are rg*b-clopen subsets of (X,τ) such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is rg*b-co-T_1.

Theorem 4.14. If f : $(X,\tau) \to (Y,\sigma)$ is totally rg*b-continuous injective function and Y is a T_2-space, then X is rg*b-co-T_2.
Proof: For any distinct points x and y in X, there exists disjoint open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$ and $U \cap V = \phi$. Since f is totally rg*b-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are rg*b-clopen in X containing x and y respectively. Therefore $f^{-1}(U) \cap f^{-1}(V) = \phi$ because $U \cap V = \phi$. This shows that X is rg*b-co-T_2.

Theorem 4.15. If f : $(X,\tau) \to (Y,\sigma)$ is totally rg*b-continuous injective open function from a rg*b-co-normal space X onto a space Y, then Y is normal.
Proof: Let F_1 and F_2 be disjoint closed subsets of Y. Since f is totally rg*b-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are rg*b-clopen sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \phi$. Since X is rg*b-co-normal, there exists disjoint open sets A and B such that $U \subset A$ and $V \subset B$. We obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that f(A) and f(B) are disjoint open sets. Thus, Y is normal.

Theorem 4.16. If f : $(X,\tau) \to (Y,\sigma)$ is totally rg*b-continuous injective open function from a rg*b-co-regular space X onto a space Y, then Y is regular.
Proof: Let F be closed set in Y and $y \notin F$. Take $y = f(x)$. Since f is totally rg*b-continuous,
$f^{-1}(F)$ is a rg*b-clopen set. Take $G = f^{-1}(F)$, we have $x \notin G$. Since X is a rg*b-co-regular space there exists disjoint open sets U and V such that $G \subset U$ and $x \in V$. We obtain $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$ such that f(U) and f(V) are disjoint open sets. This shows that Y is regular.

Theorem 4.17. Let f : $(X,\tau) \to (Y,\sigma)$ be a totally rg*b-continuous injective function. If Y is hausdorff, then X is rg*b-co-Hausdorff .
Proof: Let x_1 and x_2 be two distinct points of X. Since f is injective and Y is Hausdorff, there exists open sets V_1 and V_2 in Y such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$. By Theorem 3.2, $x_i \in f^{-1}(V_i) \in$ rg*b-clopen(X) for i=1,2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus X is rg*b-co-Hausdorff.

Definition 4.18. A space X is said to be
1. rg*b-co-compact if every rg*b-clopen cover of X has a finite subcover.
2. rg*b-co-compact relative to X if every cover of a rg*b-clopen set of X has a finite sub cover.
3. countably rg*b-co-compact if every countable cover of X by rg*b-clopen sets has a finite subcover.
4. rg*b-co-Lindelof if every rg*b-clopen cover of X has a countable subcover.

Theorem 4.19. Let f : $(X,\tau) \to (Y,\sigma)$ be a totally rg*b-continuous surjective function. Then the following statements hold.
1. If X is rg*b-co-Lindelof then Y is Lindelof.
2. If X is countably rg*b-co-compact then Y is countably compact.
3. If X is rg*b-co-compact then Y is compact.
4. If X is countably rg*b-co-compact then Y is countably compact.

Proof: Let $\{V_\alpha : \alpha \in I\}$ be an open cover of Y. Since f is totally rg*b-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a rg*b-clopen cover of X. Since X is rg*b-co-Lindelof, there exists a countable subset I_0 of I such that $X = \cup\{ V_\alpha : \alpha \in I_0\}$ and hence Y is Lindelof.
Proof of 2 to 4 is similar.

REFERENCES

1. A.Al-Omari and M.SM. Noorani, On generalized b-closed sets. *Bull.Malays. Math. Sci. Soc.*, 32(1) (2009) 19-30
2. D.Andrijevic, b-open sets, *Mat.Vesink*, 48 (1996) 59-64.
3. M.Caldas, S.Jafari and N.Rajesh, Properties of totally b-continuous functions, ANALELE S¸TIINT¸ IFICE ALE UNIVERSITAT¸ II “AL.I. CUZA” DIN IAS¸I (S.N.) ˘MATEMATICA, Tomul LV, (2009), 119-130.
4. K.Indirani and G.Sindhu, On regular generalized star b closed sets, *IJMA*, 4 (10) (2013) 1-8.
5. K.Indirani and G.Sindhu, On regular generalized star b-closed functions, *Proceedings of NARC*, (2013) 74-78.
6. R.C.Jain, The role of regularly open sets in general topology, Ph.D. Thesis, Meerut University, Institute of Advanced Studies, Meerut, India, 1980.
7. N.Levine, Strong continuity in topological spaces, *Amer. Math. Monthly,* 67 (1960) 269.
8. K.Mariappa and S.Sekar, On regular generalized b-closed set, *Int. Journal of Math. Analysis*, 7 (2013) 613-624.
9. T.M.Nour, Totally semi-continuous functions, *Indian J. Pure Appl. Math.*, 26(7) (1995) 300-304.
10. N.Palaniappan and K.C.Rao, Regular generalized closed sets, *Kyungpook Math. J.*, 33 (1993) 211-219.
11. M.H.Stone, Applications of the theory of Boolean rings to the general topology, *Trans.A.M.S.*, 41 (1937) 375-481.