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On L-R Type Interval Valued Fuzzy Numbers in Critical Path Analysis

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Abstract. In this paper, new arithmetic operations of L-R type interval valued fuzzy numbers are defined. A new ranking function and distance function are also defined with the aid of L-R type interval valued fuzzy numbers. Some more properties of both ranking and distance functions are also discussed. The proposed notion is applied in the domain of critical path analysis and a relevant numerical example of it is also included to justify the notion.

Keywords: Interval valued fuzzy numbers, l-r type interval valued fuzzy numbers, distance function, project network, critical path

AMS Mathematics Subject Classification (2010): 03E72, 90B99

1. Introduction

Critical path method is a network based method designed for planning and managing of complicated projects in real world applications.The main purpose of critical path method is to evaluating project performance and to identifying the critical activities on the critical path so that the available resources could be utilized on these activities in the project network in order to reduce project completion time. The use of fuzzy variables in PERT was proposed by Chanas and Kamburowski [2]. They presented the project completion time in the form of a fuzzy set in the time space. Lin [5] developed fuzzy network of a priori unknown project to estimate the activity duration and fuzzy algebraic operator to calculate the duration of the project and its critical path.The fuzzy networking was proposed by Nasution [8] and Lorterapong and Moselhi [6] following on this, Mccahon [7] Chang et al [3] and Lin and Yao [5] presented three methodologies to calculate fuzzy completion project time. Other resources such as Ravishankar [10] and oliveros and Rabinson [9] using fuzzy numbers presented other methods to obtain fuzzy critical paths, critical activities and activity delay. Previous work on network scheduling using fuzzy theory provides methods for scheduling projects. Chen and Huang [4], applied fuzzy method for measuring criticality in project network. Anusuya and Sathya [1] proposed complement of type-2 fuzzy shortest path using possibility measure. Stephen Dinagar and Abirami [11] proposed an analytical method for finding critical path using IVFNS in a fuzzy project network. In this paper, a new approach of ranking value of L-R type interval valued fuzzy numbers to fuzzy critical analysis are introduced. It is also assumed that the uncertain parameters are represented by IVFNS. An algorithm to tackle the problem in fuzzy project decision analysis is proposed. Finally an illustrative numerical example is given to demonstrate the validity of the proposed methods.

This paper is organized as follows:In section 1, Introduction is introductory in nature.In section 2, we introduce some basic definitions which are useful for our work.In section 3, the definition of L-R type interval valued fuzzy numbers is proposed with arithmetic operations.In section 4, the properties of L-R type interval valued fuzzy numbers have been discussed.A new distance function for L-R type interval valued fuzzy numbers is proposed in section 5.In section 6,an application part of this work have been included. Finally the conclusion part is also given.

2. Preliminaries

Definition 2.1. A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} \equiv \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$.Here $\mu_{\tilde{A}} : x \to [0,1]$ is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$.

Definition 2.2. The $\alpha -$level set(or interval of confidence at level α (or $\alpha -$cut) of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values in A greater than or equal to .i.e., $A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X, \alpha \in [0,1]\}$

Definition 2.3. A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics.

(i) $\tilde{A}$ is convex. i.e., $\tilde{A}(\lambda X_1 + (1-\lambda) X_2) = \min imum\{\tilde{A}(X_1), \tilde{A}(X_2)\}$, for all $X_2, X_1 \in R$ and $\lambda \in [0,1]$

(ii) $\tilde{A}$ is normal i.e., there exists an $X_0 \in R$ such that $\tilde{A}(X_0) = 1$

(iii) $\tilde{A}$ is piecewise continuous.

Definition 2.4. A fuzzy number $\tilde{A}$ is said to be a trapezoidal fuzzy number if its membership function $\mu_{\tilde{A}}(x) : R \to [0,1]$ has the following characteristics.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{(x-a_4)}{(a_3-a_4)}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

Definition 2.5. An IVFNS $\tilde{A}$ is called as zero – equivalent fuzzy number if R($\tilde{A}$)= 0and denoted by $\hat{0}$.

Definition 2.6. An IVFNS $\tilde{A}$ is called as zero fuzzy number if $\tilde{A}$ =[(0,0,0,0),(0,0,0,0)] and denoted by $\tilde{0}$

3. L-R type interval valued fuzzy numbers

Definition 3.1. The L-R type interval valued fuzzy number is of the form $\tilde{A}_{LR} = [(a_1^L, a_2^L, \alpha_1^L, \alpha_2^L), (a_1^U, a_2^U, \alpha_1^U, \alpha_2^U)]$ where $a_1^L \geq a_1^U$, $a_2^L \leq a_2^U$ also $\alpha_1^L, \alpha_2^L, \alpha_1^U, \alpha_1^U \geq 0$

Definition 3.2. The Ranking for $\tilde{A}$ of L-R type interval valued fuzzy number is defined by $R(\tilde{A}_{LR}) = \frac{2(a_1^L + a_2^L + a_1^U + a_2^U) + (\alpha_2^L + \alpha_2^L) - (\alpha_1^L + \alpha_1^U)}{8}$ (3.1)

Definition 3.3. [Arithmetic Operations on L-R type IVFNS]

Let $\tilde{A}_{LR} = [(a_1^L, a_2^L, \alpha_1^L, \alpha_2^L), (a_1^U, a_2^U, \alpha_1^U, \alpha_2^U)$, $\tilde{B}_{LR} = [(b_1^L, b_2^L, \beta_1^L, \beta_2^L), (b_1^U, b_2^U, \beta_1^U, \beta_2^U)$ then we have

(i) Addition:

$\tilde{A}_{LR} \oplus \tilde{B}_{LR} = \left[\left(a_1^L + b_1^L, a_2^L + b_2^L, \alpha_1^L + \beta_1^L, \alpha_2^L + \beta_2^L\right), \left(a_1^U + b_1^U, a_2^U + b_2^U, \alpha_1^U + \beta_1^U, \alpha_2^U + \beta_2^U\right)\right]$ (3.2)

(ii) Subtraction:

$\tilde{A}_{LR} \ominus \tilde{B}_{LR} = \left[\left(a_1^L - b_2^L, a_2^L - b_1^L, \alpha_1^L + \beta_2^L, \alpha_2^L + \beta_1^L\right), \left(a_1^U - b_2^U, a_2^U - b_1^U, \alpha_1^U + \beta_2^U, \alpha_2^U + \beta_1^U\right)\right]$ (3.3)

(iii) Scalar Multiplication:

$k\,\tilde{A}_{LR} = [(ka_1^L, ka_2^L, k\alpha_1^L, k\alpha_2^L), (ka_1^U, ka_2^U, k\alpha_1^U, k\alpha_2^U)] if k \geq 0$

$k\,\tilde{A}_{LR} = [(ka_2^L, ka_1^L, k\alpha_1^L, k\alpha_2^L), (ka_2^U, ka_1^U, k\alpha_1^U, k\alpha_2^U)] if k < 0$ (3.4)

Example 3.1. Let $\tilde{A}_{LR}$=[(4,5,2,2),(3,6,2,2)] $\tilde{B}_{LR}$ = [(8,10,4,4),(6,12,4,4)] be L-R type interval valued fuzzy numbers then,

R($\tilde{A}_{LR}$) = 4.5 and R($\tilde{B}_{LR}$) = 9 by (3.1)

$\tilde{A}_{LR} \oplus \tilde{B}_{LR}$ = [(12,15,6,6),(9,18,6,6)] by (3.2)

$\tilde{A}_{LR} \ominus \tilde{B}_{LR}$ = [(-6,-3,6,6),(-9,0,6,6)] by (3.3)

R($\tilde{A}_{LR} \oplus \tilde{B}_{LR}$) = 13.5 and R($\tilde{A}_{LR} \ominus \tilde{B}_{LR}$) = -4.5 by (3.1)

k $\tilde{A}_{LR}$ =[(8,10,4,4),(6,12,4,4)] and k $\tilde{B}_{LR}$ = [(16,20,8,8),(12,24,8,8)] by (3.4)

3.1. Properties of L-R type interval valued fuzzy numbers

Property 3.1.1. R($\tilde{A}_{LR} \oplus \tilde{B}_{LR}$) = R($\tilde{A}_{LR}$)+ R($\tilde{B}_{LR}$)

Property 3.1.2. R($\tilde{A}_{LR} \ominus \tilde{B}_{LR}$) = R($\tilde{A}_{LR}$)- R($\tilde{B}_{LR}$)

Property 3.1.3. R(k $\tilde{A}_{LR}$) = k(R($\tilde{A}_{LR}$)).

3.2. A distance function for L-R type interval valued fuzzy numbers

Let $\tilde{A}_{LR} = [(a_1^L, a_2^L, \alpha_1^L, \alpha_2^L), (a_1^U, a_2^U, \alpha_1^U, \alpha_2^U)$ $\tilde{B}_{LR} = [(b_1^L, b_2^L, \beta_1^L, \beta_2^L), (b_1^U, b_2^U, \beta_1^U, \beta_2^U)$ then we have,

(i) D($\tilde{A}_{LR}, \tilde{B}_{LR}$)=$\frac{1}{4}\max\{\left|(a_1^L + a_1^U) - (b_1^L + b_1^U) - (\alpha_1^L + \alpha_1^U) + (\beta_1^L + \beta_1^U)\right| + \left|(a_2^L + a_2^U) - (b_2^L + b_2^U) + (\alpha_2^L + \alpha_2^U) - (\beta_2^L + \beta_2^U)\right|, \left|(a_1^L + a_1^U) - (b_1^L + b_1^U)\right| + \left|(a_2^L + a_2^U) - (b_2^L + b_2^U)\right|\}$ (5.1)

(ii) D($\tilde{A}_{LR}$,0) = D($\tilde{A}_{LR}$) (5.2)

(iii) D($\tilde{A}_{LR} \oplus \tilde{B}_{LR}$)=D($\tilde{A}_{LR}$)+D($\tilde{B}_{LR}$) (5.3)

(iv) D(k $\tilde{A}_{LR}$)=kD($\tilde{A}_{LR}$) (5.4)

(v) D($\tilde{A}_{LR} \ominus \tilde{B}_{LR}$) $\neq$ D($\tilde{A}_{LR}$) - D($\tilde{B}_{LR}$) (5.5)

Example 3.2.1. Let $\tilde{A}_{LR}$=[(4,5,2,2),(3,6,2,2)] $\tilde{B}_{LR}$ = [(8,10,4,4),(6,12,4,4)] be L-R type interval valued fuzzy numbers then,

(i) D($\tilde{A}_{LR}$,0) = 4.5 and D($\tilde{B}_{LR}$) = 9 by (5.2)

(ii) D($\tilde{A}_{LR}$, $\tilde{B}_{LR}$) = 4.5 by(5.1)

(iii) D($\tilde{A}_{LR} \oplus \tilde{B}_{LR}$) =13.5 by (5.3)

3.3. A distance function for L-R type interval valued fuzzy numbers

A fuzzy project network is an acyclic digraph where the vertices represents events and the directed edges represents the activities to be performed in a project.

Notations 3.3.1. The notations that will be used in the presented methods are as follows.
N: The set of all nodes in a project network.
$\tilde{A}_{ij}$: The activity between nodes i and j
$F\tilde{E}T_{ij}$: The fuzzy activity time of A_{ij}.
$F\tilde{E}S_j$: The earliest fuzzy time of node j.
$F\tilde{L}F_j$: The latest fuzzy time of node j.
$F\tilde{T}F_i$: The total floats fuzzy time of A_{ij}.

Algorithm for fuzzy critical activity 3.3.1

Let $F\tilde{E}S_i$ and $F\tilde{L}S_i$ be the earliest fuzzy event time, and the latest Fuzzy event time for event i, respectively Functions that define the earliest starting times, latest starting times and floats in terms of fuzzy activity durations are convex, normal whose membership functions are piecewise continues, hence the quantities such as earliest fuzzy event time $F\tilde{E}S_i$, the latest fuzzy event time $F\tilde{L}S_i$ and the floats $\tilde{T}_i$ are also IVFNS for an event i respectively.

Step 1: Identify fuzzy activities in a fuzzy project

Step2: Establish precedence relationship of all fuzzy activities; by applying fuzzy ranking function of L-R type IVFNS

Step 3: Construct the fuzzy project network with IVFNS as fuzzy activity times.

Step 4: Let $F\tilde{E}S_1$ be the earliest Fuzzy event time and $F\tilde{L}S_1$ be the latest fuzzy event time for the initial event $\tilde{V}_1$ of the project network and assume that $F\tilde{E}S_1 = F\tilde{L}S_1 = \tilde{0}$ Compute the earliest fuzzy event time $F\tilde{E}Sj$ of the event $\tilde{V}_j$ by using the formula $F\tilde{E}S_j = \max_{i \in N: i \to j}\{F\tilde{E}Si + \tilde{A}_{ij}\}$ (6.1)

Step 5: Let $F\tilde{E}S_n$ be the earliest fuzzy event time and $F\tilde{L}S_n$ be the latest fuzzy event time for the terminal event $\tilde{V}_n$ of the fuzzy project network and assume that

$F\widetilde{E}S_n = F\widetilde{L}S_n$. Compute the latest fuzzy event time $F\widetilde{L}S_i$ by using the following equation $F\widetilde{L}S_i = \min_{i\in N}\{ F\widetilde{L}S_j - \widetilde{A}_{ij} \}$ (6.2)

Step 6: compute the total float $F\widetilde{T}F_{ij}$of each fuzzy activity $\widetilde{A}_{ij}$ by using the following equation $F\widetilde{T}F_{ij} = \{ F\widetilde{L}S_j - F\widetilde{E}S_i - \widetilde{A}_{ij} \}$ (6.3)

Hence we can obtain the earliest fuzzy event time, latest fuzzy event time, and the total float of every fuzzy activity by using equations (6.1), (6.2) and (6.3).

Step 7: If $F\widetilde{T}F_{ij} = \hat{0}$ then the activity $\widetilde{A}_{ij}$ is said to be a fuzzy critical activity. That is activities with zero equivalent fuzzy total float and they are equal fuzzy numbers and are always found on one or more fuzzy critical paths.

Step 8: The length of the longest fuzzy critical path from the start of the fuzzy project to its finish is the minimum time required to complete the fuzzy project. This (or these) fuzzy project duration.

3.4. Illustration

Let us consider the following project network as:

$\tilde{V} = \{1,2,3,4,5,6\}$, the fuzzy activity time for each activity as shown in the table (3.4.1). All the durations are in hours. Find the fuzzy critical path for the given network.

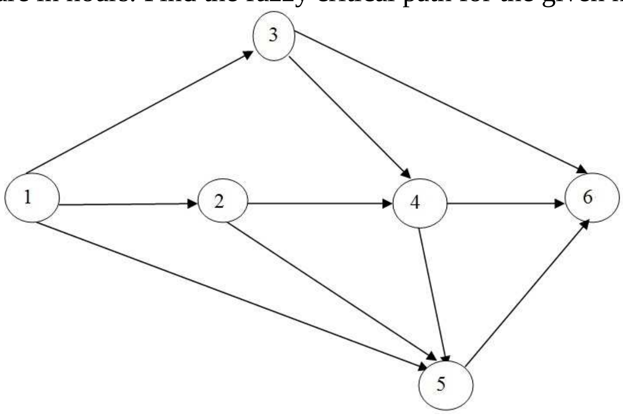

Figure 3.4.1: A fuzzy project network

Activity A_{ij}	**Fuzzy activity times (Hours) $F\tilde{E}T_{ij}$**
A_{12}	Approximately 2 and 3 Hours [(2,3,0,1), (2,3,1,2)]
A_{13}	Approximately 2 and 4 Hours [(3,3,1,3), (2,4,1,3)]
A_{15}	Approximately between 3 and 4 Hours [(3,4,1,1),(3,4,2,2)]
A_{24}	Approximately between 2 and 4 Hours [(2,4,0,1),(2,4,1,2)]
A_{25}	Approximately between 4 and 5 Hours [(4,5,2,3),(4,5,3,4)]

A_{34}	Approximately between 1 and 2 Hours [(1,2,0,0),(1,2,1,1)]
A_{36}	Approximately between 7 and 12 Hours [(8,11,1,4),(7,12,1,4)]
A_{45}	Approximately between 2 and 4 Hours [(3,3,1,2),(2,4,1,2)]
A_{46}	Approximately between 3 and 4 Hours [(3,4,0,2),(3,4,1,3)]
A_{56}	Approximately between 1 Hours [(1,1,0,1),(1,1,1,2)]

	A_{34}	A_{36}	A_{45}	A_{46}	A_{56}
Duration	[(1,2,0,0), (1,2,1,1)]	[(8,11,1,4), (7,14,1,4)]	[(3,3,1,2), (2,4,1,2)]	[(3,4,0,2), (3,4,1,3)]	[(1,1,0,1), (1,1,1,2)]
Earliest Start	[(3,3,1,4), (2,4,1,3)]	[(3,3,1,4), (2,4,1,3)]	[(4,7,0,2), (4,7,2,4)]	[(4,7,0,2), (4,7,2,4)]	[(7,10,1,4), (6,11,3,6)]
Earliest Finish	[(4,5,1,3), (3,6,2,4)]	**[(11,14,2,7), (9,16,2,7)]**	[(7,10,1,4), (6,11,3,6)]	[(7,11,0,4), (7,11,3,7)]	[(8,11,1,6), (7,12,4,8)]
Latest Start	[(4,7,0,2), (4,7,2,4)]	**[(11,14,2,7), (9,16,2,6)]**	[(7,10,1,4), (6,11,3,6)]	[(11,14,2,7), (9,16,2,7)]	[(11,14,2,7), (9,16,2,7)]
Latest Finish	[(2,6,0,2), (2,6,3,5)]	[(0,6,6,8), (-3,9,6,8)]	[(4,7,3,5), (2,9,5,7)]	[(7,11,4,7), (5,13,5,8)]	[(10,13,3,7), (8,15,4,8)]
Total Float	[(-1,3,6,6), (-4,6,9,9)]	[(-3,3,9,9), (-7,7,9,9)]	[(0,6,7,8), (-3,9,10,11)]	[(0,7,6,7), (-2,9,9,10)]	[(0,6,7,8), (-3,9,10,11)]

Table 3.4.2: Fuzzy critical activity

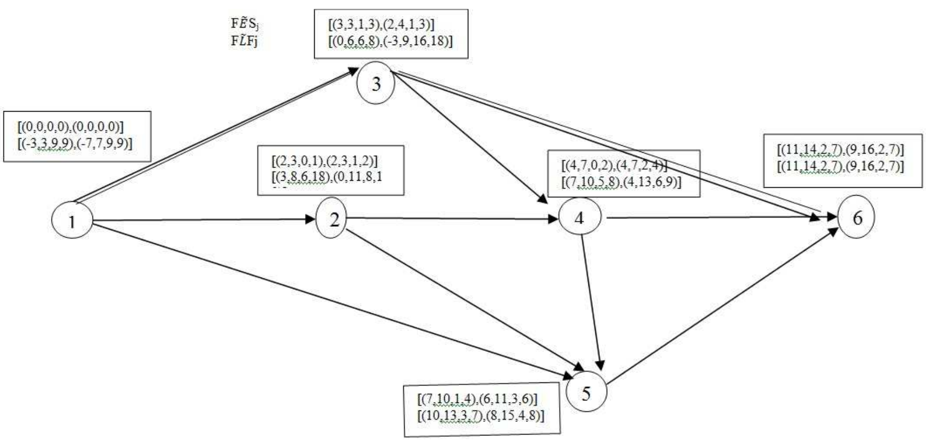

Figure 3.4.2: Fuzzy critical path

Fuzzy critical path is $1 \rightarrow 3 \rightarrow 6$. The minimum fuzzy project duration is the length of the fuzzy critical path. The fuzzy project duration is in fuzzy hours.[(11,14,2,7),(9,16,2,6)].

In addition, when we utilize the notion proposed 'Distance Function', we have
$D(\tilde{A}_{12}) = 7, D(\tilde{A}_{13}) = 14, D(\tilde{A}_{24}) = 11, D(\tilde{A}_{36}) = 14$.
From this, It is note that the fuzzy critical path is $1 \rightarrow 3 \rightarrow 6$,which is same as the critical path of the previous one.

8. Conclusion

In this paper a ranking function for L-R type interval valued fuzzy numbers to fuzzy critical analysis are introduced. Also it is important to note that a relevant numerical illustration was utilized to justify the proposed notions. The project characteristics like earliest time, latest time and total float time in terms of L-R type interval valued fuzzy numbers are calculated without converting the fuzzy nature to classical nature.

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