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On Fuzzy Ward Continuity in an Intuitionistic 2-Fuzzy 2-Normed Linear Space

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Abstract. In this paper not only the fuzzy ward continuity but also some other kinds of continuities are investigated in intuitionistic 2-fuzzy 2-normed linear space. It turns out that uniform limit of fuzzy ward continuous functions is again fuzzy ward continuous.

Keywords: Fuzzy ward continuity, fuzzy ward compactness, quasi Cauchy

AMS Mathematics Subject Classification (2010): 03E72, 03F55

1. Introduction

The concepts involving continuity play a major role not only in pure mathematics but also in other branches of sciences like computer science, information theory etc., Menger [6] introduced the notion called a generalized metric in 1928. On the other hand Vulich [11] in 1938 defined a notion of higher dimensional norm in linear spaces. The concept of 2-normed space was developed by Gahler in the middle of 1960's [5]. Recently many mathematicians came out with results in 2-normed spaces, analogous with that in classical normed spaces and Banach spaces [4,7,8]. Using the main idea in the definition of sequential continuity, many kinds of continuities were introduced and investigated in [1,2,3,10] not all but only some of them. The concept of ward continuity of real functions and ward compactness of a subset E of R are introduced by Cakalli in [2]. In 1965, Zadeh [12] introduced the concept of the fuzzy set in this seminal paper. Somasundaram and Beaula [9] have newly coined 2-fuzzy normed linear space and proved many important theorems. In this paper our aim is to investigate the concept of fuzzy ward continuity in an intuitionistic 2-fuzzy 2-normed linear space and prove some interesting theorems.

2. Preliminaries

Definition 2.1. A binary operation $*: [0,1] \times [0, 1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

1. $*$ is commutative and associative
2. $*$ is continuous
3. $a * 1 = a$, for all $a \in [0,1]$
4. $a * b \le c * d$ whenever $a \le c$ and $b \le d$ and $a,b,c,d \in [0,1]$

Definition 2.2. A binary operation $\lozenge : [0,1] \times [0,1] \to [0,1]$ is a continuous t – conorm if it satisfies the following conditions:

1. $\lozenge$ is commutative and associative
2. $\lozenge$ is continuous
3. $a \lozenge 0 = a$, for all $a \in [0,1]$
4. $a \lozenge b \le c \lozenge d$ whenever $a \le c$ and $b \le d$ and $a, b, c, d \in [0,1]$

Definition 2.3. An intuitionistic fuzzy 2- normed linear space (i.2f-2-NLS) is of the form $A = \{F(X), N(f_1, f_2, t), M(f_1, f_2, t) / (f_1, f_2) \in F[(X)]^2\}$ where F(X) is a linear space over a field K, * is a continuous t-norm, $\lozenge$ is a continuous t-conorm, N and M are fuzzy sets on $[F(X)]^2 \times (0,\infty)$ such that N denotes the degree of membership and M denotes the degree of non-membership of $(f_1, f_2, t) \in [F(X)]^2 \times (0, \infty)$ satisfying the following conditions:

(1) $N(f_1, f_2, t) + M(f_1, f_2, t) \le 1$
(2) $N(f_1, f_2, t) > 0$
(3) $N(f_1, f_2, t) = 1$ if and only if f_1, f_2 are linearly dependent
(4) $N(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2
(5) $N(f_1, f_2, t) : (0, \infty) \to [0,1]$ is continuous in t.
(6) $N(f_1, cf_2, t) = N(f_1, f_2, \frac{t}{|c|})$, if $c \neq 0$, $c \in K$
(7) $N(f_1, f_2, s) * N(f_1, f_3, t) \le N(f_1, f_2 + f_3, s + t)$
(8) $M(f_1, f_2, t) > 0$
(9) $M(f_1, f_2, t) = 0$ if and only if f_1, f_2 are linearly dependent
(10) $M(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2
(11) $M(f_1, cf_2, t) = M(f_1, f_2, \frac{t}{|c|})$, if $c \neq 0$, $c \in k$
(12) $M(f_1, f_2, s) \lozenge M(f_1, f_3, t) \ge M(f_1, f_2 + f_2, s + t)$
(13) $M(f_1, f_2, t) : (0, \infty) \to [0,1]$ is continuous in t.

3. Fuzzy ward continuity and fuzzy ward compactness

Definition 3.1. A sequence $\{f_n\}$ of points in an intuitionistic 2-fuzzy 2-normed linear space $(F(X), N, M)$ is said to be quasi-Cauchy if $\lim_{n\to\infty} N(\Delta f_n, g, t) = 1$ and $\lim_{n\to\infty} M(\Delta f_n, g, t) = 0$ for every $g \in F(X), t \in (0,1)$, where $\Delta f_n = f_{n+1} - f_n$ for n in set of natural numbers.

Definition 3.2. A subspace A of F(X) is said to be fuzzy ward compact if any sequence in A has a quasi-Cauchy subsequence.

Definition 3.3. Let $(F(X), N_1, M_1)$ and $(F(Y), N_2, M_2)$ be intuitionistic 2-fuzzy 2-normed linear spaces.

A function $\varphi : F(X) \to F(Y)$ is said to be fuzzy ward continuous if it preserves quasi-Cauchy property. That is, $\lim_{n\to\infty} N_2(\Delta\varphi(f_n), g, t) = 1$ and $\lim_{n\to\infty} M_2(\Delta\varphi(f_n), g, t) = 0$ for every $g \in F(Y)$ whenever $\lim_{n\to\infty} N_1(\Delta f_n, h, t) = 1$ and $\lim_{n\to\infty} M_1(\Delta f_n, h, t) = 0$ for every $h \in F(X)$.

Definition 3.4. A function φ on a subspace A of an intuitionistic 2-fuzzy 2-normed linear space $(F(X), N_1, M_1)$ is said to be sequentially continuous at f_0 if for any sequence $\{f_n\}$ in A converging to f_0, $\varphi(f_n)$ converges to $\varphi(f_0)$ in $(F(Y), N_2, M_2)$.

Theorem 3.1. If $\varphi : F(X) \to F(Y)$ is fuzzy ward continuous on A of F(X) then it is sequentially continuous on A.

Proof: Let $\{f_n\}$ be a convergent sequence in A.

Then $\lim_{n\to\infty} N_1(f_n - f_0, h, t) = 1$ and $\lim_{n\to\infty} M_1(f_n - f_0, h, t) = 0$ where $h \in F(X), t \in (0,1)$

Construct a sequence $\{g_n\}$ as $g_n = \begin{cases} f_n & \text{if } n = 2k\text{-}1 \text{ where } k \text{ is a positive integer} \\ f_0 & \text{if n is even} \end{cases}$

Consider $N_1(g_n - f_0, h, t) = N_1\left(g_n - f_n + f_n - f_0, h, \frac{t}{2} + \frac{t}{2}\right)$

$$\geq N_1\left(g_n - f_n, h, \frac{t}{2}\right) * N_1\left(f_n - f_0, h, \frac{t}{2}\right) \tag{1}$$

When n is even, (1) becomes

$$N_1(g_n - f_0, h, t) \geq N_1\left(f_n - f_0, h, \frac{t}{2}\right) * N_1\left(f_n - f_0, h, \frac{t}{2}\right) = N_1(f_0 - f_n, h, t)$$

Again when n is odd, $N_1(g_n - f_0, h, t) \geq N_1(f_n - f_0, h, t)$

Again $M_1(g_n - f_0, h, t) = M_1\left(g_n - f_n + f_n - f_0, h, \frac{t}{2} + \frac{t}{2}\right)$

$$\leq M_1\left(g_n - f_n, h, \frac{t}{2}\right) \Diamond M_1\left(f_n - f_0, h, \frac{t}{2}\right) \tag{2}$$

when n is odd, (2) becomes $M_1(g_n - f_0, h, t) \leq M_1(f_n - f_0, h, t)$

Again when n is even,

$$M_1(g_n - f_0, h, t) \leq M_1\left(f_0 - f_n, h, \frac{t}{2}\right) \Diamond M_1\left(f_n - f_0, h, \frac{t}{2}\right) = M_1(f_0 - f_n, h, t)$$

In either case, $\{g_n\}$ converges to f_0 and $\{g_n\}$ is a quasi-Cauchy sequence. As φ is fuzzy ward continuous, define the transformed sequence $\varphi(g_n)$ as

$$\varphi(g_n) = \begin{cases} \varphi(f_n) & \text{if } n = 2k - 1 \text{ where } k \text{ is a positive integer} \\ \varphi(f_0) & \text{if } n \text{ is even} \end{cases}$$

Again $\varphi(g_n)$ is quasi-Cauchy.

For , $N_2(\varphi(g_{n+1}) - \varphi(g_n), h^*, t)$

$$= N_2\left(\varphi(g_{n+1}) - \varphi(f_{n+1}) + \varphi(f_{n+1}) - \varphi(g_n) + \varphi(f_n) - \varphi(f_n), h^*, \frac{t}{3} + \frac{t}{3} + \frac{t}{3}\right)$$

$$\geq N_2\left(\varphi(g_{n+1})-\varphi(f_{n+1}),h^*,\frac{t}{3}\right)*N_2\left(\varphi(f_n)-\varphi(g_n),h^*,\frac{t}{3}\right)$$
$$*N_2\left(\varphi(f_{n+1})-\varphi(f_n),h^*,\frac{t}{3}\right) \tag{3}$$

If n is odd, (3) becomes

$$N_2(\varphi(g_{n+1})-\varphi(g_n),h^*,t)\geq N_2\left(\varphi(f_{n+1})-\varphi(f_n),h^*,\frac{t}{3}\right)$$

Again if n is even,

$$N_2\left(\varphi(g_{n+1})-\varphi(g_n),h^*,\frac{t}{3}+\frac{t}{3}+\frac{t}{3}\right)$$
$$\geq N_2\left(\varphi(f_0)-\varphi(f_{n+1}),h^*,\frac{t}{3}\right)*N_2\left(\varphi(f_n)-\varphi(f_0),h^*,\frac{t}{3}\right)*N_2\left(\varphi(f_{n+1})-\varphi(f_n),h^*,\frac{t}{3}\right)$$
$$\geq N_2\left(\varphi(f_{n+1})-\varphi(f_n),h^*,\frac{t}{3}\right)$$

$$M_2(\varphi(g_{n+1})-\varphi(g_n),h^*,t)$$
$$\leq M_2\left(\varphi(g_{n+1})-\varphi(f_{n+1}),h^*,\frac{t}{3}\right)\Diamond M_2\left(\varphi(f_n)-\varphi(g_n),h^*,\frac{t}{3}\right)\Diamond\left(\varphi(f_{n+1})-\varphi(f_n),h^*,\frac{t}{3}\right)$$

If n is odd,

$$M_2(\varphi(g_{n+1})-\varphi(g_n),h^*,t)\leq M_2\left(\varphi(f_{n+1})-\varphi(f_n),h^*,\frac{t}{3}\right)$$

If n is even,

$$M_2(\varphi(g_{n+1})-\varphi(g_n),h^*,t)$$
$$\leq M_2\left(\varphi(f_0)-\varphi(f_{n+1}),h^*,\frac{t}{3}\right)\Diamond M_2\left(\varphi(f_n)-\varphi(f_0),h^*,\frac{t}{3}\right)\Diamond M_2\left(\varphi(f_{n+1})-\varphi(f_n),h^*,\frac{t}{3}\right)$$
$$\leq M_2\left(\varphi(f_{n+1})-\varphi(f_n),h^*,\frac{t}{3}\right)$$

Hence $\{\varphi(f_n)\}$ is quasi-Cauchy sequence.

Now, $\lim_{n\to\infty} N_2\left(\varphi(g_{n+1})-\varphi(g_n),h^*,t\right)=1$ and $\lim_{n\to\infty} M_2\left(\varphi(g_{n+1})-\varphi(g_n),h^*,t\right)=0$ (4)

From (4) and by construction of $\{g_n\}$ we get,

$\lim_{n\to\infty} N_2\left(\varphi(f_{n+1})-\varphi(f_0),h^*,t\right)=1$ and $\lim_{n\to\infty} M_2\left(\varphi(f_{n+1})-\varphi(f_0),h^*,t\right)=0$

Thus $\varphi(f_{n+1})$ converges to $\varphi(f_0)$and so φ is sequentially continuous on A.

Theorem 3.2. Let $(F(X),N_1,M_1)$ and $(F(Y),N_2,M_2)$ be intuitionistic 2-fuzzy 2-normed linear spaces and A be a fuzzy ward compact subspace of $F(X)$. If $\varphi: F(X)\to F(Y)$ is fuzzy ward continuous on A then $\varphi(A)$ is fuzzy ward compact.

Proof: Ward compactness of A implies that there is a subsequence $\{g_{n_k}\}$ of $\{g_n\}$ with

$\lim_{n_k\to\infty} N_1\left(\Delta(g_{n_k}),h,t\right)=1$ and $\lim_{n_k\to\infty} M_1\left(\Delta(g_{n_k}),h,t\right)=0,$ for every $h\in F(X), t\in(0,1)$.

Let $\left\{\varphi(g_{n_k})\right\}=\left\{f_{n_k}\right\}$. Then $\left\{f_{n_k}\right\}$ is a subsequence of the sequence $\{\varphi(g_n)\}$ with

$\lim_{n_k\to\infty} N_2\left(\Delta(f_{n_k}),\varphi(h),t\right)=1$ and $\lim_{n_k\to\infty} M_2\left(\Delta(f_{n_k}),\varphi(h),t\right)=0$

Satisfies the requirements as desired.

3.1. Uniform continuity

Definition 3.1.1. A function $\varphi: F(X)\to F(Y)$ is said to be uniformly continuous on a subspace A of $F(X)$ if for any $\varepsilon>0$ there exists $\delta>0$ such that $N_2\left(\varphi(f_1)-\varphi(f_2),h,t\right)>1-\varepsilon$ $M_2\left(\varphi(f_1)-\varphi(f_2),h,t\right)<\varepsilon$ for any $h\in F(Y), t\in(0,1)$ whenever $N_1\left(f_1-f_2,g,t\right)>1-\delta$ $M_1\left(f_1-f_2,g,t\right)<\delta$ for every $f_1, f_2\in A$ and $g\in F(X)$.

Theorem 3.1.1. If a function $\varphi: F(X)\to F(Y)$ is uniformly continuous on a subspace A of $F(X)$ then it is fuzzy ward continuous on A.

Proof: Let φ be uniformly continuous on A and $\left\{f_n\right\}$ be any quasi-Cauchy sequence of points in A. Then for given $\varepsilon>0$, there exists $\delta>0$ such that

$N_2\left(\varphi(f_1)-\varphi(f_2),h,t\right)>1-\varepsilon$ $M_2\left(\varphi(f_1)-\varphi(f_2),h,t\right)<\varepsilon$ for any $h\in F(Y), t\in(0,1)$

whenever $N_1\left(f_1-f_2,g,t\right)>1-\delta$

$M_1\left(f_1-f_2,g,t\right)<\delta$ for any $f_1, f_2\in A$ and $g\in F(X)$

For the choice of δ, there exists N depending on both ε and δ.

Since $\left\{f_n\right\}$ is quasi-Cauchy,

$\lim_{n\to\infty} N_1\left(\Delta f_n,g,t\right)=1$ and so $N_1\left(\Delta f_n,g,t\right)>1-\delta$ for all $n>N$

$\lim_{n\to\infty} M_1\left(\Delta f_n,g,t\right)=0$ and so $M_1\left(\Delta f_n,g,t\right)<\delta$ for all $n>N$

So $N_2\left(\Delta\varphi(f_n),h,t\right)>1-\varepsilon$ for all $n>N$ and $M_2\left(\Delta\varphi(f_n),h,t\right)<\varepsilon$

Hence φ is ward continuous.

Theorem 3.1.2. The image of a fuzzy ward compact space under a uniform continuous map is fuzzy ward compact.

Proof: Let $\varphi: F(X)\to F(Y)$ be uniform continuous; let A be a fuzzy ward compact subspace of $F(X)$.We assert that $\varphi(A)$ is a fuzzy ward compact subspace of $F(Y)$.

Consider a sequence $\left\{g_n\right\}$ in $\varphi(A)$, provided $g_n=\varphi(f_n)$ where $\left\{f_n\right\}$ is a sequence in A.

Since A is ward compact, $\left\{f_n\right\}$ has a quasi-Cauchy subsequence $\left\{f_{n_k}\right\}$ because A is ward compact. Therefore, $\lim_{n_k\to\infty} N_1\left(\Delta f_{n_k},g,t\right)=1$ and $\lim_{n_k\to\infty} M_1\left(\Delta f_{n_k},g,t\right)=0$. For a given $\varepsilon>0$, there exists δ satisfying the required conditions of uniform continuity.

Choose a positive number N depending on both ε and δ such that

$\lim_{n_k\to\infty} N_2\left(\Delta\varphi(f_{n_k}),h,t\right)=1$ and $\lim_{n_k\to\infty} M_2\left(\Delta\varphi(f_{n_k}),h,t\right)=0$ provided $\lim_{n_k\to\infty} N_1\left(\Delta f_{n_k},g,t\right)=1$ and $\lim_{n_k\to\infty} M_1\left(\Delta f_{n_k},g,t\right)=0$ for all $n_k > N$. Thus $\varphi(A)$ is fuzzy ward compact.

Theorem 3.1.3. Let $\{\varphi_n\}$ be a sequence of uniformly continuous functions defined on a subspace A of $F(X)$ to $F(Y)$, and if $\{\varphi_n\}$ converges uniformly to φ then φ is uniformly continuous.

Proof: Using the uniform convergence of $\{\varphi_n\}$, for a given $\varepsilon > 0$, there exists a positive integer N such that $N_2\left(\varphi_n(f)-\varphi(f),h,t\right)>1-\varepsilon$ where $n\geq N, f,h\in A$

$M_2\left(\varphi_n(f)-\varphi(f),h,t\right)<\varepsilon$

Using the uniform continuity of φ_n on A for a given $\in > 0$, there exists $\delta > 0$ such that $N_2\left(\varphi_N(f_1)-\varphi_N(f_2),h,t\right)>1-\varepsilon$ and $M_2\left(\varphi_N(f_1)-\varphi_N(f_2),h,t\right)<\varepsilon$ for $f_1, f_2\in A$ and $h\in F(Y)$ provided $N_1\left(f_1-f_2,g,t\right)>1-\delta$, $M_1\left(f_1-f_2,g,t\right)<\delta$ (by the choice of N). Then whenever $N_1\left(f_1-f_2,g,t\right)>1-\delta$,

$N_2\left(\varphi(f_1)-\varphi(f_2),h,t\right) = N_2\left(\varphi(f_1)-\varphi_N(f_1)+\varphi_N(f_1)-\varphi_N(f_2)+\varphi_N(f_2)-\varphi(f_2),h,t\right)$

$\geq N_2\left(\varphi(f_1)-\varphi_N(f_1),h,\frac{t}{3}\right)*N_2\left(\varphi_N(f_1)-\varphi_N(f_2),h,\frac{t}{3}\right)*N_2\left(\varphi_N(f_2)-\varphi(f_2),h,\frac{t}{3}\right)$

$\geq(1-\varepsilon)*(1-\varepsilon)*(1-\varepsilon)$

$>(1-\varepsilon)$ for every $h\in F(Y)$

Also, when $M_1\left(f_1-f_2,g,t\right)<\delta$

$M_2\left(\varphi(f_1)-\varphi(f_2),h,t\right) = M_2\left(\varphi(f_1)-\varphi_N(f_1)+\varphi_N(f_1)-\varphi_N(f_2)+\varphi_N(f_2)-\varphi(f_2),h,t\right)$

$\geq M_2\left(\varphi(f_1)-\varphi_N(f_1),h,\frac{t}{3}\right)\lozenge M_2\left(\varphi_N(f_1)-\varphi_N(f_2),h,\frac{t}{3}\right)\lozenge M_2\left(\varphi_N(f_2)-\varphi(f_2),h,\frac{t}{3}\right)$

$\leq\varepsilon\lozenge\varepsilon\lozenge\varepsilon$

$<\varepsilon$ for every $h\in F(Y)$

So φ is uniformly continuous on A as desired.

Theorem 3.1.4. Let $\{\varphi_n\}$ be a sequence of fuzzy ward continuous functions defined on a subspace A of an intuitionistic 2-fuzzy 2-normed linear space $F(X)$ to $F(Y)$ and $\{\varphi_n\}$ is uniformly convergent to a function φ then φ is fuzzy ward continuous.

Proof: We assert that φ is fuzzy ward continuous on A. Take any quasi-Cauchy sequence $\{f_n\}$ in A. As $\{\varphi_n\}$ is uniformly convergent to φ for $\varepsilon\in(0,1)$ there exists a positive integer N such that $N_2\left(\varphi_n(f_n)-\varphi(f_n),h,t\right)>1-\varepsilon$ and $M_2\left(\varphi_n(f_n)-\varphi(f_n),h,t\right)<\varepsilon$ provided $n\geq N$. By the choice of N, as φ_N is ward continuous on A, there exists a positive integer $N_0\geq N$ such that

$N_2\left(\varphi_N(f_{n+1})-\varphi_N(f_n),h,t\right)>1-\varepsilon$ and $M_2\left(\varphi_N(f_{n+1})-\varphi_N(f_n),h,t\right)<\varepsilon,$ for every $n\in N_0$ and $h\in F(Y)$

$$N_2\left(\varphi(f_{n+1})-\varphi(f_n),h,t\right)$$
$$=N_2\left(\varphi(f_{n+1})-\varphi_N(f_{n+1})+\varphi_N(f_{n+1})-\varphi_N(f_n)+\varphi_N(f_n)-\varphi(f_n),h,\frac{t}{3}+\frac{t}{3}+\frac{t}{3}\right)$$
$$\geq N_2\left(\varphi(f_{n+1})-\varphi_N(f_{n+1}),h,\frac{t}{3}\right)*N_2\left(\varphi_N(f_{n+1})-\varphi_N(f_n),h,\frac{t}{3}\right)*N_2\left(\varphi_N(f_n)-\varphi(f_n),h,\frac{t}{3}\right)$$
$$\geq(1-\varepsilon)*(1-\varepsilon)*(1-\varepsilon)$$
$$=1-\varepsilon \text{ for every } n\geq N_0$$

and $M_2\left(\varphi(f_{n+1})-\varphi(f_n),h,t\right)$

$$=M_2\left(\varphi(f_{n+1})-\varphi_N(f_{n+1})+\varphi_N(f_{n+1})-\varphi_N(f_n)+\varphi_N(f_n)-\varphi(f_n),h,\frac{t}{3}+\frac{t}{3}+\frac{t}{3}\right)$$
$$\leq M_2\left(\varphi(f_{n+1})-\varphi_N(f_{n+1}),h,\frac{t}{3}\right)\Diamond M_2\left(\varphi_N(f_{n+1})-\varphi_N(f_n),h,\frac{t}{3}\right)\Diamond M_2\left(\varphi_N(f_n)-\varphi(f_n),h,\frac{t}{3}\right)$$
$$<\varepsilon\Diamond\varepsilon\Diamond\varepsilon=\varepsilon \text{ for every } n\geq N_0$$

Hence φ is fuzzy ward continuous.

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