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Majority Neighbourhood Polynomialsofa Graph

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Abstract. In this article, majority neighborhood polynomial of graph G is introduced. The majority neighborhood polynomial $N_M(G,x)$ of a graph G of order n is defined as $N_M(G,x) = \sum_{i=N_M}^{|V(G)|} n_M(G,i)x^l$ where $N_M(G)$ is the majority neighborhood number of a graph G. For this majority neighborhood number$N_M(G)$, majority neighborhood polynomial of G is defined and studied for some standard graphs. Also coefficients of majority neighborhood polynomials of a graph G are obtained.

Keywords: Neighborhood polynomial and Majority neighborhood polynomial

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1. Introduction

By a graph, we mean a finite, simple, undirected and connected graph with p vertices and q edges. We follow the notation and terminology given by Haynes et al. [2]. The open neighborhood $N(v)$ of a vertex v is the set of vertices adjacent to v and the closed neighborhood of a vertex v is $N[v] = N(v) \cup \{v\}$. Let $\Delta(G)$ *and* $\delta(G)$ be the maximum and minimum degree of G respectively. A set S of vertices in a graph G is a neighborhood set if $G = \bigcup_{v \in S} \langle N[v] \rangle$, where $\langle N[v] \rangle$ is the subgraph of G induced by v and all vertices adjacent to v. The neighborhood number $n_0(G)$ of G is the minimum number of vertices in a neighborhood set of G. These parameter has been studied by E. Sampathkumar et al. [5]. Let $G = (V,E)$ be a finite graph with p vertices and q edges. A subset $S \subseteq V(G)$ of vertices in a graph G is called a majority dominating set if at least half of the vertices of $V(G)$ are either in S or adjacent to vertices of S, i.e. $|N[S]| \geq \left\lceil \frac{|V(G)|}{2} \right\rceil$.

The minimum cardinality of the minimal majority dominating set is called the majority domination number and it is denoted by $\gamma_M(G)$. A set $S \subseteq V(G)$ is called a majority neighborhood set if $G_M = \bigcup_{v \in S} \langle N[v] \rangle$ contains at least $\left\lceil \frac{p}{2} \right\rceil$ vertices and at least $\left\lceil \frac{q}{2} \right\rceil$ edges.

A majority set S is called a minimal majority neighborhood set if no proper subset of S is a majority neighborhood set. The minimum cardinality of a majority neighborhood set is called the majority neighborhood number of G and is denoted by $n_M(G)$.These parameters are studied by Swaminathan and Joseline Manora [6,7].

Proposition 1.1. [8] For $G = K_P, K_{1,p-1}, p \geq 2, F_p \geq 3$ and $W_p, P \geq 5$ then $n_m(G) = 1$.

Proposition 1.2. [8] For a graph $G = C_p$ a cycle on p vertices, $P \geq 5$ $n_m(G) = \left\lceil \frac{p}{4} \right\rceil$

Corollary 1.1. [8] Let $G = P_p$ be a path on p vertices then $n_m(G) = \left\lceil \frac{p-1}{4} \right\rceil$.

Definition 1.1. [9] Let $N(G, i)$ be the family of neighborhood sets of a graph G with cardinality iand let $n(G, i) = |N(G, i)|$. Then the neighborhood sets polynomial $N(G, x)$ of G is defined as $N(G, x) = \sum_{i=n_0}^{|V(G)|} n(G, x)x^i$, where $n_0(G)$ is the neighborhood number of G.

2. Majority neighborhood polynomial

Definition 2.1. Let $N_m(G,i)$ be the family of majority neighborhood sets of a graph G with cardinality i and let $N_M(G,i) = |N_M(G,i)|$. Then the majority neighborhood sets polynomial $N_M(G,x)$ ofG is defined as $N_M(G, x) = \sum_{i=n_M}^{|V(G)|} n_M(G, x)x^i$, where $n_M(G)$ is the majority neighborhood number of G.

Example 2.1.

i. Let G be the Cycle with n=5 then the majority neighborhood polynomial is $N_M(C_5, x) = x^5 + 5x^4 + 10x^3 + 10x^2$.
ii. Let G be the Path with n=5 then the majority neighborhood polynomial is $N_M(P_5, x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 3x$.

Proposition 2.2. Let $G = K_p, p \geq 3$ be a complete graph then $n_M(G) = 1$. The majority neighborhood polynomial of G is $n_M(K_p, x) = (1+x)^p - 1$.

Theorem 2.1. If a graph G consist of m components $G_1, G_2, G_3 ... G_m$ then $N_M(G,x) = \prod_{i=1}^{m} N_M(G_i, x)$.

Proof: Suppose G has two components G_1 and G_2. For $k \geq n_M(G)$, a majority neighborhood set of k vertices in G. The majority neighborhood set of k vertices in G arises by choosing a majority neighborhood set of j vertices in G_1 and majority neighborhood set of k-j vertices in G_2. The number of way doing this over all

$j = n_M(G_1).....|V(G_1)|$ is exactly the coefficient of x^k in $N_M(G_1,x)N(G_2,x)$. Hence both side of the above equation have the same coefficient, so the identical polynomial.

Corollary 2.1. Let $\overline{K_p}$ be the totally disconnected graph with p vertices. Then $N_M(\overline{K_p},x) = x^p$.

Join of two connected graphs

Theorem 2.4. Let G_1 and G_2 be the connected graphs of order p_1 and p_2 respectively. Then $N_M(G_1 \vee G_2) = ((1+x)^{p_1} - 1)((1+x)^{p_2} - 1) + N_M(G_1,x) + N_M(G_2,x)$.

Proof: Let G_1 and G_2 be the connected graphs of order p_1 and p_2 respectively. Let $1 \le i \le p_1 + p_2$. To determine the majority neighborhood sets $N_M(G_1 \vee G_2, i)$. Let i_1, i_2 be the natural numbers such that $i_1 + i_2 = i$. Then clearly for every majority neighborhood sets of G_1 and G_2, $N_1 \subseteq V(G_1)$ and $N_2 \subseteq V(G_2)$ such that $|N_j| = i_j$, j = 1,2. Every majority neighborhood set of $G_1 \vee G_2$ of size i contains sum of the degree of the vertices is p-1, where p $=|V(G_1 \vee G_2)|$. Moreover if $N_M \in N_M(G_1, i)$ then N_M is the majority neighborhood sets of $(G_1 \vee G_2)$ of size i. The same is true for every $N \in N_M(G_2, i)$.
Thus $N_M(G_1 \vee G_2) = ((1+x)^{p_1} - 1)((1+x)^{p_2} - 1) + N_M(G_1,x) + N_M(G_2,x)$.

Corollary 2.2.

(i) $N_M(K_{1,p-1},x) = x^{p-1} + x(1+x)^{p-1}$

(ii) If p ≥4 then $N_M(W_p,x) = x(1+x)^{p-1} + N_M(C_{p-1},x)$

(iii) $N_M(F_p,x) = x((1+x)^{p-1} - 1) + N_M(P_{p-1},x) + x$

Proof:

(i) By theorem 2.4, for $G_1 = K_1$ and $G_2 = \overline{K_{p-1}}$

(ii) Since $W_p = K_1 \vee C_{p-1}$, $p \ge 5$. $G_1 = K_1$ and $G_2 = C_{p-1}$

(iii) Since $F_p = K_1 \vee P_{p-1}$, $p \ge 3$. $G_1 = K_1$ and $G_2 = P_{p-1}$ we get the result.

Proposition 2.1. If the graph G is a Double star graph then the majority neighborhood polynomial of a graph is

$$N_M(D_{r,s},x) = \begin{cases} (1+x)^p - \sum_{k=1}^{\lceil \frac{q}{2} \rceil - 1} \binom{t}{k} x^k & , if\ s \ge r+2 \\ (1+x)^p - \sum_{k=1}^{\lceil \frac{q}{2} \rceil - 1} \binom{t+1}{k} x^k, & if\ s \le r+1 \end{cases}$$

Proof: Let G be a double star (Dr,s). p = r + s + 2 ; q = r + s + 1. Let u_1 and u_2 be the centers of G which have r, s pendent vertices respectively in its neighborhood. Let t be the total number of pendent vertices and t = r + s. For a double star Dr,s , $n_M(D_{r,s}) = 1$.

Case (i) $s \le r+1$.

Let $t=\{t_1,t_2,t_3.....t_n\}$ be the pendent vertices.If $s \le r+1$ then $d(u) \ge \left\lceil \frac{q}{2} \right\rceil$ and $(v) \ge \left\lceil \frac{q}{2} \right\rceil$, therefore all the combinations of the sets which contains the centre vertex with cardinality 1 to p. $d(t_i) \le \left\lceil \frac{q}{2} \right\rceil, i=1,2,3....n$, therefore the combination of the sets which contains only the pendent vertices with the cardinality 1 to $\left\lceil \frac{q}{2} \right\rceil - 1$ is not a majority neighborhood set. Hence $pC_1x^1 + pC_2x^2 + pC_3x^3 + ... + pC_px^p - tC_1x^1 - tC_2x^2 - .... - tC_{\left\lceil \frac{q}{2} \right\rceil - 1}$

Case (ii) $s \ge r+2$

If $s \ge r+2$, then the any one of the center vertex of the degree $d(u_i) < \left\lceil \frac{q}{2} \right\rceil$, i =1 or 2. It does not cover the at least $\left\lceil \frac{q}{2} \right\rceil$ edges. Therefore the combinations of the set which contains the pendent vertices and center vertex u_i, $i = 1 or 2$ not a majority neighborhood sets.

Hence $pC_1x^1 + pC_2x^2 + pC_3x^3 + ... + pC_px^p - (t+1)C_1x^1 - (t+1)C_2x^2 - .... - (t+1)C_{\left\lceil \frac{q}{2} \right\rceil - 1}$

$$\therefore N_M(D_{r,s},x) = \begin{cases} (1+x)^p - \left(\left(tC_1x^1\right) + \left(tC_2x^2\right) + ... + \left(tC_{\left\lceil \frac{q}{2} \right\rceil - 1} x^{\left\lceil \frac{q}{2} \right\rceil - 1} \right) \right) & ,if\ s \ge r+2 \\ (1+x)^p - \left(\left((t+1)C_1x^1\right) + \left((t+1)C_2x^2\right) + ... + \left((t+1)C_{\left\lceil \frac{q}{2} \right\rceil - 1} x^{\left\lceil \frac{q}{2} \right\rceil - 1} \right) \right), & if\ s \le r+1 \end{cases}$$

Remark: The join of two totally disconnected graphs is a complete bipartite graph and the resultant graph has no triangles.

3. Majority neighborhood sets polynomial of a complete bipartite graph

Proposition 3.1. [7] If G is a bipartite graph without isolates, with bipartition $\{V_1, V_2\}$ of $V(G)$ the $n_M(G) \le \min\left\{\left\lceil \frac{m}{2} \right\rceil, \left\lceil \frac{n}{2} \right\rceil\right\}$. Let $G = k_{m,n}$, $m \le n$ be the complete bipartite graph and $N_M(G,i)$ be the family of majority neighborhood sets of $K_{m,n}$ with cardinality i. We can determine the family of majority neighborhood sets of $K_{m,n}$ as follows.

The complete bipartite graph $G = K_{m,n}$, $m \le n$ is the join of two totally disconnected graphs $G_1 = \overline{K_m}$ and $G_2 = \overline{K_n}$ with vertex set $V(G_1)$ and $V(G_2)$.

Since $n_M(G) = \min\{|V_1|, |V_2|\}$, $m = |V_1|, n = |V_2|$, $n_M(G) = m$ for $m \le n$. Observe that every majority neighborhood set of $G \subseteq V_1$ or V_2 or both V_1 and V_2.

Some majority neighborhood polynomial of Complete bipartite given

$$N_M(k_{2,2},x) = x^4 + 4x^3 + 6x^2 + 4x$$

$$N_M(k_{2,3},x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 2x$$

$$N_M(k_{2,4},x) = x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 2x$$

$$N_M(k_{2,5},x) = x^7 + 7x^6 + 21x^5 + 35x^4 + 25x^3 + 11x^2 + 2x$$

$N_M\left(k_{2,6},x\right)=x^8+8x^7+28x^6+56x^5+70x^4+56x^3+13x^2+2x$

$N_M\left(k_{2,7},x\right)=x^9+9x^8+36x^7+84x^6+126x^5+126x^4+49x^3+15x^2+2x$

$N_M\left(k_{3,3},x\right)=x^6+6x^5+15x^4+20x^3+15x^2$

$N_M\left(k_{3,4},x\right)=x^7+7x^6+21x^5+35x^4+35x^3+21x^2$

$N_M\left(k_{3,5},x\right)=x^8+8x^7+28x^6+56x^5+70x^4+56x^3+3x^2$

$N_M\left(k_{3,6},x\right)=x^9+9x^8+36x^7+84x^6+126x^5+126x^4+84x^3+3x^2$

$N_M\left(k_{3,7},x\right)=x^{10}+10x^9+45x^8+120x^7+210x^6+252x^5+210x^4+85x^3+3x^2$

$N_M\left(k_{3,8},x\right)=x^{11}+11x^{10}+55x^9+165x^8+330x^7+462x^6+462x^5+330x^4+108x^3+3x^2$

$N_M\left(k_{4,5},x\right)=x^9+9x^8+36x^7+84x^6+126x^5+126x^4+84x^3+6x^2$

$N_M\left(k_{4,6},x\right)=x^{10}+10x^9+45x^8+120x^7+210x^6+252x^5+210x^4+120x^3+6x^2$

For example: Let $G=K_{4,6}$ we investigated the n_M set if G with different cardinality i=1,2,3… where i = $n_M\left(G\right)$. The majority neighborhood polynomial G is obtained as

$N_M\left(K_{4,6},x\right)=x^{10}+10x^9+45x^8+120x^7+210x^6+252x^5+210x^4+120x^3+6x^2$

In this polynomial there are $6\,n_M$ set of cardinality 2, 120 n_M set of cardinality 3, 252, 210,45,10,1 n_M sets of cardinality 4,5,6,7,8,9 and 10 respectively.

Observation 3.1. Total number of minimum cardinality majority neighborhood sets is $mC_{\left\lceil \frac{m}{2}\right\rceil}$.

Theorem 3.2. Let G_1 and G_2 be the totally disconnected graph with the vertex set V_1 and V_2 respectively.

Then $$N_M\left(K_{m,n},x\right)=\begin{cases}\left(1+x\right)^{m+n}-\displaystyle\sum_{k=1}^{\left\lceil \frac{m+n}{2}\right\rceil-1}\binom{m+n}{k}x^k & ,\ if\ m=n\\ \displaystyle\sum_{k=\left\lceil \frac{n}{2}\right\rceil}^{m+n}\binom{m+n}{k}x^k+\binom{m}{\left\lceil \frac{m}{2}\right\rceil}+\sum_{k=\left\lceil \frac{m}{2}\right\rceil+1}^{\left\lceil \frac{n}{2}\right\rceil-1}\left(\binom{m+n}{k}-\binom{n}{k}\right)x^k & ,\ m\le n\end{cases}$$

3.1. Coefficients of majority neighborhood sets polynomial

In this section we obtain some properties of the coefficient of the majority neighborhood polynomial of a graph. By the definition of majority neighborhood set polynomial, we have the following results.

Theorem 3.1.1. Let G be a graph with $|V(G)|=p$. Then

(i) If G is connected , then $N_M(G,p)=1$ and $N_M(G,p-1)=p$

(ii) $N_M(G, i) = 0$if and only if $i < N_M(G)$ or $i > p$.

(iii) $N_M(G, x)$ has no constant term.

(iv) $N_M(G, x)$ is strictly increasing function in $[0, \infty)$.

(v) Let G be a Graph and H be any induced subgraph of G.

then $\deg\big(N_M(G, x)\big) \geq \deg\ (N_M(H, x))$.

4. Conclusion

In this paper, we have introduced new type of neighborhood polynomial of a graph. For further investigation corresponding product of two graphs, cycle, path, K- regular graph majority neighborhood polynomials.

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