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Restrained Triple Connected Domination Number of Cardinal, Strong and Equivalent Products of Graphs

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Abstract. In this paper, we commence the restrained triple connected domination number of the product of path graphs as cardinal, strong and equivalent products. the restrained dominating set is said to be restrained triple connected dominating set, if the <S> is triple connected. The minimum cordiality taken over all the restrained triple connected dominating sets is called the restrained triple connected domination number and is denoted by $\gamma_{rtc}(G)$. We determine the domination numbers of $P_m \times P_n, P_m \otimes P_n$ and$P_m \circ P_n$.

Keywords: Restrained Triple connected domination number of a graph, cardinal, strong and equivalent product of paths

AMS Mathematics Subject Classification (2010): 05C69

1. Introduction

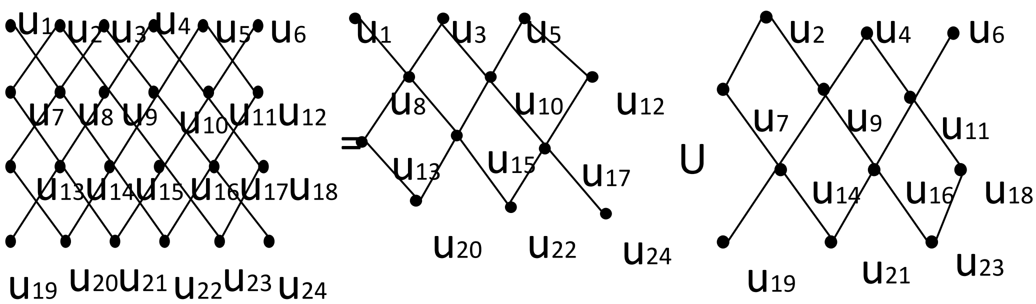
By a graph we mean a finite, simple, connected and undirected graph G (V, E). A subset S of V of a nontrivial graph G is called a dominating set of G if every vertex in V − S is adjacent to at least one vertex in S. The domination number γ(G) of G is the minimum cardinality taken over all dominating sets in G. A graph G is said to be triple connected if any three vertices lie on a path in G. A dominating set is said to be restrained dominating set if every vertex in V – S is adjacent to at least one vertex in S as well as another vertex in V - S. The minimum cardinality taken over all restrained dominating sets is called the restrained domination number and is denoted by γ_r(G). The restrained dominating set is said to be restrained triple connected dominating set, if the <S> is triple connected. The minimum cordiality taken over all the restrained triple connected dominating sets is called the restrained triple connected domination number and is denoted by γ_{rtc}(G).The product of path graphs of four types such as cardinal, Cartesian, strong and equivalent products. In this paper we afford the restrained triple connected domination number of cardinal, strong and equivalent products. A two dimensional complete grid graph $G_{m,n} = P_m \circ P_n$, is the product of path graphs on m and n vertices. For a fixedi, the set

$(P_m)_i = P_m \diamond i$ is called a column of $P_m \circ P_n$(i^{th}column of$P_{m,n}$), the set $j(P_n) = j \diamond P_n$is called a row of $P_m \circ P_n$(j^{th}row of$P_{m,n}$).$(j, i)P_m$ denotes the row by column format.
Let C_1 and C_2 be two cycles of vertices 4. Suppose C_1 has vertex set $\{x_1, x_2, x_3, x_4\}$ and C_2 has vertex set $\{y_1, y_2, y_3, y_4\}$ then H-merging of C_1 and C_2 having the vertex $\{x_1, x_2 = y_1, x_3, x_4 = y_3, y_2, y_4\}$ edge set including all the edges C_1 and C_2 and $(x_2, x_4) = (y_1, y_3)$. Similarly V-merging of C_1 and C_2 having the vertex $\{x_1, x_2, x_3 = y_1, x_4 = y_2, y_3, y_4\}$ edge set including all the edges C_1 and C_2 and $(x_3, x_4) = (y_1, y_2)$.

2. The cardinal product of two paths

On the cardinal product $G \times H$ of two graphs G and H, $(u_1, v_1), (u_2, v_2) \in E(G \times H)$ if and only if $(u_1, u_2) \in E(G)$ and $(v_1, v_2) \in E(H)$.

Theorem 2.1. The RTCD number does not exists on the cardinal product of two paths.
Proof: Let us prove by induction on m, If $m = 2$, then the grid graph $G_{2,n} = P_2 \times P_n$,$n \geq 3$. For $n = 3$, $G_{2,3} = P_2 \times P_3$, is the union of two P_2's which is contradiction to the triple connected graphs. Similarly, $G_{2,4} = P_2 \times P_4$, is the union of P_4's, generally $G_{2,n} = P_2 \times P_n$, is the union of P_n's. , If $m = 3$, then the grid graph $G_{3,n} = P_3 \times P_n$, $n \geq 3$. For $n = 3$, $G_{3,3} = P_3 \times P_3$, is the union of C_4 and $K_{1,4}$which is contradiction to the triple connected graphs. Similarly $G_{3,4} = P_3 \times P_4$, is the union of $C_4(2P_2)$ and $C_4(0, 2P_2, 2P_2, 0)$, which contradiction to the triple connected graphs is. Generally$G_{m,n} = P_m \times P_n$, is the union of graphs. Hence RTCD number does not exists for cardinal product of two paths.
For instance $P_4 \times P_6$ as follows,



3. The strong product of two paths

On the strong product $G \otimes H$ of two graphs G and H, $(u_1, v_1), (u_2, v_2) \in E(G \times H)$ if and only if (i) $(u_1, u_2) \in E(G)$ and $(v_1, v_2) \in E(H)$ or (ii) $u_1 = u_2$ and $(v_1, v_2) \in E(H)$ or (iii) $v_1 = v_2$ and $(v_1, v_2) \in E(H)$

Theorem 3.1. The RTCD number of a grid graph $(P_m \otimes P_n)$ for $n \geq 3$ is $\gamma_{rtc}(P_2 \otimes P_n) = \gamma_{rtc}(G_{2,n}) = \begin{cases} 3 & \text{if } n = 3,4 \\ n-2 & \text{if } n \geq 5 \end{cases}$

Proof: It is obvious $\gamma_{rtc}(P_2 \otimes P_n) = 3$ for $n = 3,4$. On G,
$(2,2)P_m, (2,3)P_m, \dots \dots \dots (2, n-1)P_m$ dominates all the vertices.

Thus $\gamma_{rtc}(P_2 \otimes P_n) \leq n-2$. If $(2,1)P_m, (2,2)P_m, \ldots \ldots \ldots, (2,n)P_m$ dominates then

$\gamma_{rtc}(P_2 \otimes P_n) = n \geq n-2$. Hence $\gamma_{rtc}(P_2 \otimes P_n) = n-2$.

Theorem 3.2. The RTCD number of a grid graph $(P_m \otimes P_n)$ for $n \geq 2$ is

$$\gamma_{rtc}(P_3 \otimes P_n) = \gamma_{rtc}(G_{3,n}) = \begin{cases} 3 & \text{if } n = 2,3,4 \\ n-2 & \text{if } n \geq 5 \end{cases}$$

Proof: It is obvious that $\gamma_{rtc}(P_3 \otimes P_n) = 3$ for $n = 2,3,4$. The possible ways for RTCD sets are $[(2,2)P_m, (2,3)P_m, \ldots \ldots \ldots (2, n-1)P_m]$ or
$[(1,2)P_m, (1,3)P_m, \ldots \ldots \ldots (1,n)P_m] \cup [(3,n)P_m, (3, n-1)P_m, \ldots \ldots (3,2)P_m] \cup (2,n)P_m$.

If $[(2,2)P_m, (2,3)P_m, \ldots \ldots (2, n-1)P_m]$ is a dominating set then $\gamma_{rtc}(P_3 \otimes P_n)) \leq n-2$.

If $[(1,2)P_m, (1,3)P_m, \ldots \ldots (1,n)P_m] \cup [(3,n)P_m, (3, n-1)P_m, \ldots \ldots (3,2)P_m] \cup (2,n)P_m$ is a dominating set then $\gamma_{rtc}(P_3 \otimes P_n) = n-1+n-1+1 = 2n-1 \geq n-2$.

Hence $\gamma_{rtc}(P_3 \otimes P_n) = n-2$.

Theorem 3.3. The RTCD number of a grid graph $(P_m \otimes P_n)$ for $n \geq 4$ is $\gamma_{rtc}(P_4 \otimes P_n) = \gamma_{rtc}(G_{4,n}) = \begin{cases} 3 & \text{if } n = 2,3 \\ 2n-4 & \text{if } n \geq 4 \end{cases}$

Proof: It is obvious that $\gamma_{rtc}(P_2 \otimes P_n) = 3$ for $n = 3,4$.The possible ways for RTCD sets are $[(2,2)P_m, (2,3)P_m, \ldots \ldots \ldots (2, n-1)P_m] \cup [(3,2)P_m, (3,2)P_m, \ldots \ldots (3, n-1)P_m]$ or
$[(1,2)P_m, (1,3)P_m, \ldots (1,n)P_m] \cup [(4,2)P_m, (4,3)P_m, \ldots (4,n)P_m] \cup (2,n)P_m \cup (3,n)P_m$
If $[(2,2)P_m, (2,3)P_m, \ldots \ldots \ldots (2, n-1)P_m] \cup [(3,2)P_m, (3,2)P_m, \ldots \ldots \ldots (3, n-1)P_m]$is a dominating set then

$\gamma_{rtc}(P_4 \otimes P_n) = n-2+n-2 \leq 2n-4$.If$[(1,2)P_m, (1,3)P_m, \ldots (1,n)P_m] \cup$
$[(4,2)P_m, (4,3)P_m, \ldots (4,n)P_m] \cup (2,n)P_m \cup (3,n)P_m$ as dominating set

then $\gamma_{rtc}(P_4 \otimes P_n) = n+n+1+1 = 2n+2 \geq 2n-4$.

Hence $\gamma_{rtc}(P_4 \otimes P_n) = 2n-4$.

Theorem 3.4. $\gamma_{rtc}(G_{5,n}) = \gamma_{rtc}(P_5 \otimes P_n) = 2n-2$.

Proof: Consider the grid graph $P_5 \otimes P_n$, each row dominates its two neighbouring rows. Thus $(n-2)\left\lceil\frac{m}{3}\right\rceil$ rows dominate all the rows. For satisfying RTCD set, in between vertices of $(P_m)_1$ and $(P_m)_n$ will also be in RTCD set, i.e., $\lceil n+2 \rceil$. Thus $\gamma_{rtc}(P_5 \otimes P_n) = (n-2)\left\lceil\frac{m}{3}\right\rceil + \lceil n+2 \rceil$. For the complete grid graph $P_5 \otimes P_n$, $2(P_n)$ and $4(P_n)$, then

$\gamma_{rtc}(P_5 \otimes P_n) \leq 2(n-2)+2 = 2n-2$. If $1(P_n)$ and $4(P_n)$ or $2(P_n)$ and $5(P_n)$ as RTCD then $\gamma_{rtc}(P_5 \otimes P_n) = 2(n-2)+4 \geq 2n-2$.

Hence $\gamma_{rtc}(G_{5,n}) = \gamma_{rtc}(P_5 \otimes P_n) = 2n-2$.

Generalizing this result for even values onm, i.e., $m = 2k$, for all even values of m, each row $i(P_m)$ dominates two neighbouring rows. Thus, the RTCD number includes $\left\lceil \frac{m}{3} \right\rceil$, and the merging pattern of $G_{2k,n} = G_{k,n} G_{k,n} \ldots\ldots\ldots\ldots G_{k-1,n}$.

$$\gamma_{rtc}(G_{2k,n}) = \text{n times of } (k-1) + \left\lceil \frac{m}{3} \right\rceil - \frac{m}{3}$$
$$= n(k-1) + \left\lceil \frac{m}{3} \right\rceil \text{. For all odd values of m,}$$

iem $= 2k+1$ same as previous $G_{2k+1,n} = G_{k,n} G_{k,n} \ldots\ldots\ldots\ldots G_{k,n}$.

$$\gamma_{rtc}(G_{2k,n}) = \text{n times of } k + \left\lceil \frac{m}{3} \right\rceil - \frac{m}{3}$$
$$= nk + \left\lceil \frac{m}{3} \right\rceil = nk + \left\lceil \frac{2k+1}{3} \right\rceil$$

By combining all the above results, the merging pattern and RTCD number are tabulated as follows.

Complete Grid Graph	Merging pattern	RTCD number
$G_{2,n}, n \geq 3$	$G_{2,n}$	$n-2$
$G_{3,n}$	$G_{2,n} \otimes G_{2,n}$	$n-2$
$G_{4,n}$	$G_{3,n} \otimes G_{2,n}$	$2n-4$
$G_{5,n}$	$G_{3,n} \otimes G_{3,n}$	$2n-2$
$G_{6,n}$	$G_{3,n} \otimes G_{2,n} \otimes G_{2,n}$	$2n$
$G_{7,n}$	$G_{4,n} \otimes G_{4,n}$	$3n-2$
.	.	.
.	.	.
$G_{2k,n}$	$G_{k,n} \otimes G_{k,n} \otimes \ldots\ldots\ldots\ldots\ldots \otimes G_{k,n}$	$n(k-1) + \left\lceil \frac{m}{3} \right\rceil$
$G_{2k+1,n}$	$G_{k,n} \otimes G_{k,n} \otimes \ldots\ldots\ldots\ldots\ldots \otimes G_{k,n}$	$nk + \left\lceil \frac{2k+1}{3} \right\rceil$

Theorem 3.5. The RTCD number of a grid graph $G_{m,n} = P_m \otimes P_n$ for $n \geq 3$ is,

$$\gamma_{rtc}(P_m \otimes P_n) = \begin{cases} \left\lceil \frac{m(n+1)}{3} \right\rceil, m = 3k \\ \left\lceil \frac{(m+2)(n+2)}{3} - \frac{m}{3} \right\rceil, m = 3k+1 \\ \left\lceil \frac{(m+1)(n+2)}{3} - \frac{m}{3} \right\rceil, m = 3k+2 \end{cases}$$

Proof:

Case1: If $m = 3k, k \geq 2$. $G_{3k,n}$ can be formed by V- merging of $G_{k,n} \otimes G_{k,n} \otimes \ldots \otimes G_{k+1,n}$ and each component is a complete grape with vertex 4. Each $i(P_n)$ dominates its two neighboring rows. For satisfying the triple connected domination the in between vertices of the first and last column will also be in RTCD set.

$$\gamma_{rtc}(G_{3k,n}) = \text{k times of } (n+2) - \frac{m}{3}$$
$$= k(n+2) - \frac{m}{3} = \left\lceil \frac{m}{3}(n+2) \right\rceil - \frac{m}{3}$$
$$= \left\lceil \frac{m}{3}(n+2) - \frac{m}{3} \right\rceil$$
$$= \left\lceil \frac{m(n+1)}{3} \right\rceil, m = 3k$$

Case 2: If $m = 3k+1, k \geq 2,$ means one row added to the grid graph. Thus the RTCD set is k times of (n+2)+(n+2).

$$\gamma_{rtc}(G_{3k+1,n}) = \text{k times of } (n+2) + (n+2) - \frac{m}{3} = (n+2)(k+1) - \frac{m}{3}$$
$$= (n+2)\left\lceil \frac{(m-1)}{3} + 1 \right\rceil - \frac{m}{3}$$
$$= \left\lceil \frac{(m+2)(n+2)}{3} - \frac{m}{3} \right\rceil.$$

Case 3: If $m = 3k+2, k \geq 2,$ means two ro ws added to the grid graph. Thus,

$$\gamma_{rtc}(G_{3k+1,n}) = \text{k times of } (n+2) + (n+2) = (n+2)(k+1) - \frac{m}{3}$$
$$= (n+2)\left\lceil \frac{(m-2)}{3} + 1 \right\rceil - \frac{m}{3}$$
$$= \left\lceil \frac{(m+1)(n+2)}{3} - \frac{m}{3} \right\rceil$$

3.1. The equivalent product of two paths

On the equivalent product $G \circ H$ of two graphs G and H, $(u_1, v_1), (u_2, v_2) \in E(G \circ H)$ if and only if (i) $(u_1, u_2) \in E(G)$ and $(v_1, v_2) \in E(H)$ or (ii) $u_1 = u_2$ and $(v_1, v_2) \in E(H)$ or (iii) $v_1 = v_2$ and $(v_1, v_2) \in E(H)$ (iv) $(u_1, u_2) \in E(G')$ and $(v_1, v_2) \in E(H')$

Observation: (i)$\gamma_{rtc}(P_1 \circ P_n) = \gamma_{rtc}(P_n) = n$,

(ii)$\gamma_{rtc}(P_2 \circ P_n) = \gamma_{rtc}(P_2 \otimes P_n) = \gamma_{rtc}(G_{2,n}) = \begin{cases} 3 & \text{if } n = 3,4 \\ n-2 & \text{if } n \geq 5 \end{cases}$

Theorem 3.1.1. The RTCD number of a grid graph $(P_3 \circ P_n)$ for $n \geq 2$ is

$$\gamma_{rtc}(P_3 \circ P_n) = \gamma_{rtc}(G_{3,n}) = \begin{cases} 3 & \text{if } n = 2 \\ n & \text{if } n = 3,4,5 \\ n-2 & \text{if } n \geq 6 \end{cases}$$

Proof: It is obvious that $\gamma_{rtc}(P_3 \circ P_n) = 3$ if $n = 2$ and n if $n = 3,4,5$ and $P_1 \circ P_n$ contains all the edges of $P_1 \otimes P_n$ including some more edges.

$\gamma_{rtc}(P_3 \circ P_n) \leq \gamma_{rtc}(P_2 \otimes P_n) = n-2$ then $\gamma_{rtc}(P_3 \circ P_n) \leq n-2.$ Let D be the dominating set and the vertex set $(i, j) \in D, i = \{1,2,3\}$ and $j = \{1,2,3, \dots \dots, n\}$.

Case 1: $i = 1$ or $i = 3$. The vertex set $(1, j)$ dominates$(1, j - 1), (1, j), (1, j + 1), (2, j - 1), (2, j), (2, j + 1), (3,1), (3,2), \dots. (3, j - 2), \dots \dots \dots (3, n)$, if j=1 then $(1, j - 1), (2, j - 1), (3, j - 1)$ does not exists. Except the above mentioned vertices, the remaining vertices are dominated by the vertices are third row or the vertices of 1st and the 2nd row. For satisfying triple connected condition, the best way by taking the 3rd row as a dominating set. Hence $\gamma_{rtc}(P_3 \circ P_n) = n - 2 + 3 = n + 1 \geq n - 2$.

Case 2: $i = 2$, the vertices dominates $(j - 1)$th, jth, $(j + 1)$th vertices on $(P_3 \circ P_n)$ and for triple connected domination set, $(2,2), (2,3) \dots \dots (2, n - 2)$. thus D has at least $n - 2$ vertices. Hence $\gamma_{rtc}(P_3 \circ P_n) \geq n - 2$. Hence $\gamma_{rtc}(P_3 \circ P_n) = n - 2$.

Theorem 3.1.2. The RTCD number of $\gamma_{rtc}(P_m \circ P_n) = n$, where $m \geq 4$.

Proof: By taking the vertex set $(1, j), j = 1,2, \dots \dots, m$ as a dominating set then it dominates

$$(1, i - 1), (1, i), (1, i + 1), (2, i - 1), (2, i), (2, i + 1), (3,1), (3,2), \dots. (3, i - 2), \dots \dots \dots (3, n), (4,1), (4,2), (4, i + 1), (4, i - 1), (4, i), (4, i + 1), \dots \dots \dots. (n, 1), (n, 2), \dots. (n, i - 2), \dots \dots \dots (n, n)$$

if $i = 1$ then the dominating set do not include $(1, i - 1), (2, i - 1), (3, i - 1), (4, i - 1), \dots \dots \dots.. (n - 1, i - 1), (n, i - 1)$. Hence for getting dominating 1st row and 2nd row …… and (n-1)th row will on the dominating set. Thus $\gamma_{rtc}(P_m \circ P_n) \leq n$. If 1st row is taken to be a dominating set then the dominating set must at least n. $\gamma_{rtc}(P_3 \circ P_n) \geq n$.

Hence $\gamma_{rtc}(P_m \circ P_n) = n$.

4. Conclusion

In this paper, we afford the restrained triple connected domination number of cardinal, strong and equivalent products of path graphs. The authors obtained the continual exploration of Restrained triple connected domination number of Cartesian product of path will be reported in the subsequent papers.

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