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A Note on Maximize Fuzzy Net Present Value with New Ranking

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Abstract. In this paper, a new procedure for project scheduling called 'fuzzy cash flow weight' and 'fuzzy discounted cash flow weight' is studied with the aid of generalized trapezoidal fuzzy numbers. A new ranking procedure namely 'SD of PILOT' (Shortest Distance of the Point of Intersection of Legs of Trapezium) is proposed to find out the output of the above said notion. Relevant arithmetic operations of generalized trapezoidal fuzzy numbers are also utilized to justify the notion. A numerical example is also included.

Keywords: Trapezoidal fuzzy number, project scheduling, cash flow, net present value, SD of PILOT

AMS Mathematics Subject Classification (2010): 03E72, 90B99

1. Introduction

Net present value (*NPV*) is the most commonly used method of evaluating the effectiveness of investment projects. *NPV* is the sum of discounted net benefits (net cash flows) over the whole life cycle of the investment project. Cash flow analysis is one of the most popular methods for investigating the outcome of an economical project. In this paper, we present a simple method to calculate the net present value of a cash flow when both costs and benefits are given as generalized trapezoidal fuzzy numbers. Project scheduling is the process where the various activities that need to be undertaken during a projects lifetime should be scheduled. It is concerned with the techniques that can be employed to manage the activities that need to be undertaken during the development of a project. It is primarily concerned with attaching a timescale and sequence to the activities to be conducted within the project. This paper will focus on *project scheduling* that is the subset of project management [14]. There are many recent works on project scheduling to maximize net present value of the project. The approaches offered for the solution of the problem of maximizing the net present value of a project through the manipulation of the times of realization of its key events are reviewed in [6]. An integer programming algorithm for project scheduling subject to resource limitations during each period of the schedule duration is described in [16]. An activity scheduling problem for a project where cash inflows and outflows are given and availability restrictions are imposed on capital and renewable resources is presented in [13]. The problem of scheduling activities in a project to maximize the net present value of the project is solved for the case where the activity cash flows are independent of the time of activity realization in [7]. The unconstrained project scheduling problem with discounted cash flows where the net cash flows are assumed to be dependent on the completion times of the corresponding activities to maximize the net present value of the project subject to the precedence constraints and a fixed deadline are examined in [15]. PeddiPhaniBushanRao and Nowpada Ravi Shankar found a ranking procedure based on Area, Mode, Spreads and Weight for Generalized Fuzzy Numbers [11]. Bih-SheueShieh developed ranking procedure in an approach to Centroids of Fuzzy Numbers [2]. Allahviranloo et al. developed a ranking procedure based on a new distance measure for Generalized Trapezoidal Fuzzy Numbers [1]. İrem UÇAL and Kuchta [8] developed project scheduling procedure for the projects which have fuzzy cash flows very first, however there are lots of studies on project scheduling to maximize net present value of the project. It seems to be useful to adopt some of the to the fuzzy case, so that they can be used to build a project schedule with a maximal *NPV* taking into account the risk and uncertainty connected to the cash flow estimation in practice. In this paper we are introducing a new ranking function to defuzzify the generalized trapezoidal fuzzy number. We are applying Generalized Trapezoidal fuzzy numbers in project management, especially in network scheduling in maximizing net present value for which İrem UÇAL and Kuchta had worked on Triangular fuzzy numbers. Also a fuzzy version of a procedure for project scheduling is modified from accordingly to maximize the fuzzy net present value of projects with fuzzy cash flows. Fuzzy equivalents of cash flow weight and discounted cash flow weight are defined which are used to find the importance of the activities with respect to the fuzzy net present value of the project. The procedure is applied to an example and results are discussed in conclusions. There are two methods are used to determine net present value: 1. Cash flow weight heuristics and 2. Discounted cash flow weight heuristics [4]. Section 1 deals with preliminaries of fuzzy numbers, Section 3 deals with new ranking procedure we developed, Section 4 consists of cash flow weight heuristics, Section 5 has Fuzzy cash flow weight heuristics, Section 6 has its application and we concluded in Section 7.

2. Preliminaries

Definition 2.1. Chen (1985, 1990) represented a Generalized Trapezoidal Fuzzy Number (GTrFN) $\tilde{A}$ as $\tilde{A} = (a_1,b_1,c_1,d_1;w)$, $0 < w \leq 1$ and a_1,b_1, $c_1$and $d_1$are real numbers. The generalized fuzzy number $\tilde{A}$ is a fuzzy subset of real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions:

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_{\tilde{A}}(x) = 0$,where $-\infty \leq x \leq a_1$.

(iii) $\mu_{\tilde{A}}(x)$ isstrictly increasing with constant rate on $a_1 \leq x \leq b_1$.

(iv) $\mu_{\tilde{A}}(x)$ = w, where $b_1 \leq x \leq c_1$.

(v) $\mu_{\tilde{A}}(x)$ isstrictly decreasing with constant rate on $c_1 \leq x \leq d_1$.

(vi) $\mu_{\tilde{A}}(x) = 0$ where $d_1 \le x \le \infty$.

Definition 2.2. A GTrFN $\tilde{A}$ = (a_1, b_1, c_1, d_1; w) is a fuzzy set of the real line R whose membership function $\mu_{\tilde{A}}(x)$: $R \to [0, w]$ is defined as

$$\mu_{\tilde{A}}^{w}(x) = \begin{cases} \mu_{L\tilde{A}}^{w}(x) = w\left(\dfrac{x-a_1}{b_1-a_1}\right), \; for a_1 \le x \le b_1 \\ w, \; for b_1 \le x \le c_1 \\ \mu_{R\tilde{A}}(x) = w\left(\dfrac{d_1-x}{d_1-c_1}\right), \; for c_1 \le x \le d_1 \\ 0, Otherwise \end{cases}$$

, where $a_1 < b_1 < c_1 < d_1$ and $w \in (0,1]$

2.3. Arithmetic operations

Algebraic operations for G*TrFN*s are given by Property (1) to Property (7) where all the fuzzy numbers are positive [8].

Addition rule: If $\tilde{A}$ = (a_1, b_1, c_1, d_1; $w_{\tilde{A}}$) and $\tilde{B}$ = (a_2, b_2, c_2, d_2; $w_{\tilde{B}}$), then

$\tilde{A} \oplus \tilde{B}$ = ($a_1 + a_2$, $b_1 + b_2$, $c_1 + c_2$, $d_1 + d_2$; min($w_{\tilde{A}}$, $w_{\tilde{B}}$)).

Subtraction rule: If $\tilde{A}$ = (a_1, b_1, c_1, d_1; $w_{\tilde{A}}$) and $\tilde{B}$ = (a_2, b_2, c_2, d_2; $w_{\tilde{B}}$), then

$\tilde{A} \ominus \tilde{B}$ = ($a_1 - d_2$, $b_1 - c_2$, $c_1 - b_2$, $d_1 - a_2$; min($w_{\tilde{A}}$, $w_{\tilde{B}}$)).

Multiplication rule: If $\tilde{A}$ = (a_1, b_1, c_1, d_1; $w_{\tilde{A}}$) and $\tilde{B}$ = (a_2, b_2, c_2, d_2; $w_{\tilde{B}}$), then

$\tilde{A} \otimes \tilde{B}$ = ($a_1 a_2$, $b_1 b_2$, $c_1 c_2$, $d_1 d_2$; min($w_{\tilde{A}}$, $w_{\tilde{B}}$)).

Division rule: If $\tilde{A}$ = (a_1, b_1, c_1, d_1; $w_{\tilde{A}}$) and $\tilde{B}$ = (a_2, b_2, c_2, d_2; $w_{\tilde{B}}$), then

$\tilde{A} \div \tilde{B}$ = ($\dfrac{a_1}{d_2}, \dfrac{b_1}{c_2}, \dfrac{c_1}{b_2}, \dfrac{d_1}{a_2}$; min($w_{\tilde{A}}$, $w_{\tilde{B}}$)).

Scalar multiplication rule: If $\tilde{A}$ = (a, b, c, d; w) and λ is any scalar, then

$$\lambda \otimes (a,b,c,d;w) = \begin{cases} (\lambda a, \lambda b, \lambda c, \lambda d; w), if \lambda \ge 0 \\ (\lambda d, \lambda c, \lambda b, \lambda a; w), if \lambda \le 0 \end{cases}, \forall \lambda \in \Re$$

Scalar division rule: If $\tilde{A}$ = (a, b, c, d; w) and λ is any scalar, then

$$\lambda \div (a,b,c,d;w) = \begin{cases} \left(\dfrac{\lambda}{d}, \dfrac{\lambda}{c}, \dfrac{\lambda}{b}, \dfrac{\lambda}{a}; w\right), if \lambda \ge 0 \\ \left(\dfrac{\lambda}{a}, \dfrac{\lambda}{b}, \dfrac{\lambda}{c}, \dfrac{\lambda}{d}; w\right), if \lambda \le 0 \end{cases}, \forall \lambda \in \Re$$

Power rule: If $\tilde{A}$ = (a, b, c, d; w) and λ is any scalar, then

$$(a,b,c,d;w)^{\lambda} = \begin{cases} (a^{\lambda}, b^{\lambda}, c^{\lambda}, d^{\lambda}; w), if \lambda \ge 0 \\ (d^{\lambda}, c^{\lambda}, b^{\lambda}, a^{\lambda}; w), if \lambda \le 0 \end{cases}, \forall \lambda \in \Re$$

3. SD of PILOT ranking procedure

Ranking fuzzy numbers has been an indispensable area of research especially for its applications in decision making analysis to represent uncertain value. Many ranking fuzzy number approaches have been suggested in literature for multi attribute fuzzy decision making problems, data analysis and artificial intelligence, thus make ranking fuzzy numbers is imperative because the measurements are imprecise in nature [5, 9].

Proposition 3.1. The point of intersection of the two non parallel lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is $\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$.

Proposition 3.2. The equation of the line joining the points (x_1, y_1) and (x_2, y_2) is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$.

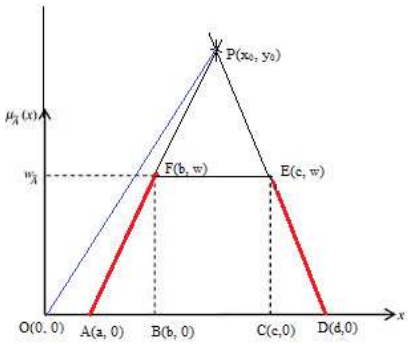


Figure 3.2.1: A generalized trapezoidal fuzzy number $\tilde{A}$= (*a*, *b*, *c*, *d*; *w*).

In Figure 3.2.1, the generalized trapezoidal fuzzy number covers the trapezoid ADEF, in which the parallel sides AD and EF are called bases and the non-parallel sides AF and DE are called legs. The generalized trapezoidal fuzzy number is covered by a trapezium ADEF by the points A(a, 0), D(d,0), E(c, w), F(b, w) in which the legs AF and DE will meet in a point P(x_0, y_0) when they are extended. The shortest distance between the origin O(0, 0) and P(x_0, y_0) is the ranking function defined using "**SD of PILOT Ranking Procedure**".Using proposition (2), the equation of the line segment

AF = $\frac{y-0}{w-0} = \frac{x-a}{b-a} \Rightarrow \frac{y}{w} = \frac{x-a}{b-a} \Rightarrow y(b-a) = w(x-a) \Rightarrow (b-a)y - wx + aw = 0$

$$\Rightarrow wx - (b-a)y - aw = 0 \qquad (3.1)$$

Similarly the equation of the line segment DE is $wx - (c-d)y - dw = 0$ (3.2)

The straight lines in the equation (3.1) and (3.2) will meet in the point $P(x_0, y_0)$ can be found using Proposition (1) as follows:

$$P(x_0, y_0) = \left(\frac{(b-a)dw - (c-d)aw}{-w(c-d) + w(b-a)}, \frac{-aw^2 + dw^2}{-w(c-d) + w(b-a)}\right) = \left(\frac{bd - ad - ac + ad}{(b-a) + (d-c)}, \frac{w(d-a)}{(b-a) + (d-c)}\right)$$

$$P(x_0, y_0) = \left(\frac{bd - ac}{(b-a) + (d-c)}, \frac{w(d-a)}{(b-a) + (d-c)}\right)$$

The distance between the origin $O(0, 0)$ and $P(x_0, y_0)$ can be found as follows:

$$O\overline{P} = \sqrt{\left(\left[\frac{bd-ac}{(b-a)+(d-c)}-0\right]^2+\left[\frac{w(d-a)}{(b-a)+(d-c)}-0\right]^2\right)} = \sqrt{\frac{(bd-ac)^2+w^2(d-a)^2}{((b-a)+(d-c))^2}}$$

$$= \frac{1}{(b-a)+(d-c)}\sqrt{\left([bd-ac]^2+w^2[d-a]^2\right)} = \frac{\sqrt{\left[(bd-ac)^2+w^2(d-a)^2\right]}}{(b+d)-(a+c)}$$

The ranking function of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$is

$$\Re(\tilde{A}) = \frac{\sqrt{\left[(bd-ac)^2+w^2(d-a)^2\right]}}{(b+d)-(a+c)} \tag{3.3}$$

3.1. Cash flow weighing heuristics

A project is a network with activities (A , i = 1,2,...,N) represented as nodes, relations between activities represented as arcs, the resources required by activities denoted by r_{ik}(i = 1,2,...,N and k = 1,2,...,m) the total resources available for the project denoted by r_{tk}(k =1,2,...,m), and durations of the activities denoted by d_i(i = 1,2,..., N). Net cash flows of activities occur at the beginning or end of the related activity and the value of it is independent of the starting or ending moment of the activity. The sum of all the cash flows from different activities starting or finishing in moment j will be denoted as CFj(j = 1,2,...,$T^{H)}$where T^H denotes time horizon). Present value (PV) of a single future payment occurred in the end of n^{th} year from now is given in (3.1.1) where F stands for amount of the payment and r denotes the interest rate (cost of capital). $PV = \frac{F}{(1+r)^n}$ (3.1.1)

The goal is to find a schedule with a maximal NPV which is sum of all discounted cash flows formulated on (3.1.2): $NPV = \sum_{j=0}^{n} \frac{CF_j}{(1+r)^j}$ (3.1.2)

Cash flow weight (CFW) heuristic is a heuristic which dynamically selects a high priority activity from available activities for the assignment of resources. In the considered heuristic procedure, the priority of an activity is linked to the cash flows linked to the very activity and all the activities which follow it. The priority is measured by means of cash flow weighting [8].

3.1.1. Cash flow weighting

Cash flow weighting is an assignment of a weight to each activity with respect to the cash flow creating potential of the activity which means the sum of the cash flows occurred from the activity and its successor activities. The cash flow weight heuristic is a forward pass heuristic which selects the activity with the largest CFW from the list of available activities and attempts to assign it to the earliest possible period with considering precedence and resource constraints [8]. After assignment of an activity, the resource constraints are updated. When the last activity is assigned, the procedure stops [3].

3.1.2. Cash flow weight algorithm

There are three steps on cash flow weight procedure. In the first step, the cash flow weights of each activity are determined and all activities are included to the list of available activities in an order of i (i= 1,2,..., N) without taking into account the predecessors. In the second step, the activity with the highest CFW is selected from the top of the list of available activities. In case of a tie, the lowest numbered task is assigned first. If the selected task has predecessors, in order to assign the selected activity as soon as possible, the predecessors of the selected activity are assigned respectively in the increasing order of their indices *i* (i = 1,2,..., *N)* and as soon as possible with respect to the resources available. After assignment of the selected activity the available resources are updated. In the third step if there is any unassigned activity second step is repeated, otherwise the project schedule is completed [3].

3.1.3. Discounted cash flow weight algorithm

Discounted cash flow algorithm has the same procedure with cash flow weight algorithm while it deals with discounted cash flow weights (*DCFW*s) instead of *CFW*s. *DCFW* for an activity is determined by the summation of cash flow of the activity and the discounted value of all future cash flows of successor activities [3].

3.2. Fuzzy cash flow weighing heuristics

We consider a project with fuzzy cash flows, linked to the beginning or ending of activities independent of their time setting, fuzzy interest rate. The goal is to find a schedule with a maximal fuzzy NPV, where in comparing the fuzzy NPV we choose one of the relations defined in Section 3.1.1. Fuzzy present value (*PV*) of a single future payment occurred in the end of *n*th year from now is given in (3.2.1) where $\tilde{F}$ stands for fuzzy amount of the payment and $\tilde{i}$ denotes the fuzzy interest rate

$$\tilde{P}V = \frac{\tilde{F}}{(1+\tilde{i})^n} \tag{3.2.1}$$

The general formula of fuzzy net present value $N\tilde{P}V$ is given in (3.2.2), where $C\tilde{F}_j$ denotes net fuzzy cash flows occurred at time *j*, *n* denotes the useful life of the project and $\tilde{i}$ denotes the fuzzy interest rate [9]. $N\tilde{P}V = \sum_{j=0}^{n} \frac{C\tilde{F}_j}{(1+\tilde{i})^j}$ (3.2.2)

Fuzzy net present value formula for *TFN*s is generated on (3.2.1)

$$N\tilde{P}V = \sum_{\substack{j=0 \\ CF_j<0}}^{n} \frac{C\tilde{F}_{ja}}{(1+\tilde{i}_a)^j}, \frac{C\tilde{F}_{jb}}{(1+\tilde{i}_b)^j}, \frac{C\tilde{F}_{jc}}{(1+\tilde{i}_c)^j}, \frac{C\tilde{F}_{jd}}{(1+\tilde{i}_d)^j}; \min(w_1, w_2) \quad +$$

$$\sum_{\substack{j=0 \\ CF_j>0}}^{n} \frac{C\tilde{F}_{ja}}{(1+\tilde{i}_d)^j}, \frac{C\tilde{F}_{jb}}{(1+\tilde{i}_c)^j}, \frac{C\tilde{F}_{jc}}{(1+\tilde{i}_b)^j}, \frac{C\tilde{F}_{jd}}{(1+\tilde{i}_a)^j}; \min(w_1, w_2) \tag{3.2.3}$$

where $C\tilde{F}_j = (a,b,c,d;w_1)$ - cash flow $\tilde{i} = (i_a, i_b, i_c, i_d; w_2)$ - Interest rate

3.1.1. Fuzzy cash flow weighting

Fuzzy cash flow weighting is an assignment of a fuzzy weight to each activity with respect to the fuzzy cash flow creating potential of the activity which means the sum of the cash flows occurred from the activity and its successor activities. In this procedure, the cash flows of the activities are assumed as either negative or positive fuzzy numbers.

3.2.2. Fuzzy cash flow weight algorithm

There are the following four steps involved on fuzzy cash flow weight algorithm:

Step 1: The fuzzy cash flow weights of each activity which are denoted by $C\tilde{F}W_i$, are determined and all activities are added without predecessors to the available list.

Step 2: $C\tilde{F}W$ values are ordered with a method from equation (3.1).

Step 3: The activity with the highest $\tilde{CFW}$ is selected from the list of precedence available. In case of a tie, the lowest numbered task is assigned first. If the selected task has predecessors, in order to assign the selected activity as soon as possible, the predecessors of the selected activity are assigned respectively. After assignment of the selected activity the resource available list is updated.

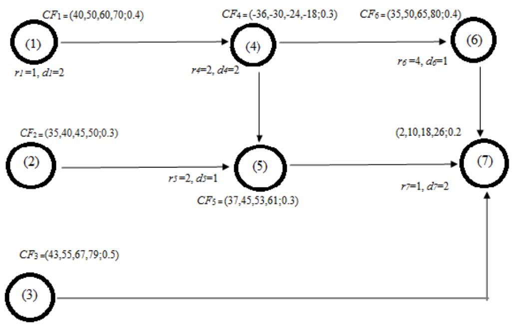
Step 4: If there is any unassigned activity the third step is repeated, otherwise the project schedule is completed.

3.2.3. Fuzzy discounted cash flow weight algorithm

Fuzzy discounted cash flow algorithm has the same procedure with fuzzy cash flow algorithm while it deals with fuzzy discounted cash flow weights $D\tilde{CFW}$ instead of $\tilde{CFW}$. $D\tilde{CFW}_i$ for an activity is determined by the summation of cash flow of the activity and the discounted value of all future cash flows of successor activities [12].

3.3. Application

The fuzzy cash flows occurred at the beginning of the activity, immediate predecessors, durations, and resource requirements for each task are given in Table 3.3.1. The number of available resources for this project is determined as 5. A network diagram of a project is given in Fig. 3.3.1 with the cash flows, resource requirements, and durations of the tasks. The project has just one type of resource which is limited to 5 over the project realization time.



CF_i = Fuzzy cash flows; ( ) = Task number; r_i = Resource requirement; d_i = duration

Figure 3.3.1: Network diagram of the project

Task Number	Fuzzy Cash Flow	Immediate Predecessors	Duration	Resource Requirement
1	(40,50,60,70;0.4)	-	2	1
2	(35,40,45,50;0.3)	-	4	2
3	(43,55,67,79;0.5)	-	2	3
4	(-36,-30,-24,-18;0.3)	1	2	2
5	(37,45,53,61;0.3)	2,4	1	2
6	(35,50,65,80;0.4)	4	1	4
7	(2,10,18,26;0.2)	3,5,6	2	1

Table 3.3.1: Project data

3.3.1. Fuzzy cash flow weighting

The calculation of $DC\tilde{F}W$ for the tasks 1 to 7 is given below. The results, their preference values calculated using the SD of PILOT ranking function from the equation (3.3), their pessimistic and optimistic values are given on Table 3.3.2.

$C\tilde{F}W_1 = C\tilde{F}_1 + C\tilde{F}_4 + C\tilde{F}_5 + C\tilde{F}_6 + C\tilde{F}_7$ = (40,50,60,70;0.4) + (-36,-30,-24,-18;0.3) + (37,45,53,61;0.3) + (35,50,65,80;0.4) +(2,10,18,26;0.2) = (78,125,172,219;0.2).

Task No	Fuzzy Cash Flow	$C\tilde{F}W$	Preference Value	Pessimist Value	Optimist Value
1	(40,50,60,70;0.4)	(78,125,172,219;0.2)	148.50	78	219
2	(35,40,45,50;0.3)	(74,95,116,137;0.2)	105.50	74	137
3	(43,55,67,79;0.5)	(45,65,85,105;0.2)	75	45	105
4	(-36,-30,-24, -18;0.3)	(38,75,112,147;0.2)	93.50	38	147
5	(37,45,53,61;0.3)	(39,55,71,87;0.2)	63.00	39	87
6	(35,50,65,80;0.4)	(37,60,,83,106;0.2)	71.50	37	106
7	(2,10,18,26;0.2)	(2,10,18,26;0.2)	8.548	2	26

Table 3.3.2: Fuzzy cash flow weights

Ranking of $C\tilde{F}W$ values of activities are found as: $C\tilde{F}W_1 > C\tilde{F}W_2 > C\tilde{F}W_3 > C\tilde{F}W_4 > C\tilde{F}W_5 > C\tilde{F}W_6 > C\tilde{F}W_7$ for the optimistic and neutral ranking methods. Activity 1 which has the highest value is scheduled first and the available resources updated as 4 for periods 1-2. Activity 2 which has the next highest value is scheduled in periods 1-4 and available resources are updated as 2 for the periods 1-2, and as 3 for the periods 3-4. Activity 4 which has the third highest $C\tilde{F}W$ value is scheduled in periods 3-4 and available resources are updated as 1 for the 3-4. Activity 3 which has the next highest value is scheduled in periods 5-6 and available resources are updated as 2 for periods 5-6. Activity 6 which has the next highest value is scheduled in period 7 and available resources for period 7 are updated as 1. Activity 5 which has the next highest value is scheduled in period 5 and available resources for period 5 are updated as 0 and the last activity, Activity 7 is scheduled in periods 8-9 and available resources are updated for periods 8-9 as 4. After scheduling the last activity the algorithm is stopped.

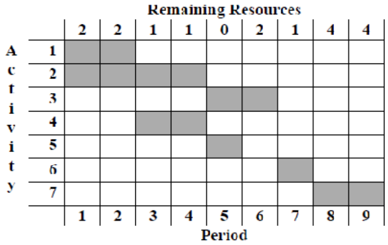


Figure 3.3.2: Project schedule resulting from $C\tilde{F}W$ heuristic by neutral and optimistic ranking methods.

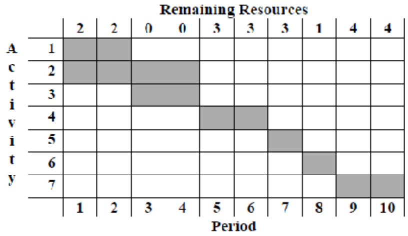


Figure 3.3.3: Project schedule resulting from $C\tilde{F}W$ heuristic by pessimistic ranking method.

The project schedules resulting from the neutral and optimistic ranking methods for $C\tilde{F}W$ heuristic is given in Fig. 6.2. The net present value based on $C\tilde{F}W$ Heuristic by Neutral and Optimistic Ranking Methods is calculated for the fuzzy interest rate $\tilde{i} = (0.06,0.08,0.10,0.12;0.2)$ as follows:

$$N\tilde{P}V = \frac{(40,50,60,70;0.4)}{(1+0.06,1+0.08,1+0.10,1+0.12;0.2)^0} + \frac{(35,40,45,50;0.3)}{(1.06,1.08,1.10,1.12;0.2)^0}$$
$$+ \frac{(43,55,67,79;0.5)}{(1.06,1.08,1.10,1.12;0.2)^4} + \frac{(-36,-30,-24,-18;0.3)}{(1.06,1.08,1.10,1.12;0.2)^2} + \frac{(37,45,53,61;0.3)}{(1.06,1.08,1.10,1.12;0.2)^4}$$
$$+ \frac{(35,50,65,80;0.4)}{(1.06,1.08,1.10,1.12;0.2)^6} + \frac{(2,10,18,26;0.2)}{(1.06,1.08,1.10,1.12;0.2)^7}$$
$$= (112.44,165.94,224.83,290.23;0.2)$$

Ranking of $C\tilde{F}W$ values of activities are found as: $C\tilde{F}W_1 > C\tilde{F}W_2 > C\tilde{F}W_3 > C\tilde{F}W_5 > C\tilde{F}W_6 > C\tilde{F}W_4 > C\tilde{F}W_7$ for the pessimistic ranking method. Activity 1 which has the highest value is scheduled first and the available resources updated as 4 for periods 1-2. Activity 2 which has the next highest value is scheduled in periods 1-4 and available resources are updated as 2 for the periods 1-2, and as 3 for the periods 3-4. Activity 3 which has the third highest $C\tilde{F}W$ value is scheduled in periods 3-4 and available resources are updated as 0 for the 3-4. Activity 5 which has the next highest value but because of the predecessors, Activity 4 is scheduled in periods 5-6 and available resources are updated as 3 for periods 5-6. Activity 5 which has no predecessor constraint any more is scheduled in period 7 and available resources are updated as 3 for period 7. Activity 6 which has the next highest value is scheduled in period 8 and available resources for period 8 are updated as 1, and the last activity, Activity 7 is scheduled in periods 9-10 and available resources are updated for periods 9-10 as 4. After scheduling the last activity the algorithm is stopped. The project schedules resulting from the pessimistic ranking method for $C\tilde{F}W$ heuristic is given in Fig. 3.3.3.

The net present value based on $C\tilde{F}W$ Heuristic by Neutral and Optimistic Ranking Methods is calculated for the fuzzy interest rate $\tilde{i} = (0.06,0.08,0.10,0.12;0.2)$ as follows:

$$N\tilde{P}V = \frac{(40,50,60,70;0.4)}{(1+0.06,1+0.08,1+0.10,1+0.12;0.2)^0} + \frac{(35,40,45,50;0.3)}{(1.06,1.08,1.10,1.12;0.2)^0}$$
$$+ \frac{(-36,-30,-24,-18;0.3)}{(1.06,1.08,1.10,1.12;0.2)^4} + \frac{(37,45,53,61;0.3)}{(1.06,1.08,1.10,1.12;0.2)^6} + \frac{(43,55,67,79;0.5)}{(1.06,1.08,1.10,1.12;0.2)^2}$$
$$+ \frac{(35,50,65,80;0.4)}{(1.06,1.08,1.10,1.12;0.2)^7} + \frac{(2,10,18,26;0.2)}{(1.06,1.08,1.10,1.12;0.2)^8}$$
$$= (116.15,169.13,227.10,291.39;0.2)$$

3.3.2. Fuzzy discounted cash flow weighting

The calculation of $DC\tilde{F}W$ for the tasks 1 to 7 is given below. The results, their preference values calculated using the ranking function from the equation (3.3), their pessimistic and optimistic values are given on Table 3.3.3.

$$DC\tilde{F}W_1 = DC\tilde{F}_1 + DC\tilde{F}_4 + DC\tilde{F}_5 + DC\tilde{F}_6 + DC\tilde{F}_7$$

$$= \frac{(40,50,60,70;0.4)}{(1+0.06,1+0.08,1+0.10,1+0.12;0.2)^1} + \frac{(-36,-30,-24,-18;0.3)}{(1.06,1.08,1.10,1.12;0.2)^4}$$

$$+ \frac{(37,45,53,61;0.3)}{(1.06,1.08,1.10,1.12;0.2)^5} + \frac{(35,50,65,80;0.4)}{(1.06,1.08,1.10,1.12;0.2)^6} + \frac{(2,10,18,26;0.2)}{(1.06,1.08,1.10,1.12;0.2)^7}$$

$$= (44.59,79.54,118.13,161.26;0.2)$$

Task No	**Fuzzy Cash Flow**	$DC\tilde{F}W$	**Preference Value**	**Pessimist Value**	**Optimist Value**
1	(40,50,60,70;0.4)	(44.59,79.54,118.13,161.26;0.2)	96.81	44.59	161.26
2	(35,40,45,50;0.3)	(46.95,61.95,79.34,99.55;0.2)	69.36	46.95	99.55
3	(43,55,67,79;0.5)	(29.82,43.67,59.57,77.88;0.2)	50.51	29.82	77.88
4	(-36,-30,-24,-18;0.3)	(9.50,34.89,63.58,96.44;0.2)	47.39	9.50	96.44
5	(37,45,53,61;0.3)	(35.96,55.39,78.84,107.10;0.2)	64.94	35.96	107.10
6	(35,50,65,80;0.4)	(16.75,29.86,45.93,65.58;0.2)	36.29	16.75	65.58
7	(2,10,18,26;0.2)	(2,10,18,26;0.2)	14.00	2	26

Table 3.3.3: Fuzzy discounted cash flow weight

Rankings of $DC\tilde{F}W$ values of activities are found as:

$DC\tilde{F}W_1 > DC\tilde{F}W_2 > DC\tilde{F}W_5 > DC\tilde{F}W_3 > DC\tilde{F}W_4 > DC\tilde{F}W_6 > DC\tilde{F}W_7$

for the neutral ranking method, and

$DC\tilde{F}W_1 > DC\tilde{F}W_5 > DC\tilde{F}W_2 > DC\tilde{F}W_4 > DC\tilde{F}W_3 > DC\tilde{F}W_6 > DC\tilde{F}W_7$ for the optimistic ranking. The difference between neutral ranking method and optimistic ranking method is on Activity 2 and Activity 5. The Activity 2 should be scheduled first due to it is predecessor of Activity 5. So these two rankings result on the same schedule which is shown in Fig 6.4. The project schedules resulting from the neutral and optimistic ranking methods for $DC\tilde{F}W$ heuristic is given in Fig. 3.3.4.

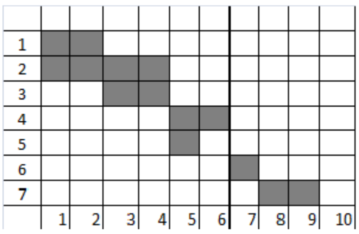


Figure 3.3.4: Project schedule resulting from $DC\tilde{F}W$ heuristic by neutral and optimistic ranking methods

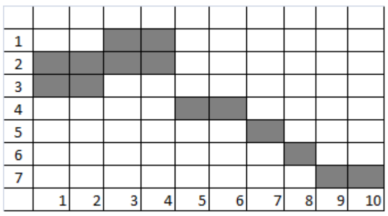


Figure 3.3.5: Project schedule resulting from $DC\tilde{F}W$ heuristic by pessimistic ranking method

The net present value based on $DC\tilde{F}W$ Heuristic by Neutral and Optimistic Ranking Methods is calculated for the fuzzy interest rate $\tilde{i} = (0.06,0.08,0.10,0.12;0.2)$ as follows:

$$N\tilde{P}V = \frac{(40,50,60,70;0.4)}{(1+0.06,1+0.08,1+0.10,1+0.12;0.2)^0} + \frac{(35,40,45,50;0.3)}{(1.06,1.08,1.10,1.12;0.2)^0}$$
$$+ \frac{(43,55,67,79;0.5)}{(1.06,1.08,1.10,1.12;0.2)^2} + \frac{(-36,-30,-24,-18;0.3)}{(1.06,1.08,1.10,1.12;0.2)^4} + \frac{(37,45,53,61;0.3)}{(1.06,1.08,1.10,1.12;0.2)^4}$$
$$+ \frac{(35,50,65,80;0.4)}{(1.06,1.08,1.10,1.12;0.2)^6} + \frac{(2,10,18,26;0.2)}{(1.06,1.08,1.10,1.12;0.2)^7}$$
$$= (122.92,177.49,236.46,300.88;0.2)$$

Ranking of $DC\tilde{F}W$ values of activities are found as:

$DC\tilde{F}W_2 > DC\tilde{F}W_1 > DC\tilde{F}W_5 > DC\tilde{F}W_3 > DC\tilde{F}W_6 > DC\tilde{F}W_4 > DC\tilde{F}W_7$ for the pessimistic ranking method. The project schedules resulting from the pessimistic ranking methods for $DC\tilde{F}W$ heuristic is given in Fig. 3.3.5.The net present value based on $DC\tilde{F}W$ Heuristic by Optimistic Method is calculated for the fuzzy interest rate $\tilde{i} = (0.06,0.08,0.10,0.12;0.2)$ as follows:

$$N\tilde{P}V = \frac{(40,50,60,70;0.4)}{(1+0.06,1+0.08,1+0.10,1+0.12;0.2)^2} + \frac{(35,40,45,50;0.3)}{(1.06,1.08,1.10,1.12;0.2)^0}$$
$$+ \frac{(43,55,67,79;0.5)}{(1.06,1.08,1.10,1.12;0.2)^0} + \frac{(-36,-30,-24,-18;0.3)}{(1.06,1.08,1.10,1.12;0.2)^4} + \frac{(37,45,53,61;0.3)}{(1.06,1.08,1.10,1.12;0.2)^6}$$
$$+ \frac{(35,50,65,80;0.4)}{(1.06,1.08,1.10,1.12;0.2)^7} + \frac{(2,10,18,26;0.2)}{(1.06,1.08,1.10,1.12;0.2)^8}$$
$$= (116.75,169.99,228.099,292.38;0.2)$$

4. Conclusion

Two different heuristic methods for project scheduling to maximize fuzzy net present value of a project are proposed. In the application section, the schedules resulting from $C\tilde{F}W$ and $DC\tilde{F}W$ heuristics are different which make differences on project's fuzzy net present value. Also the ranking method chosen for the ranking step of the algorithm could change the schedule, fuzzy net present value, and realization time of the project. The interpretation the decision maker gets from these algorithms is which activities interpretation the decision maker gets from these algorithms is which activities are critical for fuzzy net present value of the project and cannot be moved and which activities are dependent on his/her attitude. It is also worth mentioning that in our case the whole project duration is planned to be 9 or 10 time units and in one of the cases it is equal to the shortest possible project duration (which is 9) and in the other case that it is more advantageous to prolong the project realization by 1 time unit to achieve higher fuzzy net present value. As a further research the proposed models could be expanded for different ranking methods to determine the best suitable ranking method for fuzzy critical path method and maximizing NPV.

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