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Single Period Inventory System with Service Facility Having Fuzzy Demand and Fuzzy Service Time

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Abstract. In this article we considered a single period inventory system in which a specific item is maintained at service facility to supply unit item to customers with fuzzy service rate. Demand for item is assumed to be fuzzy. Customers arrived during the service time of a previous customer has to wait in a queue. For different fuzzy total cost obtained from different order quantity, a method for ranking fuzzy numbers is adopted to find the optimal order quantity in terms of cost. Grading the fuzzy cost, with the policy that when the profit gained from selling one item is less (greater) than the loss incurred due to one unsold item, the optimal order quantity lies in the interval defined for the left (right) shape function of the fuzzy demand as well as the fuzzy service time also. We used this methodology to our inventory cost model to get optimal order quantity.

Keywords: Fuzzy demand, service facility, fuzzy service time and fuzzy number ranking

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

Single period inventory system is a simple but most suitable for seasonal items sold in the festival seasons [1-5]. It is also a frequently discussed model in the literature. A single procurement is made to satisfy the customers who want to purchase them with almost care during that season. The customers accept the items only after an uncertain service time leads to fuzzy service rate. Stocking of spare parts, perishable items, style goods and special season items are practical examples of this model. The traditional names for these kinds of problems are Christmas Tree Problems or News Boy Problem. The problem here is that how many Christmas trees or Newspapers can be procured to get maximum profit for a particular season. The major difficultly faced by the decision maker, in this kind of problem is to forecast the demand. As the procurement decision highly depends on the demand behavior of customers. This demand may be for unit items or multiple items. Another difficulty faced by the vendor is that the items become obsolete at the end of the period. Exiting models (most of them) assume the probabilistic demand [17-20] and marked them as stochastic models [8-15]. The nature of the probability distribution is

estimated form historical data. In most the cases historical data is not adequately available to predict the exact distribution. But the demand for the single period seasonal items can be suitably described by linguistic terms such as 'high' or 'low' of are approximately estimated with certain amount by an expert (subjective estimation). The above descriptions lead us to confirm that the demands are fuzzy. Fuzzy sets and system is an ever growing subject matter, where we can find so many solution procedures to solve such a problem. Many research papers have been published in this field of research. The purpose of study is to find the optimal quantity Q^* for a single period inventory system in the service facility with fuzzy demand. The criteria used to find the optimal order quantity. Q^* is minimization of total cost T(Q). The approach we adopted is to find the quantity which has minimum cost is the method of Fuzzy ranking.

2. Model description

- Single period inventory in a service facility system where inventory is maintained to satisfy a customer after a fuzzy service time.
- The demand is fuzzy number.
- The service time is also fuzzy interval (fuzzy service rate).
- The lead time for replenishment constant(rate α) from a remote source.

Consider a single period inventory problem. The demand is assumed to be normal fuzzy number $\tilde{\lambda}$, defined by the membership function

$$\mu_{\tilde{\lambda}}(x) = \begin{cases} L_1(x), & l_1 \le x \le s_1 \\ 1, & s_1 \le x \le t_1 \\ R_1(x), & t_1 \le x \le u_1, \end{cases}$$

Again the service rate is to be a normal fuzzy number $\tilde{\gamma}$, described by a general membership function

$$\mu_{\tilde{\gamma}}(y) = \begin{cases} L_2(y), & l_2 \le y \le s_2 \\ 1, & s_2 \le y \le t_2 \\ R_2(y), & t_2 \le y \le u_2, \end{cases}$$

where $L_1(x)$, $L_2(x)$, $R_1(y)$ and $R_2(y)$ are left and right–shape functions respectively of $\tilde{\lambda}$ and $\tilde{\gamma}$.

The problem is to find the best order quantity in terms of the cost incurred. Suppose a quantity Q is ordered. The total cost $\tilde{T}(Q)$ can be described as

$$\tilde{T}(Q) = cQ + p \max\left(0, \tilde{\lambda} - Q\right) + h \max\left(0, Q - \tilde{\lambda}\right) + w \max\left(0, \tilde{\lambda} - \tilde{\gamma}\right)$$

where c is the unit cost for purchasing each item. The cost p (p>c) is the penalty cost per unit, the case where p<c is nonsensical because it implies that cost of purchasing the item is higher than the penalty for not providing it. Let h be the holding cost per unit remaining at end of period. Let w be the waiting cost for a customer waiting in queue per unit time. Let x be the demand rate and y be the service rate. In a traditional inventory system the inventory depletes at the rate of demand say x, but in service facility system inventory depletes at the rate of service completion say y.

$$T(x \mid Q) = \begin{cases} cQ+h(Q-x) & l \le x \le Q. \\ cQ+(p+w)(x-Q)) & Q \le x \le u. \end{cases}$$

Here, $l = max\ (l_1, l_2)$, $s = \max (s_1, s_2)$, $t = \min (t_1, t_2)$, $u = \min (u_1, u_2)$.

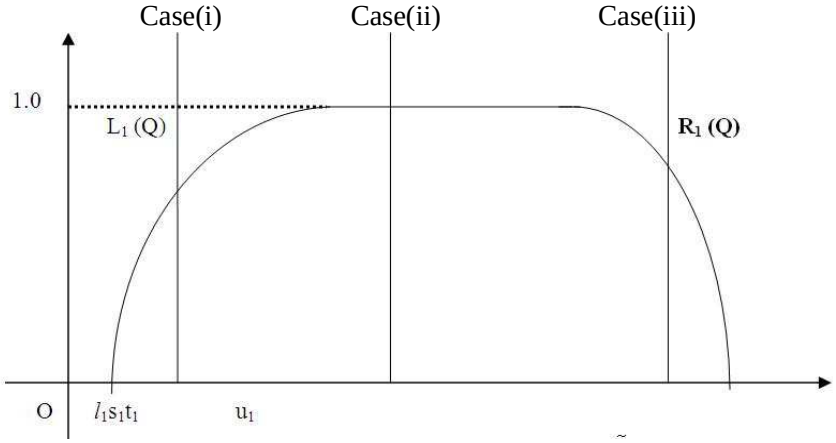

Figure 1: The membership function of $\tilde{\lambda}$

Case (i): $l \le Q \le s$

$$T(\alpha) = \begin{cases} \left[cQ+h\left(Q-L^{-1}(\alpha)\right),\ cQ+p\left(R^{-1}(\alpha)-Q\right)+w(x-Q)\right] & 0 \le \alpha \le L(Q) \\ \left[cQ+\left(p(L^{-1}(\alpha)-Q\right)+w(x-Q),\ cQ+p\left(R^{-1}(\alpha)-Q\right)+w(x-Q)\right] & L(Q) \le \alpha \le 1 \end{cases}$$

Case (ii): $s \le Q \le t$

$$T(\alpha) = \left\{\left[cQ+h\left(Q-L_1^{-1}(\alpha)\right),\ cQ+p\left(R^{-1}(\alpha)-Q\right)+w(x-Q)\right] \quad 0 \le \alpha \le 1\right.$$

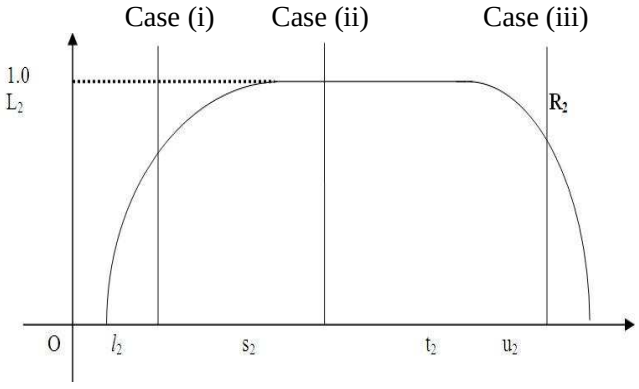

Figure 2: The membership function of $\tilde{\gamma}$

Case (iii): $t \le Q \le u$

$$T(\alpha) = \begin{cases} \left[cQ+h\left(Q-L^{-1}(\alpha)\right),\ cQ+p\left(R^{-1}(\alpha)-Q\right)+w(x-Q) \right] & 0\le\alpha\le R(Q) \\ \left[cQ+h\left((Q-L^{-1}(\alpha)\right),\ cQ+h\left(Q-R^{-1}(\alpha)\right) \right] & R(Q)\le\alpha\le 1 \end{cases}$$

3. Analysis

We considered the total cost $\tilde{T}(Q)$ which is a fuzzy number. The value of this function can be ranked (ordered) in such a way that one can get the optimal value of Q, for minimal $\tilde{T}(Q)$.

Even though lots of ranking methods are available for fuzzy numbers in the literature, we prefer the method due to Yager [21], because of its less dependence on membership functions. The area measure of the membership functions leads to the Yager ranking index which is defined as $I(\tilde{T}) = \frac{1}{2}\left[I_L(\tilde{T}) + I_R(\tilde{T}) \right]$ where $I_L(\tilde{T})$ represent the area bounded by the left – shape function of $\tilde{T}(Q)$, co-ordinate axes and the horizontal line $\mu_{\tilde{T}}$ =1 similar argument holds for right shape function of $\tilde{T}(Q)$ namely $I_R(\tilde{T})$.

Optimization 3.1. In order to get optimal Q^*, we have considered the following three cases:

Case (i): $l \le Q \le s$, here $l = \max(l_1, l_2)$, $s = \max(s_1, s_2)$

$$I(\tilde{T}) = \int_0^{L(Q)} 0.5\left[CQ + h\left(Q - L^{-1}(\alpha)\right) + CQ + p\left(R^{-1}(\alpha) - Q + w\left(x-Q\right)\right)\right] d\alpha +$$

$$\int_{L(Q)}^{1} 0.5\left[CQ + p\left(L^{-1}(\alpha) - Q\right) + w(x-Q) + CQ + p\left(R^{-1}(\alpha) - Q\right) + w\left(x-Q\right)\right] d\alpha$$

$$= CQ + \frac{Q}{2}(h-p)L(Q) - pQ(1-L(Q)) + \frac{R}{2}\int_0^1 R^{-1}(\alpha)d\alpha - \frac{h}{2}\int_0^{L(Q)} L^{-1}(\alpha)d\alpha$$

$$+ \frac{p}{2}\int_0^1 L^{-1}(\alpha)d\alpha + w(x-Q)\left[1 - \frac{L(Q)}{2}\right]$$

The first derivative of $I(\tilde{T})$ we have $\frac{\partial I(\tilde{T})}{\partial Q} = (c-p) + 0.5(p+h)L(Q)$

The second derivate of $I(\tilde{T})$ we have $\frac{\partial^2 I(\tilde{T})}{\partial Q^2} = 0.5(p+h)L'(Q)$

$Q^* = L^{-1}\left(\frac{2(p-c)}{(p+h)}\right)$ *for* $(p-c) \le (c+h)$.

Case (ii): $s \le Q \le t$

$$I(\tilde{T}) = \int_0^1 0.5\left[CQ + h\left(Q - L^{-1}(\alpha)\right) + 0.5\left[CQ + p\left(R^{-1}(\alpha) - Q\right)\right] + w(x-Q)\right] d\alpha$$

$$= c - 0.5(p-h) - \frac{h}{2}\int_0^1 L^{-1}(\alpha)d\alpha + \frac{p}{2}\int_0^1 R^{-1}(\alpha)d\alpha - w\left(\frac{x-Q}{2}\right)$$

$$Q^* \in \begin{cases} \{s\} & (p-c) < (c+h) \\ [s,t] & (p-c) = (c+h) \\ \{t\} & (p-c) > (c+h) \end{cases}$$

Case (iii): $t \le Q \le u$

$$I(\tilde{T}) = \int_0^{R(Q)} 0.5\left[CQ + h\left(Q - L^{-1}(\alpha)\right) + 0.5\left[CQ + p\left(R^{-1}(\alpha) - Q\right)\right] + w(x-Q)\right]d\alpha +$$

$$\int_{R(Q)}^1 \left\{0.5\left[CQ + h\left(Q - L^{-1}(\alpha)\right) + 0.5\left[CQ + p\left(R^{-1}(\alpha) - Q\right)\right] + w(x-Q)\right]\right.$$

$$= CQ + \frac{Q}{2}(h-p)\mathrm{R(Q)} + hQ(1-\mathrm{R(Q)}) - \frac{h}{2}\int_0^1 L^{-1}(\alpha)d\alpha + \frac{p}{2}\int_0^{R(Q)} R^{-1}(\alpha)d\alpha - \frac{h}{2}\int_{R(Q)}^1 R^{-1}(\alpha)d\alpha + \frac{w}{2}(x-\mathrm{Q})\mathrm{R(Q)}$$

The first and second derivatives of $I(\tilde{T})$ we have

$$\frac{\partial I(\tilde{T})}{\partial Q} = (c+h) - 0.5(p+h)R(\mathrm{Q}),\ \frac{\partial^2 I(\tilde{T})}{\partial Q^2} = -0.5(p+h)R'(\mathrm{Q})$$

$$Q^* = R^{-1}\left(\frac{2(c+h)}{(p+h)}\right) for\ (p-c) \ge (c+h)$$

Combining for three cases,

$$Q^* = \begin{cases} L^{-1}\left[\frac{2(p-c)}{(p+h)}\right], & (p-c) \le (c+h) \\ \in [s,t], & (p-c) = (c+h) \\ R^{-1}\left[\frac{2(c+h)}{(p+h)}\right], & (p-c) \ge (\mathrm{c+h}) \end{cases}$$

Discussion 3.2. In this case of stochastic inventory models. The demand is described by a distributions functions F(x) of the random variable x, with density f(x).

In order to minimize the expected total cost the optimal quantity to order is Q^*, given the equation

$$F(Q^*) = \frac{p-c}{p+h},\quad Q^* = F^{-1}\left(\frac{(p-c)}{(p+h)}\right)$$

One of the membership function widely used in applications in the trapezoidal function of the following form

$$\mu_{\tilde{\lambda}}(x) or \mu_{\tilde{\gamma}}(x) = \begin{cases} L(x) = \frac{(x-l)}{(s-l)} & L \le x \le s \\ 1 & s \le x \le t \\ R(x) = \frac{(u-x)}{(u-t)} & t \le x \le u \end{cases}$$

Here , l= max (l_1,l_2), s = max (s_1,s_2),t = min (t_1,t_2),u = min (u_1, u_2)

The α - cut of this fuzzy number is $\lambda(\alpha)\, or\, \gamma(\alpha) = \left[L^{-1}(\alpha), \mathrm{R}^{-1}(\alpha) \right]$

$= \left[l + \alpha(s-l),\ \ u - \alpha(\mathrm{u}-t) \right].$

$$Q^* = \begin{cases} = l + \left(\dfrac{2(p-c)}{(p+h)} \right)(s-l), (\mathrm{p}-\mathrm{c}) \le (\mathrm{c}+\mathrm{h}) \\ \in [s,t], \qquad\qquad (p-c) = (c+h) \\ = u - \left[\dfrac{2(c+h)}{(p+h)} \right](\mathrm{u}-\mathrm{t}), \quad (\mathrm{p}-\mathrm{c}) \ge (\mathrm{c}+\mathrm{h}). \end{cases}$$

4. Conclusion

The single-period inventory system with service facility problem deals with finding the product's order quantity which minimizes the expected cost of seller with random demand. However, in real world, sometimes the probability distribution of the demand for products is difficult to acquire due to lack of information and historical data. This study focuses on possibility situations, where the demand and service rate are described by a membership functions and uncertain service time causes an uncertain total cost function. This paper proposes an analytical method to obtain the exact expected value of total cost function which is composed of inventory holding, inventory shortage and unit production costs for a single-period inventory problem under uncertainty. To determine the optimum order quantity that minimizes the fuzzy total cost function, the expected value of a fuzzy function based on the credibility theory is employed. By this method, closed-form solutions to the optimum order quantities and corresponding total cost values are derived. The advantages of the closed-form solutions obtained are those: they eliminate the need for enumeration over alternative values and give the opportunity to analyze the effects of model parameters on optimum order quantity and optimum cost value. The proposed methodology, used for optimization based on the credibility theory can be applied to the solution of other complex real world problems where this complexity arises from uncertainty in the form of ambiguity. The single-period inventory system with service facility model analyzed in this paper considers only a single type of product. The model can be extended to a multi-product case and the solution procedure can be extended as a further research of this study. Another issue of interest is the examination of the proposed model with imprecise inventory cost coefficients. The analysis of single-period inventory problem with other sources of uncertainty besides imprecise demand is another area of further research.

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