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Total Degree of a Vertex in Union and Join of Some Intuitionistic Fuzzy Graphs

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Abstract. An intuitionistic fuzzy graph can be obtained from two given intuitionistic fuzzy graphs using union and join. In this paper, we discuss the total degree of a vertex in intuitionistic fuzzy graphs formed by these operations in terms of the degree of vertices in the given intuitionistic fuzzy graphs in some particular cases.

Keywords: Total degree of a vertex, union and join

AMS Mathematics Subject Classification (2010): 03E72, 03F55

1. Introduction

Intuitionistic Fuzzy Graph theory was introduced by Atanassov in [1]. In [4], Karunambigai and Parvathi introduced intuitionistic fuzzy graph as a special case of Atanassov's IFG. In [2], NagoorGani and Shajitha Begum introduced degree, order and size in intuitionistic fuzzy graph. In [5] Radha and Vijaya introduced the total degree of a vertex in some fuzzy graphs, in [3] NagoorGani and Rahman introduced the total and middle intuitionistic fuzzy graph.In this paper we discuss about union and join operations of intuitionistic fuzzy graph and some properties of intuitionistic fuzzy graph are introduced.

2. Preliminaries

Definition 2.1. An intuitionistic fuzzy graph (IFG) is of the form $\mathrm{G} = \langle \mathrm{V}, \mathrm{E} \rangle$ where

(i) $V = \{v_{1,}v_2, \dots, v_n\}$such that $\mu_1 : V \rightarrow [0,1]$ and $\nu_1 : V \rightarrow [0,1]$ denotes the degree of membership and non-membership of the element $v_i \in \mathrm{V}$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$.

(ii)$E \subseteq V \times V$where$\mu_2 : V \times V \rightarrow [0,1]$ and $\nu_2 : V \times V \rightarrow [0,1]$ such that

$$\mu_2(v_i, v_j) \leq min \left(\mu_1(v_i), \mu_1(v_j)\right)$$
$$\nu_2(v_i, v_j) \leq max \left(\nu_1(v_i), \nu_1(v_j)\right)$$

and $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, for every$(v_i, v_j) \in E$.

Here the triple $(v_i, \mu_{1i}, \nu_{1i})$ denotes the degree of membership and non-membership of the vertexv_i. The triple $\left(e_{ij}, \mu_{2ij}, \nu_{2ij}\right)$ denotes the degree of membership and non-membership of the edge relation $e_{ij} = \left(v_i, v_j\right)$ on $V \times V$.

Definition 2.2. An IFG $G = \langle V, E\rangle$ is said to be regular IFG, if there is a vertex which is adjacent to vertices with same degrees.

Definition 2.3. Let $G = \langle V, E\rangle$ be an IFG .Then the degree of a vertex v is defined by *d(v)=(*d_μ*(v),* d_ν*(v))* where $d_\mu(v)=\sum_{u\neq v}\mu_2(v,u)$ *and* $d_\nu(v)=\sum_{u\neq v}\nu_2(v,u)$.

Definition 2.4. Let $G = \langle V, E\rangle$ be an IFG. If $(d_\mu(v), d_\nu(v)) = (k_1, k_2)$ for all $v\in V$ that is if each vertex has same membership degree k_1 and same nonmembership degree k_2 then G is said to be a regular intuitionistic fuzzy graph.

Definition 2.5. Let $G = \langle V, E\rangle$ be an IFG. Then the total degree of a vertex $u\in v$ is defined by $td(u) = (td_\mu(u), td_\nu(u))$=$(\sum_{u\neq v}\mu_2\,(u,v) + \mu_1(u), \sum_{u\neq v}\nu_2\,(u,v) + \nu_1(u))$

$$= (\,d_\mu(u) + \mu_1(u), d_\nu(u) + \nu_1(u))$$

If each vertex of G has same membership total degree k_1 and same nonmembership total degree k_2, then said to be a total regular IFG.

3. Total degree of a vertex in union

Definition 3.1. Let $G_1 : (V_1\,, E_1)$ and $G_2 : (V_2\,, E_2)$ be two intuitionistic fuzzy graphs and $G = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$ be the union of $G_1$and G_2. Then the union of IFGs G_1 and G_2 is an IFG defined by

$$(\mu_1 \cup \mu_1{}')(v) = \begin{cases} \mu_1\ (v) & if\ v \in V_1 \\ \mu_1'\ (v) & if\ v \in V_2 \\ \mu_1\,(v)\quad \vee \mu_1{}'(v) & if\ v \in V_1 \cap V_2 \end{cases}$$

$$(\nu_1 \cup \nu_1{}')(v) = \begin{cases} \nu_1\ (v) & if\ v \in V_1 \\ \nu_1'\quad (v) & if\ v \in V_2 \\ \nu_1\,(v) \wedge \quad \nu_1{}'(v) & if\ v \in V_1 \cap V_2 \end{cases}$$

$$(\mu_2 \cup \mu_2{}')(v_i, v_j) = \begin{cases} \mu_{2ij} & if\ e_{ij} \in E_1 \\ \mu_{2ij}' & if\ e_{ij} \in E_2 \\ \mu_{2ij}\quad \vee \quad \mu_{2ij}{}' & if\ e_{ij} \in E_1 \cap E_2 \end{cases}$$

$$(\nu_2 \cup \nu_2{}')(v_i, v_j) = \begin{cases} \nu_{2ij} & if\ e_{ij} \in E_1 \\ \nu_{2ij}' & if\ e_{ij} \in E_2 \\ \nu_{2ij}\quad \wedge \quad \nu_{2ij}{}', & if\ e_{ij} \in E_1 \cap E_2 \end{cases}$$

where (μ_1, ν_1) and $(\mu_1{}', \nu_1{}')$ refer the vertex membership and nonmembership of$G_1$and G_2 respectively; (μ_2, ν_2) and $(\mu_2{}', \nu_2{}')$ refer the edge membership and nonmembership of $G_1$and G_2 respectively.

Theorem 3.1. Let $G_1 : (\mu_1\,, \nu_1)$ $\;and\; G_2 : (\mu_2\,, \nu_2)$ be two intuitionistic fuzzy graphs, then

1. $td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u)$; $td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_2}(u)$ *and* $td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u)$; $td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_2}(u)$, *if* $u \in V_1$ *or* $u \in V_2$ *but not both.*
2. $td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u) + td_{\mu G_2}(u) - \mu_1(u) \wedge \mu_1{}'(u)$ *and* $td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u) + td_{\nu G_2}(u) - \nu_1(u) \vee \nu_1'(u)$, *if* $u \in V_1 \cap V_2$ *but no edge incident at u lies in* $E_1 \cap E_2$.
3. $td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u) + td_{\mu G_2}(u) - \mu_1(u) \wedge \mu_1{}'(u) - \sum_{uv \in E_1 \cap E_2} \mu_2(uv) \wedge \mu_2{}'(uv)$ *and* $td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u) + td_{\nu G_2}(u) - \nu_1(u) \vee \nu_1'(u) - \sum_{uv \in E_1 \cap E_2} \nu_2(uv) \vee \nu_2'(uv)$, *if* $u \in V_1 \cap V_2$ *and some edges incident at u are in* $E_1 \cap E_2$.

Proof: For any $u \in V_1 \cup V_2$, we have three cases to consider.
Case 1: Either $u \in V_1$ or $u \in V_2$ but not both.
Then no edge incident at u lies in $E_1 \cap E_2$
So, $\mu_2 \cup \mu_2{}' = \mu_2(uv) if u \in V_1$ (i.e.) $if uv \in E_1$
$= \mu_2'(uv) if u \in V_2$ (i.e.) $if uv \in E_2$
Hence if $u \in V_1$, then the total membership degree of u is

$$td_{\mu(G_1 \cup G_2)}(u) = d_{\mu(G_1 \cup G_2)}(u) + \mu_1(u)$$
$$= \sum_{uv \in E_1} \mu_2(uv) + \mu_1(u)$$
$$= d_{\mu G_1}(u) + \mu_1(u) = td_{\mu G_1}(u)$$

Similarly if $u \in V_2$, then the total membership degree of u is $td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_2}(u)$
$\nu_2 \cup \nu_2' = \nu_2(uv) if\ u \in V_1$ (i.e.) $if\ uv \in E_1$
$= \nu_2{}'(uv)\ if\ u \in V_2$ (i.e.) $if\ uv \in E_2$
Hence, if $u \in V_1$, then the total nonmembership degree of u is

$$td_{\nu(G_1 \cup G_2)}(u) = d_{\nu(G_1 \cup G_2)}(u) + \nu_1(u) = \sum_{uv \in E_1} \nu_2(uv) + \nu_1(u)$$
$$= d_{\nu G_1}(u) + \nu_1(u) = td_{\nu G_1}(u)$$

Similarly if $u \in V_2$, then the total nonmembership degree of u is $td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_2}(u)$
Case 2: $u \in V_1 \cap V_2$ but no edge incident at u lies in $E_1 \cap E_2$.Then any edge incident at u is either in E_1 or in E_2but not both. Also all these edges will be included in $G_1 \cup G_2$
Hence the total membership degree of u is

$$td_{\mu(G_1 \cup G_2)}(u) = d_{\mu(G_1 \cup G_2)}(u) + (\mu_1 \cup \mu_1{}')(u)$$
$$= \sum_{uv \in E_1} \mu_2(uv) + \sum_{uv \in E_2} \mu_2{}'(uv) + \mu_1(u) \vee \mu_1{}'(u)$$
$$= \sum_{uv \in E_1} \mu_2(uv) + \sum_{uv \in E_2} \mu_2{}'(uv) + \mu_1(u) + \mu_1{}'(u) - \mu_1(u) \wedge \mu_1{}'(u)$$

[Since $\mu_1(u) \vee \mu_1{}'(u) + \mu_1(u) \wedge \mu_1{}'(u) = \mu_1(u) + \mu_1{}'(u)$ *]*

$$= \sum_{uv \in E_1} \mu_2(uv) + \mu_1(u) + \sum_{uv \in E_2} \mu_2{}'(uv) + \mu_1{}'(u) - \mu_1(u) \wedge \mu_1{}'(u)$$
$$= d_{\mu G_1}(u) + \mu_1(u) + d_{\mu G_2}(u) + \mu_1{}'(u) - \mu_1(u) \wedge \mu_1{}'(u)$$
$$\therefore td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u) + td_{\mu G_2}(u) - \mu_1(u) \wedge \mu_1{}'(u)$$

Hence, the total nonmembership degree of u is

$$td_{\nu(G_1 \cup G_2)}(u) = d_{\nu(G_1 \cup G_2)}(u) + (\nu_1 \cup \nu_1{}')(u)$$
$$= \sum_{uv \in E_1} \nu_2(uv) + \sum_{uv \in E_2} \nu_2{}'(uv) + \nu_1(u) \wedge \nu_1{}'(u)$$

$$= \sum_{uv\in E_1} \nu_2(uv) + \sum_{uv\in E_2} {\nu_2}'(uv) + \nu_1(u) + {\nu_1}'(u) - \nu_1(u) \vee {\nu_1}'(u)$$

[Since $\nu_1(u) \wedge {\nu_1}'(u) + \nu_1(u) \vee {\nu_1}'(u) = \nu_1(u) + {\nu_1}'(u)$ *]*

$$= \sum_{uv\in E_1} \nu_2(uv) + \nu_1(u) + \sum_{uv\in E_2} {\nu_2}'(uv) + {\nu_1}'(u) - \nu_1(u) \vee {\nu_1}'(u)$$

$$= d_{\nu G_1}(u) + \nu_1(u) + d_{\nu G_2}(u) + {\nu_1}'(u) - \nu_1(u) \vee {\nu_1}'(u)$$

$$\therefore td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u) + td_{\nu G_2}(u) - \nu_1(u) \vee {\nu_1}'(u)$$

Case 3: $u \in V_1 \cap V_2$ and some edges incident at u are in $E_1 \cap E_2$.

Any edge uv which is in $E_1 \cap E_2$ appear only once in $G_1 \cup G_2$ and for this uv,

$(\mu_2 \cup {\mu_2}')(uv) = \mu_2(uv) \vee {\mu_2}'(uv)$

By definition, the total membership degree of u is

$td_{\mu(G_1 \cup G_2)}(u) = d_{\mu(G_1 \cup G_2)}(u) + (\mu_1 \cup {\mu_1}')(u)$

$$= \sum_{uv\in E} (\mu_2 \cup {\mu_2}')(uv) + \mu_1(u) \vee {\mu_1}'(u)$$

$$= \sum_{uv\in E_1 - E_2} \mu_2(uv) + \sum_{uv\in E_2 - E_1} {\mu_2}'(uv) + \sum_{uv\in E_1 \cap E_2} \mu_2(uv) \vee {\mu_2}'(uv) + \mu_1(u) \vee {\mu_1}'(u)$$

$$= \sum_{uv\in E_1 - E_2} \mu_2(uv) + \sum_{uv\in E_2 - E_1} {\mu_2}'(uv) + \Big[\sum_{uv\in E_1 \cap E_2} \mu_2(uv) \vee {\mu_2}'(uv)$$

$$+ \sum_{uv\in E_1 \cap E_2} \mu_2(uv) \wedge {\mu_2}'(uv) \Big] - \sum_{uv\in E_1 \cap E_2} \mu_2(uv) \wedge {\mu_2}'(uv) + \mu_1(u) \vee {\mu_1}'(u)$$

$$= \Bigg[\sum_{uv\in E_1 - E_2} \mu_2(uv) + \sum_{uv\in E_2 - E_1} {\mu_2}'(uv) + \sum_{uv\in E_1 \cap E_2} \mu_2(uv) + \sum_{uv\in E_1 \cap E_2} {\mu_2}'(uv)$$

$$- \sum_{uv\in E_1 \cap E_2} \mu_2(uv) \wedge \mu_2'(uv) \Bigg] + \mu_1(u) + \mu_1'(u) - \mu_1(u) \wedge \mu_1'(u)$$

$$= \sum_{uv\in E_1} \mu_2(uv) + \mu_1(u) + \sum_{uv\in E_2} {\mu_2}'(uv) + \mu_1'(u) - \mu_1(u) \wedge \mu_1'(u)$$

$$- \sum_{uv\in E_1 \cap E_2} \mu_2(uv) \wedge {\mu_2}'(uv)$$

$$td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u) + td_{\mu G_2}(u) - \mu_1(u) \wedge {\mu_1}'(u) - \sum_{uv\in E_1 \cap E_2} \mu_2(uv) \wedge {\mu_2}'(uv)$$

$(\nu_2 \cup {\nu_2}')(uv) = \nu_2(uv) \wedge {\nu_2}'(uv)$

The total nonmembership degree of u is

$td_{\nu(G_1 \cup G_2)}(u) = d_{\nu G_1 \cup G_2}(u) + (\nu_1 \cup {\nu_1}')(u)$

$$= \sum_{uv\in E} (\nu_2 \cup {\nu_2}')(uv) + \nu_1(u) \wedge {\nu_1}'(u)$$

$$= \sum_{uv\in E_1 - E_2} \nu_2(uv) + \sum_{uv\in E_2 - E_1} {\nu_2}'(uv) + \sum_{uv\in E_1 \cap E_2} \nu_2(uv) \wedge {\nu_2}'(uv) + \nu_1(u) \wedge {\nu_1}'(u)$$

$$= \sum_{uv \in E_1 - E_2} \nu_2(uv) + \sum_{uv \in E_2 - E_1} \nu_2{}'(uv) + \Big[\sum_{uv \in E_1 \cap E_2} \nu_2(uv) \wedge \nu_2{}'(uv)$$
$$+ \sum_{uv \in E_1 \cap E_2} \nu_2(uv) \vee \nu_2{}'(uv) \Big] - \sum_{uv \in E_1 \cap E_2} \nu_2(uv) \vee \nu_2{}'(uv)$$
$$+ \nu_1(u) \wedge \nu_1{}'(u)$$
$$= \Big[\sum_{uv \in E_1 - E_2} \nu_2(uv) + \sum_{uv \in E_2 - E_1} \nu_2{}'(uv) + \sum_{uv \in E_1 \cap E_2} \nu_2(uv) + \sum_{uv \in E_1 \cap E_2} \nu_2{}'(uv)$$
$$- \sum_{uv \in E_1 \cap E_2} \nu_2(uv) \vee \nu_2{}'(uv) \Big] + \nu_1(u) + \nu_1{}'(u) - \nu_1(u) \vee \nu_1{}'(u)$$
$$= \sum_{uv \in E_1} \nu_2(uv) + \nu_1(u) + \sum_{uv \in E_2} \nu_2{}'(uv) + \nu_1'(u) - \nu_1(u) \vee \nu_1'(u)$$
$$- \sum_{uv \in E_1 \cap E_2} \nu_2(uv) \vee \nu_2{}'(uv)$$
$$td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u) + td_{\nu G_2}(u) - \nu_1(u) \vee \nu_1{}'(u) - \sum_{uv \in E_1 \cap E_2} \nu_2(uv) \vee \nu_2{}'(uv)$$

Example 3.1. Consider $G_1 : (\mu_1, \nu_1)$ *and* $G_2 : (\mu_2, \nu_2)$ be two intuitionistic fuzzy graphs

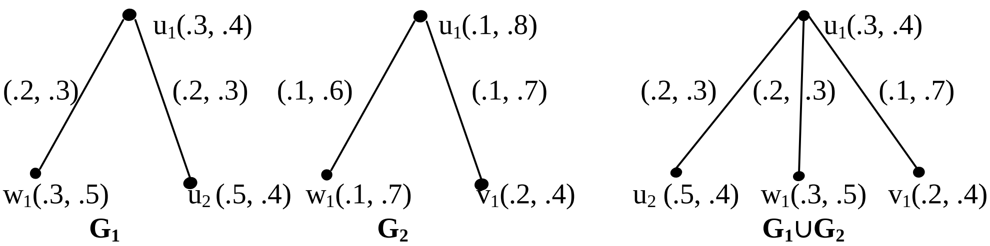

Figure 1:

In G_1: $td_{G_1}(u_1) = (.7,1); td_{G_1}(u_2) = (.7,.7); td_{G_1}(w_1) = (.5,.8)$.

G_2: $td_{G_2}(u_1) = (.3,2.1); td_{G_2}(v_1) = (.3,1.1); td_{G_2}(w_1) = (.2,1.3)$.

$G_1 \cup G_2$: $td_{G_1 \cup G_2}(u_1) = (.8,1.7); td_{G_1 \cup G_2}(u_2) = (.7,.7); td_{G_1 \cup G_2}(v_1) = (.3,1.1)$; $td_{G_1 \cup G_2}(w_1) = (.5,.8)$.

3.1. Total degree of a vertex in join

Definition 3.1.1. The join of two IFGs G_1 and G_2 is an IFG $G = G_1 + G_2 = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$ *defined by*

$(\mu_1 + \mu_1{}')(v) = (\mu_1 \cup \mu_1{}')(v) \quad if\ v \in V_1 \cup V_2$

$(\nu_1 + \nu_1{}')(v) = (\nu_1 \cup \nu_1{}')(v) \quad if\ v \in V_1 \cup V_2$

$(\mu_2 + \mu_2{}')(v_i, v_j) = (\mu_2 \cup \mu_2{}')(v_i, v_j) \quad if\ v_i, v_j \in E_1 \cup E_2$

$\qquad = \mu_1(v_i) \wedge \mu_1{}'(v_j) if\ v_i, v_j \in E'$

$(\nu_2 + \nu_2{}')(v_i, v_j) = (\nu_2 \cup \nu_2{}')(v_i, v_j) \quad if\ v_i, v_j \in E_1 \cup E_2$

$\qquad = \nu_1(v_i) \vee \nu_1{}'(v_j) if\ v_i, v_j \in E'$

Theorem 3.1.1. Let $G_1 : (\mu_1, \nu_1)$ *and* $G_2 : (\mu_2, \nu_2)$ be two intuitionistic fuzzy graphs with $V_1 \cap V_2 = \phi$. Then

$$1.\ td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v)\ , for\ any\ u \in V_1\ and$$

$$td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_2}(u) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v)\ , for\ any\ u \in V_2$$

$$2.\ td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v), for\ any\ u \in V_1\ and$$

$$td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_2}(u) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v)\ , for\ any\ u \in V_2$$

Proof: Here $V_1 \cap V_2 = \phi$. Hence $E_1 \cap E_2 = \phi$.

So $\mu_1 \cup \mu_2'(uv) = \mu_2(uv)\ if\ u \in V_1$ (i.e.) $if\ uv \in E_1$

$= \mu_2'(uv)\ if\ u \in V_2$ (i.e.)$if\ uv \in E_2$

By definition,

$$td_{\mu(G_1 \cup G_2)}(u) = \sum_{uv \in E_1 \cup E_2} (\mu_2 \cup \mu_2')(uv) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v) + (\mu_1 \wedge \mu_1')(u)$$

For any $u \in V_1$, $td_{\mu(G_1 \cup G_2)}(u) = \sum_{uv \in E_1} \mu_2(uv) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v) + \mu_1(u)$

$$= d_{\mu G_1}(u) + \mu_1(u) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v)$$

$td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v)$ *(3.1.1)*

Similarly, For any $u \in V_2$,

$td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_2}(u) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v)$ *(3.1.2)*

$\nu_1 \cup \nu_2'(uv) = \nu_2(uv)\ if\ u \in V_1$ *(i.e.)* $if\ uv \in E_1$

$= \nu_2'(uv)\ if\ u \in V_2$ *(i.e.)* $if\ uv \in E_2$

By definition,

$$td_{\nu(G_1 \cup G_2)}(u) = \sum_{uv \in E_1 \cup E_2} (\nu_2 \cup \nu_2')(uv) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v) + (\nu_1 \vee \nu_1')(u)$$

For any $u \in V_1$, $td_{\nu(G_1 \cup G_2)}(u) = \sum_{uv \in E_1} \nu_2(uv) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v) + \nu_1(u)$

$$= d_{\nu G_1}(u) + \nu_1(u) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v)$$

$td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v)$ *(3.1.3)*

Similarly, For any $u \in V_2$,

$td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_2}(u) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v)$ *(3.1.4)*

Theorem 3.1.2. Let $G_1 : (\mu_1, \nu_1)\ and\ G_2 : (\mu_2, \nu_2)$ be two intuitionistic fuzzy graphs with $V_1 \cap V_2 = \phi$. Then

1. $\mu_1 \geq \mu_1'\ then\ td_{\mu(G_1 + G_2)}(u) = td_{\mu G_1}(u) + O_\mu(G_2), if u \in V_1$

$= td_{\mu G_2}(u) + p_1\, \mu_1'(u)\ , if u \in V_2$

2. $\mu_1' \geq \mu_1\ \ then\ td_{\mu(G_1 + G_2)}(u) = td_{\mu G_1}(u) + p_2\, \mu_1(u)\ , if u \in V_1$

$= td_{\mu G_2}(u) + O_\mu(G_1), if u \in V_2$

3. $\nu_1 \geq \nu_1'\ then\ td_{\nu(G_1 + G_2)}(u) = td_{\nu G_1}(u) + O_\nu(G_2), if u \in V_1$

$$= td_{\nu G_2}(u) + p_1 \nu_1'(u), if u \in V_2$$

$$4.\nu_1' \geq \nu_1 \;\; then\; td_{\nu(G_1 + G_2)}(u) = td_{\nu G_1}(u) + p_2 \nu_1(u), if u \in V_1$$

$$= td_{\nu G_2}(u) + O_\nu(G_1), if u \in V_2$$

Proof: 1. If $\mu_1 \geq \mu_1'$, From (3.1.1), for any $u \in V_1$,

$$td_{\mu(G_1 + G_2)}(u) = td_{\mu G_2}(u) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v)$$

$$td_{\mu(G_1 + G_2)}(u) = td_{\mu G_1}(u) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v)$$

$$= td_{\mu G_1}(u) + \sum_{v \in V_2} \mu_1'(v)$$

$$= td_{\mu G_1}(u) + O_\mu(G_2)$$

From (3.1.2), for any $u \in V_2$, $td_{\mu(G_1 + G_2)}(u) = td_{\mu G_2}(u) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1'(v)$

$$= td_{\mu G_2}(u) + \sum_{v \in V_1} \mu_1'(u)$$

$$= td_{\mu G_2}(u) + p_1 \mu_1'(u)$$

Similarly,we get

2. For any $u \in V_1$, $\mu_1' \geq \mu_1$ $then\; td_{\mu(G_1 + G_2)}(u) = td_{\mu G_1}(u) + p_2 \mu_1(u)$

For any $u \in V_2$, $\mu_1' \geq \mu_1$ $then\; td_{\mu(G_1 + G_2)}(u) = td_{\mu G_2}(u) + O_\mu(G_1)$

3. If $\nu_1 \geq \nu_1'$,

From (3.1.3), for any $u \in V_1$, $td_{\nu(G_1 + G_2)}(u) = td_{\nu G_2}(u) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v)$

$$td_{\nu(G_1 + G_2)}(u) = td_{\nu G_1}(u) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v)$$

$$= td_{\nu G_1}(u) + \sum_{v \in V_2} \nu_1'(v)$$

$$= td_{\nu G_1}(u) + O_\nu(G_2)$$

From (3.1.4), for any $u \in V_2$, $td_{\nu(G_1 + G_2)}(u) = td_{\nu G_2}(u) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1'(v)$

$$= td_{\nu G_2}(u) + \sum_{v \in V_1} \nu_1'(u)$$

$$= td_{\nu G_2}(u) + p_1 \nu_1'(u)$$

Similarly,we get

4. For any $u \in V_1$, $\nu_1' \geq \nu_1$ $then\; td_{\nu(G_1 + G_2)}(u) = td_{\nu G_1}(u) + p_2 \nu_1(u)$

For any $u \in V_2$, $\nu_1' \geq \nu_1$ $then\; td_{\nu(G_1 + G_2)}(u) = td_{\nu G_2}(u) + O_\nu(G_1)$.

Example 3.1.1. Consider $G_1 : (\mu_1, \nu_1)$ *and* $G_2 : (\mu_2, \nu_2)$ be two intuitionistic fuzzy graphs with $V_1 \cap V_2 = \phi$.

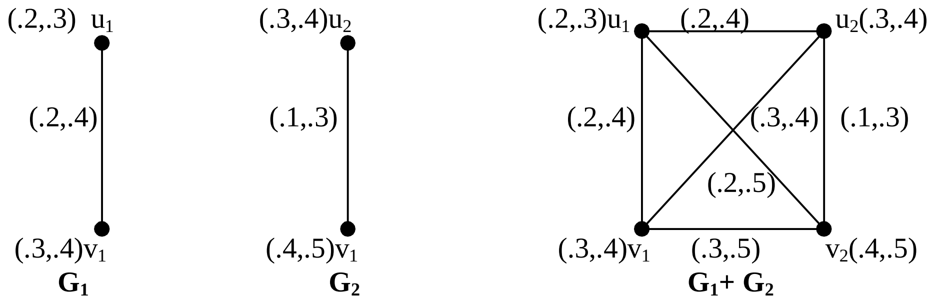

Figure 2:

In G_1: $td_{G_1}(u_1) = (.4,.7); td_{G_1}(v_1) = (.5,.8)$.

G_2: $td_{G_2}(u_2) = (.4,.7); td_{G_2}(v_2) = (.5,.8)$.

$G_1 U\, G_2$: $td_{G_1+G_2}(u_1) = (.8,1.6); td_{G_1+G_2}(u_2) = (.9,1.5); td_{G_1+G_2}(v_1) = (1.1,1.7)$; $td_{G_1+G_2}(v_2) = (1,1.8)$.

Theorem 3.1.3. Let $G_1 : (\mu_1, \nu_1)$ *and* $G_2 : (\mu_2, \nu_2)$ be two intuitionistic fuzzy graphs with $V_1 \cap V_2 = \phi$ such that $\mu_1 \wedge \mu_1'$ and $\nu_1 \vee \nu_1'$ are constant functions.

1. $td_{\mu(G_1+G_2)}(u) = td_{\mu G_1}(u) + c\, p_2$, if $u \in V_1$ and $td_{\mu(G_1+G_2)}(u) = td_{\mu G_2}(u) + c\, p_1$, if $u \in V_2$ where c is a constant value of $\mu_1 \wedge \mu_1'$.

2. $td_{\nu(G_1+G_2)}(u) = td_{\nu G_1}(u) + c\, p_2$, if $u \in V_1$ and $td_{\nu(G_1+G_2)}(u) = td_{\nu G_2}(u) + c\, p_1$, if $u \in V_2$ where c is a constant value of $\nu_1 \vee \nu_1'$.

Proof: 1. Let $\mu_1(u) \wedge \mu_1'(v) = c$ be a constant, for all $u \in V_1$ and $v \in V_2$.

From (3.1.1), for any $u \in V_1$, $td_{\mu(G_1+G_2)}(u) = td_{\mu G_1}(u) + \sum_{v \in V_2} \mu_1(u) \wedge \mu_1'(v)$

$$= td_{\mu G_1}(u) + \sum_{v \in V_2} c$$
$$= td_{\mu G_1}(u) + c\, p_2$$

From (3.1.2), for any $u \in V_2$, $td_{\mu(G_1+G_2)}(u) = td_{\mu G_2}(u) + \sum_{v \in V_1} \mu_1(u) \wedge \mu_1'(v)$

$$= td_{\mu G_2}(u) + \sum_{v \in V_1} c$$
$$= td_{\mu G_2}(u) + c\, p_1$$

2. Let $\nu_1(u) \vee \nu_1'(v) = c$ be a constant, for all $u \in V_1$ and $v \in V_2$.

From (3.1.3), for any $u \in V_1$, $td_{\nu(G_1+G_2)}(u) = td_{\nu G_1}(u) + \sum_{v \in V_2} \nu_1(u) \vee \nu_1'(v)$

$$= td_{\nu G_1}(u) + \sum_{v \in V_2} c$$
$$= td_{\nu G_1}(u) + cp_2$$

From (3.1.4), for any $u \in V_2, td_{\nu(G_1 + G_2)}(u) = td_{\nu G_2}(u) + \sum_{v \in V_1} \nu_1(u) \vee \nu_1{}'(v)$

$$= td_{\nu G_2}(u) + \sum_{v \in V_1} c$$

$$= td_{\nu G_2}(u) + cp_1$$

5. Conclusion

In this paper, we have found the total degree of vertices in $G_1 \cup G_2$ and $G_1 + G_2$ in terms of the total degree of vertices in G_1 and G_2 under some conditions and illustrated them through examples. They will be useful in studying various properties of Union and Join of two intuitionistic fuzzy graphs.

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