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Edge Regular Property of Tensor Product and Normal Product of Two Fuzzy Graphs

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Abstract. A fuzzy graph can be obtained from two given fuzzy graphs using tensor product and normal product. In this paper, we determined that the tensor product and normal product of two edge regular fuzzy graphs need not be edge regular and that if these operations of two fuzzy graphs are edge regular, then G_1 (or) G_2 need not be edge regular. A necessary and sufficient condition for tensor product and normal product of two fuzzy graphs to be edge regular fuzzy graph is determined.

Keywords: Tensor product, normal product, degree of a vertex, degree of an edge

AMS Mathematics Subject Classification (2010): 03E72, 05C72, 05C76

1. Introduction

Fuzzy graph theory was introduced by Rosenfeld in 1975 [10]. Mordeson and Peng introduced the concept of operations on fuzzy graphs [2]. Conjunction (tensor product) of two fuzzy graphs was defined by Nagoorgani and Radha [3]. The degree of a vertex in fuzzy graphs which are obtained from two given fuzzy graphs using the operation of normal product was discussed by Nirmala and Vijaya [9]. Radha and Kumaravel introduced the concept of degree of an edge and total degree of an edge in fuzzy graphs [6]. We study about the degree of an edge in fuzzy graphs which are obtained from two given fuzzy graphs using the operations of tensor product and normal product. A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, (i.e.) $\mu(xy)\leq\sigma(x)\wedge\sigma(y)$ for all $x,y\in V$. The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*:(V,E)$ where $E\subseteq V\times V$. Throughout this paper, $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ denote two fuzzy graphs with underlying crisp graphs $G_1^*:(V_1,E_1)$ and $G_2^*:(V_2,E_2)$ with $|V_i|=p_i, i=1,2$. Also $d_{G_i^*}(u_i)$ denotes the degree of u_i in G_i^*. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The degree of a vertex u is $d_G(u)=\sum_{u\neq v}\mu(u,v)$. The minimum degree of G is $\delta(G)=\wedge\{d_G(v),\forall\ v\in V\}$ and the maximum degree of G is $\Delta(G)=\vee\{d_G(v),\forall\ v\in V\}$. The total degree of a vertex $u\in V$ is defined by $td_G(u)=\sum_{u\neq v}\mu(u,v)+\sigma(u)$. The order and size of a fuzzy graph G are defined by $O(G)=\sum_{u\in V}\sigma(u)$ and $S(G)=\sum_{uv\in E}\mu(uv)$. Let $G^*:(V,E)$ be a graph and let $e=uv$ be an edge in G^*. Then the degree of an edge $e=uv\in E$ is defined by $d_{G^*}(uv)=d_{G^*}(u)+d_{G^*}(v)-2$. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The degree of an edge uv is $d_G(uv)=d_G(u)+d_G(v)-2\mu(uv)=\sum_{\substack{uw\in E\\ w\neq v}}\mu(uw)+\sum_{\substack{wv\in E\\ w\neq u}}\mu(wv)$. The total degree of an edge $uv\in E$ is defined by $td_G(uv)=d_G(u)+d_G(v)-\mu(uv)$. This is equivalent to $td_G(uv)=\sum_{\substack{uw\in E\\ w\neq v}}\mu(uw)+\sum_{\substack{wv\in E\\ w\neq u}}\mu(wv)+\mu(uv)=d_G(uv)+\mu(uv)$. The minimum edge degree and maximum edge degree of G are $\delta_E(G)=\wedge\{d_G(uv),\forall\ uv\in E\}$ and $\Delta_E(G)=\vee\{d_G(uv),\forall\ uv\in E\}$. A fuzzy graph $G:(\sigma,\mu)$ is strong, if $\mu(xy)=\sigma(x)\wedge\sigma(y)$ for all $xy\in E$.

2. Preliminaries

Definition 2.1. [3] Let $G^*=G_1^*\wedge G_2^*=(V,E)$ be the tensor product (conjunction) of two graphs G_1^* and G_2^*, where $V=V_1\times V_2$ and $E=\{(u_1,u_2)(v_1,v_2):u_1v_1\in E_1,u_2v_2\in E_2\}$.
Then the tensor product of two fuzzy graphs G_1 and G_2 is a fuzzy graph
$G=G_1\wedge G_2=G_1\wedge G_2:(\sigma_1\wedge\sigma_2,\mu_1\wedge\mu_2)$ defined by
$(\sigma_1\wedge\sigma_2)(u_1,u_2)=\sigma_1(u_1)\wedge\sigma_2(u_2),\forall(u_1,u_2)\in V$ and
$(\mu_1\wedge\mu_2)((u_1,u_2)(v_1,v_2))=\mu_1(u_1v_1)\wedge\mu_2(u_2v_2),\ \forall u_1v_1\in E_1,\forall u_2v_2\in E_2$.

Definition 2.2. [9] Let $G^*=G_1^*\bullet G_2^*=(V,E)$ be the normal product of two graphs G_1^* and G_2^*, where $V=V_1\times V_2$ and $E=\{(u,u_2)(u,v_2):u\in V_1,u_2v_2\in E_2\}\cup\{(u_1,w)(v_1,w):u_1v_1\in E_1,w\in V_2\}\cup\{(u_1,u_2)(v_1,v_2):u_1v_1\in E_1,u_2v_2\in E_2\}$. Then the normal product of two fuzzy graphs G_1 and G_2 is a fuzzy graph
$G=G_1\bullet G_2:(\sigma_1\bullet\sigma_2,\mu_1\bullet\mu_2)$ defined by $(\sigma_1\bullet\sigma_2)(u_1,u_2)=\sigma_1(u_1)\wedge\sigma_2(u_2),\forall(u_1,u_2)\in V$ and
$(\mu_1\bullet\mu_2)((u,u_2)(u,v_2))=\sigma_1(u)\wedge\mu_2(u_2v_2),\forall u\in V_1,\forall u_2v_2\in E_2$,
$(\mu_1\bullet\mu_2)((u_1,w)(v_1,w))=\sigma_2(w)\wedge\mu_1(u_1v_1),\forall w\in V_2,\forall u_1v_1\in E_1$, and
$(\mu_1\bullet\mu_2)((u_1,u_2)(v_1,v_2))=\mu_1(u_1v_1)\wedge\mu_2(u_2v_2),\forall u_1v_1\in E_1,\forall u_2v_2\in E_2$.

Notation 2.1. The relation $\sigma_1\leq\mu_2$ means that $\sigma_1(u)\leq\mu_2(e)$ for every $u\in V_1$ and $e\in E_2$, where σ_1 is a fuzzy subset of V_1 and μ_2 is a fuzzy subset of E_2.

Theorem 2.1. [4] If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$, then $\sigma_2 \ge \mu_1$ and vice versa.

Theorem 2.2. [7] If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$, then $\mu_1 \le \mu_2$.

3. Edge regular properties of tensor product of two fuzzy graphs

3.1. The degree of an edge in tensor product of two fuzzy graphs

By definition, for any $((u_1,u_2)(v_1,v_2)) \in E$ with $u_1v_1 \in E_1$ & $u_2v_2 \in E_2$.

$$d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1\wedge G_2}(u_1,u_2) + d_{G_1\wedge G_2}(v_1,v_2) - 2\mu((u_1,u_2)(v_1,v_2))$$

$$= \sum_{(u_1,u_2)(w_1,w_2)\in E} \mu((u_1,u_2)(w_1,w_2)) + \sum_{(w_1,w_2)(v_1,v_2)\in E} \mu((w_1,w_2)(v_1,v_2)) - 2(\mu_1(u_1v_1)\wedge\mu_2(u_2v_2))$$

$$= \sum_{u_1w_1\in E_1,u_2w_2\in E_2} \mu_1(u_1w_1)\wedge\mu_2(u_2w_2) + \sum_{w_1v_1\in E_1,w_2v_2\in E_2} \mu_1(w_1v_1)\wedge\mu_2(w_2v_2) - 2(\mu_1(u_1v_1)\wedge\mu_2(u_2v_2)) \quad (3.1.1)$$

In the following theorems, we find the degree of $((u_1,u_2)(v_1,v_2))$ in $G_1 \wedge G_2$ in terms of those in G_1 and G_2 in some particular cases.

Theorem 3.1.1. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\mu_1 \ge \mu_2$. Then for any $(u_1,u_2)(v_1,v_2) \in E$,

$$d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1^*}(u_1)d_{G_2}(u_2) + d_{G_1^*}(v_1)d_{G_2}(v_2) - 2\mu_2(u_2v_2).$$

Proof: We have, $\mu_1 \ge \mu_2$. From (3.1.1), for any $(u_1,u_2)(v_1,v_2) \in E$,

$$d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2))$$

$$= \sum_{u_1w_1\in E_1,u_2w_2\in E_2} \mu_1(u_1w_1)\wedge\mu_2(u_2w_2) + \sum_{w_1v_1\in E_1,w_2v_2\in E_2} \mu_1(w_1v_1)\wedge\mu_2(w_2v_2) - 2(\mu_1(u_1v_1)\wedge\mu_2(u_2v_2))$$

$$= \sum_{u_1w_1\in E_1,u_2w_2\in E_2} \mu_2(u_2w_2) + \sum_{w_1v_1\in E_1,w_2v_2\in E_2} \mu_2(w_2v_2) - 2\mu_2(u_2v_2)$$

$$= d_{G_1^*}(u_1)\sum_{u_2w_2\in E_2} \mu_2(u_2w_2) + d_{G_1^*}(v_1)\sum_{w_2v_2\in E_2} \mu_2(w_2v_2) - 2\mu_2(u_2v_2)$$

$$\therefore d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1^*}(u_1)d_{G_2}(u_2) + d_{G_1^*}(v_1)d_{G_2}(v_2) - 2\mu_2(u_2v_2).$$

Theorem 3.1.2. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs.
If $G_1:(\sigma_1,\mu_1)$ is a strong fuzzy graph with $\sigma_1 \le \mu_2$ and σ_1 is a constant function with $\sigma_1(u) = c_1$ for all $u \in V_1$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

$$d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2)) = c_1(d_{G_2^*}(u_2)d_{G_1^*}(u_1) + d_{G_2^*}(v_2)d_{G_1^*}(v_1) - 2).$$

Proof: We have $\sigma_1 \le \mu_2$ and $\sigma_1(u) = c_1$. Since $G_1:(\sigma_1,\mu_1)$ is a strong fuzzy graph.
Then $\mu_1 = c_1$ & $\mu_1 \le \mu_2$, $\forall uv \in E_1$.
From (3.1.1), for any $(u_1,u_2)(v_1,v_2) \in E$,

$$d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2)) = \sum_{\substack{u_1w_1\in E_1,u_2w_2\in E_2,\\ w_1\ne v_1 (or) w_2\ne v_2}} \mu_1(u_1w_1)\wedge\mu_2(u_2w_2) + \sum_{\substack{w_1v_1\in E_1,w_2v_2\in E_2,\\ w_1\ne u_1 (or) w_2\ne u_2}} \mu_1(w_1v_1)\wedge\mu_2(w_2v_2)$$

$$= \sum_{u_1w_1\in E_1, u_2w_2\in E_2} \mu_1(u_1w_1) - \mu_1(u_1v_1) + \sum_{w_1v_1\in E_1, w_2v_2\in E_2} \mu_1(w_1v_1) - \mu_1(u_1v_1)$$

$$= d_{G_2^*}(u_2) \sum_{u_1w_1\in E_1} c_1 + d_{G_2^*}(v_2) \sum_{w_1v_1\in E_1} c_1 - 2c_1$$

$$\therefore d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2)) = c_1(d_{G_2^*}(u_2)d_{G_1^*}(u_1) + d_{G_2^*}(v_2)d_{G_1^*}(v_1) - 2).$$

3.2. Edge regular property of tensor product of two fuzzy graphs

Remark 3.2.1. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two edge regular fuzzy graphs, then $G_1\wedge G_2$ need not be edge regular fuzzy graph. For example, in figure 3.2.1 G_1 and G_2 are edge regular fuzzy graphs, but $G_1\wedge G_2$ is not an edge regular fuzzy graph.

Remark 3.2.2. If $G_1\wedge G_2$ is an edge regular fuzzy graph, then $G_1:(\sigma_1,\mu_1)$ or $G_2:(\sigma_2,\mu_2)$ need not be edge regular fuzzy graph.For example, in figure 3.2.2 $G_1\wedge G_2$ is 3 – edge regular fuzzy graph. But G_2 is not an edge regular fuzzy graph.

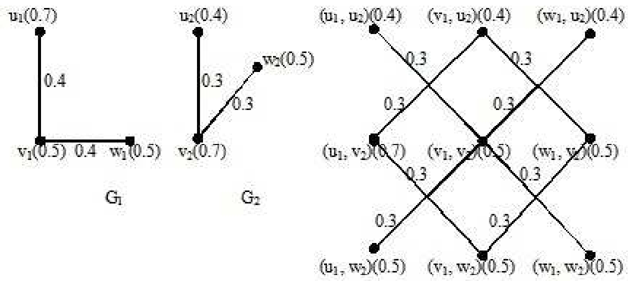

Figure 3.2.1: $G_1\wedge G_2$

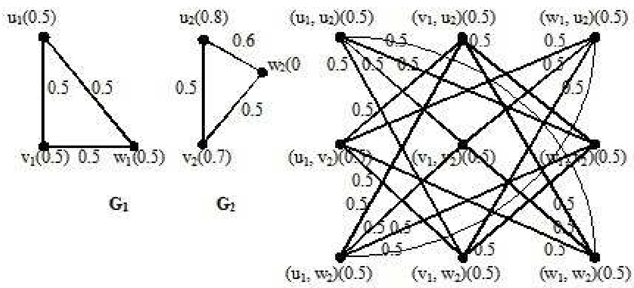

Figure 3.2.2: $G_1\wedge G_2$

Theorem 3.2.1. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs with $\mu_1 \ge \mu_2$ and μ_2 is a constant function. Let $G_1^*:(V_1,E_1)$ be regular graph. Then $G_1\wedge G_2$ is an edge regular fuzzy graph if and only if G_2 is an edge regular fuzzy graph.

Proof: Let $G_1^*:(V_1,E_1)$ be regular graph. Then $d_{G_1^*}(u_1)=m, \forall\, u_1\in V_1$, where m is a constant. Since μ_2 is a constant function. Then $\mu_2 = c$, where c is a constant.

Assume that G_2 is an edge regular fuzzy graph. Then $d_{G_2}(u_2v_2)=k, \forall\, u_2v_2 \in E_2$, where k is a constant. By theorem 3.1.1, If $\mu_1 \geq \mu_2$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

$d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1^*}(u_1)d_{G_2}(u_2)+d_{G_1^*}(v_1)d_{G_2}(v_2)-2\mu_2(u_2v_2)$

$= md_{G_2}(u_2)+md_{G_2}(v_2)-2c = m(d_{G_2}(u_2)+d_{G_2}(v_2)-2\mu_2(u_2v_2)+2\mu_2(u_2v_2))-2c$

$= m(d_{G_2}(u_2v_2))+2mc-2c = mk+2c(m-1)$ (Since $u_2v_2 \in E_2$)

Hence $G_1 \wedge G_2$ is an edge regular fuzzy graph.

Conversely, assume that $G_1 \wedge G_2$ is an edge regular fuzzy graph. Then $d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1\wedge G_2}((w_1,w_2)(x_1,x_2))$, for any $(u_1,u_2)(v_1,v_2)$ and $(w_1,w_2)(x_1,x_2) \in E$

$d_{G_1^*}(u_1)d_{G_2}(u_2)+d_{G_1^*}(v_1)d_{G_2}(v_2)-2\mu_2(u_2v_2) = d_{G_1^*}(w_1)d_{G_2}(w_2)+d_{G_1^*}(x_1)d_{G_2}(x_2)-2\mu_2(x_2x_2)$

$\Rightarrow md_{G_2}(u_2)+md_{G_2}(v_2)-2c = md_{G_2}(u_2)+md_{G_2}(v_2)-2c$

$\Rightarrow m(d_{G_2}(u_2)+d_{G_2}(v_2)-2\mu_2(u_2v_2)+2\mu_2(u_2v_2))$

$= m(d_{G_2}(w_2)+d_{G_2}(x_2)-2\mu_2(w_2x_2)+2\mu_2(w_2x_2))$

$\Rightarrow md_{G_2}(u_2v_2)+2mc = md_{G_2}(w_2x_2)+2mc$ (Since u_2v_2 and $w_2x_2 \in E_2$)

$\Rightarrow d_{G_2}(u_2v_2) = d_{G_2}(w_2x_2)$

Hence G_2 is an edge regular fuzzy graph.

Theorem 3.2.2. Let $G_1:(\sigma_1,\mu_1)$ be a strong fuzzy graph with $\sigma_1 \leq \mu_2$, σ_1 be a constant function with $\sigma_1(u)=c_1$ for all $u \in V_1$ and $G_2:(\sigma_2,\mu_2)$ be a fuzzy graph. If $G_1^*:(V_1,E_1)$ and $G_2^*:(V_2,E_2)$ are regular graphs, then $G_1 \wedge G_2$ is an edge regular fuzzy graph.

Proof: Assume $G_1^*:(V_1,E_1)$ and $G_2^*:(V_2,E_2)$ are regular graphs.

Then $d_{G_1^*}(u_1)=m, \forall\, u_1 \in V_1$ *and* $d_{G_2^*}(u_2)=n, \forall\, u_2 \in V_2$, where m and n are constants.

By theorem 3.1.2, for any $(u_1,u_2)(v_1,v_2) \in E$,

$$d_{G_1\wedge G_2}((u_1,u_2)(v_1,v_2)) = c_1(d_{G_2^*}(u_2)d_{G_1^*}(u_1)+d_{G_2^*}(v_2)d_{G_1^*}(v_1)-2)$$

$$= c_1(nm+nm-2) = 2c_1(nm-1)$$

Hence $G_1 \wedge G_2$ is an edge regular fuzzy graph.

4. Edge regular properties of normal product of two fuzzy graphs

4.1. The degree of an edge in normal product of two fuzzy graphs

By definition, for any $((u_1,u_2)(v_1,v_2)) \in E$,

$$d_{G_1\bullet G_2}((u_1,u_2)(v_1,v_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2)\in E,\\ (w_1,w_2)\neq(v_1,v_2)}}(\mu_1\bullet\mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(v_1,v_2)\in E,\\ (w_1,w_2)\neq(u_1,u_2)}}(\mu_1\bullet\mu_2)((w_1,w_2)(v_1,v_2)).$$

(1). If $u_1=v_1, u_2v_2 \in E_2$, then

$$d_{G_1\bullet G_2}((u_1,u_2)(u_1,v_2)) = \sum_{\substack{u_2w_2\in E_2, u_1=w_1,\\ w_2\neq v_2}}\sigma_1(u_1)\wedge\mu_2(u_2w_2) + \sum_{u_1w_1\in E_1, u_2=w_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)$$

$$+\sum_{u_1w_1\in E_1,u_2w_2\in E_2}\mu_1(u_1w_1)\wedge\mu_2(u_2w_2)+\sum_{\substack{w_2v_2\in E_2,w_1=u_1,\\ w_2\neq u_2}}\sigma_1(u_1)\wedge\mu_2(w_2v_2)+\sum_{w_1u_1\in E_1,w_2=v_2}\mu_1(w_1u_1)\wedge\sigma_2(v_2)$$

$$+\sum_{w_1u_1\in E_1,w_2v_2\in E_2}\mu_1(w_1u_1)\wedge\mu_2(w_2v_2) \tag{4.1}$$

(2). If $u_1v_1\in E_1, u_2=v_2$, then

$$d_{G_1\bullet G_2}((u_1,u_2)(v_1,u_2))=\sum_{u_2w_2\in E_2,u_1=w_1}\sigma_1(u_1)\wedge\mu_2(u_2w_2)+\sum_{\substack{u_1w_1\in E_1,u_2=w_2,\\ w_1\neq v_1}}\mu_1(u_1w_1)\wedge\sigma_2(u_2)$$

$$+\sum_{u_1w_1\in E_1,u_2w_2\in E_2}\mu_1(u_1w_1)\wedge\mu_2(u_2w_2)+\sum_{w_2u_2\in E_2,w_1=u_1}\sigma_1(v_1)\wedge\mu_2(w_2u_2)+\sum_{\substack{w_1v_1\in E_1,w_2=u_2,\\ w_1\neq u_1}}\mu_1(w_1v_1)\wedge\sigma_2(u_2)$$

$$+\sum_{w_1v_1\in E_1,w_2u_2\in E_2}\mu_1(w_1v_1)\wedge\mu_2(w_2u_2) \tag{4.2}$$

(3). If $u_1v_1\in E_1, u_2v_2\in E_2$, then

$$d_{G_1\bullet G_2}((u_1,u_2)(v_1,v_2))=\sum_{(u_1,u_2)(w_1,w_2)\in E}(\mu_1\bullet\mu_2)((u_1,u_2)(w_1,w_2))+\sum_{(w_1,w_2)(v_1,u_2)\in E}(\mu_1\bullet\mu_2)((w_1,w_2)(v_1,v_2))$$

$$-\ 2(\mu_1\bullet\mu_2)((u_1,u_2)(v_1,v_2))$$

$$=\sum_{u_2w_2\in E_2,u_1=w_1}\sigma_1(u_1)\wedge\mu_2(u_2w_2)+\sum_{u_1w_1\in E_1,u_2=w_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)+\sum_{u_1w_1\in E_1,u_2w_2\in E_2}\mu_1(u_1w_1)\wedge\mu_2(u_2w_2)$$

$$+\sum_{w_2v_2\in E_2,w_1=u_1}\sigma_1(v_1)\wedge\mu_2(w_2v_2)+\sum_{w_1v_1\in E_1,w_2=v_2}\mu_1(w_1v_1)\wedge\sigma_2(v_2)+\sum_{w_1v_1\in E_1,w_2v_2\in E_2}\mu_1(w_1v_1)\wedge\mu_2(w_2v_2)$$

$$-2(\mu_1\bullet\mu_2)((u_1,u_2)(v_1,v_2)) \tag{4.3}$$

Theorem 4.1.1. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs.
If $\sigma_1\geq\mu_2$, $\sigma_2\geq\mu_1$ and $\mu_1\geq\mu_2$, then for any $(u_1,u_2)(v_1,v_2)\in E$,
(1). $d_{G_1\bullet G_2}((u_1,u_2)(u_1,v_2))=d_{G_2}(u_2v_2)+2d_{G_1}(u_1)+d_{G_1^*}(u_1)(d_{G_2}(u_2)+d_{G_2}(v_2))$,
(2). $d_{G_1\bullet G_2}((u_1,u_2)(v_1,u_2))=d_{G_1}(u_1v_1)+d_{G_2}(u_2)(d_{G_1^*}(u_1)+d_{G_1^*}(v_1)+2)$,
(3). $d_{G_1\bullet G_2}((u_1,u_2)(v_1,v_2))=d_{G_1}(u_1)+d_{G_1}(v_1)+d_{G_2}(u_2v_2)+d_{G_1^*}(u_1)d_{G_2}(u_2)$
$+d_{G_1^*}(v_1)d_{G_2}(v_2)$.

Proof: We have $\sigma_1\geq\mu_2$, $\sigma_2\geq\mu_1$ and $\mu_1\geq\mu_2$.
(1). From (4.1), for any $(u_1,u_2)(u_1,v_2)\in E$,

$$d_{G_1\bullet G_2}((u_1,u_2)(u_1,v_2))=\sum_{u_2w_2\in E_2,w_2\neq v_2}\mu_2(u_2w_2)+\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+\sum_{u_1w_1\in E_1,u_2w_2\in E_2}\mu_2(u_2w_2)$$

$$+\sum_{w_2v_2\in E_2,w_2\neq u_2}\mu_2(w_2v_2)+\sum_{w_1u_1\in E_1}\mu_1(w_1u_1)+\sum_{w_1u_1\in E_1,w_2v_2\in E_2}\mu_2(w_2v_2)$$

$$=d_{G_2}(u_2v_2)+d_{G_1}(u_1)+d_{G_1^*}(u_1)d_{G_2}(u_2)+d_{G_1}(u_1)+d_{G_1^*}(u_1)d_{G_2}(v_2)$$

$$\therefore d_{G_1\bullet G_2}((u_1,u_2)(u_1,v_2))=d_{G_2}(u_2v_2)+2d_{G_1}(u_1)+d_{G_1^*}(u_1)(d_{G_2}(u_2)+d_{G_2}(v_2)).$$

(2). From (4.2), for any $(u_1,u_2)(v_1,u_2)\in E$,

$$d_{G_1\bullet G_2}((u_1,u_2)(v_1,u_2)) = \sum_{u_2w_2\in E_2}\mu_2(u_2w_2) + \sum_{u_1w_1\in E_1, w_1\neq v_1}\mu_1(u_1w_1) + \sum_{u_1w_1\in E_1, w_2\in V_2}\mu_2(u_2w_2)$$

$$+ \sum_{w_2u_2\in E_2}\mu_2(w_2u_2) + \sum_{w_1v_1\in E_1, w_1\neq u_1}\mu_1(w_1v_1) + \sum_{w_1v_1\in E_1, w_2\in V_2}\mu_2(w_2u_2)$$

$$= d_{G_2}(u_2) + d_{G_1}(u_1v_1) + d_{G_1^*}(u_1)d_{G_2}(u_2) + d_{G_2}(u_2) + d_{G_1^*}(v_1)d_{G_2}(u_2)$$

$$\therefore d_{G_1\bullet G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + d_{G_2}(u_2)(d_{G_1^*}(u_1) + d_{G_1^*}(v_1) + 2).$$

(3). From (4.3), for any $(u_1,u_2)(v_1,v_2)\in E$,

$$d_{G_1\bullet G_2}((u_1,u_2)(v_1,v_2)) = \sum_{u_2w_2\in E_2}\mu_2(u_2w_2) + \sum_{u_1w_1\in E_1}\mu_1(u_1w_1) + \sum_{u_1w_1\in E_1, u_2w_2\in E_2}\mu_2(u_2w_2)$$

$$+ \sum_{w_2v_2\in E_2}\mu_2(w_2v_2) + \sum_{w_1v_1\in E_1}\mu_1(w_1v_1) + \sum_{w_1v_1\in E_1, w_2v_2\in E_2}\mu_2(w_2v_2) - 2\mu_2(u_2v_2)$$

$$= d_{G_2}(u_2) + d_{G_1}(u_1) + d_{G_1^*}(u_1)\sum_{u_2w_2\in E_2}\mu_2(u_2w_2) + d_{G_2}(v_2) + d_{G_1}(v_1) + d_{G_1^*}(v_1)\sum_{w_2v_2\in E_2}\mu_2(w_2v_2) - 2\mu_2(u_2v_2)$$

$$\therefore d_{G_1\bullet G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + d_{G_2}(u_2v_2) + d_{G_1^*}(u_1)d_{G_2}(u_2)$$

$$+ d_{G_1^*}(v_1)d_{G_2}(v_2).$$

Theorem 4.1.2. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs.If $\sigma_1\le\mu_2$ and σ_1 is a constant function with $\sigma_1(u)=c_1$ for all $u\in V_1$, then for any $(u_1,u_2)(v_1,v_2)\in E$,

(1). $d_{G_1\bullet G_2}((u_1,u_2)(u_1,v_2)) = (d_{G_2^*}(u_2) + d_{G_2^*}(v_2) + 2)(c_1 + d_{G_1}(u_1)) - 4c_1$,

(2). $d_{G_1\bullet G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + d_{G_2^*}(u_2)(d_{G_1}(u_1) + d_{G_1}(v_1) + 2c_1)$,

(3). $d_{G_1\bullet G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + d_{G_2^*}(u_2)(d_{G_1}(u_1) + c_1) + d_{G_2^*}(v_2)(d_{G_1}(v_1) + c_1)$.

Proof: We have $\sigma_1\le\mu_2$. Then by theorem 2.4 and theorem 2.5, $\sigma_2\ge\mu_1$ and $\mu_1\le\mu_2$.

(1). From (4.1.1), for any $(u_1,u_2)(u_1,v_2)\in E$,

$$d_{G_1\bullet G_2}((u_1,u_2)(u_1,v_2)) = \sum_{u_2w_2\in E_2, w_2\neq v_2}\sigma_1(u_1) + \sum_{u_1w_1\in E_1}\mu_1(u_1w_1) + \sum_{u_1w_1\in E_1, u_2w_2\in E_2}\mu_1(u_1w_1) + \sum_{w_2v_2\in E_2, w_2\neq u_2}\sigma_1(u_1)$$

$$+ \sum_{w_1u_1\in E_1}\mu_1(w_1u_1) + \sum_{w_1u_1\in E_1, w_2v_2\in E_2}\mu_1(w_1u_1)$$

$$= c_1(d_{G_2^*}(u_2)-1) + d_{G_1}(u_1) + d_{G_2^*}(u_2)d_{G_1}(u_1) + c_1(d_{G_2^*}(v_2)-1) + d_{G_1}(u_1) + d_{G_2^*}(v_2)d_{G_1}(u_1)$$

$$= c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) - 2) + 2d_{G_1}(u_1) + d_{G_1}(u_1)(d_{G_2^*}(u_2) + d_{G_2^*}(v_2))$$

$$= c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) - 2) + d_{G_1}(u_1)(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) + 2)$$

$$= c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) + 2 - 4) + d_{G_1}(u_1)(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) + 2)$$

$$\therefore d_{G_1\bullet G_2}((u_1,u_2)(u_1,v_2)) = (d_{G_2^*}(u_2) + d_{G_2^*}(v_2) + 2)(c_1 + d_{G_1}(u_1)) - 4c_1.$$

(2). From (4.1.2), for any $(u_1,u_2)(v_1,u_2)\in E$,

$$d_{G_1\bullet G_2}((u_1,u_2)(v_1,u_2)) = \sum_{u_2w_2\in E_2}\sigma_1(u_1) + \sum_{u_1w_1\in E_1, w_1\neq v_1}\mu_1(u_1w_1) + \sum_{u_1w_1\in E_1, w_2u_2\in E_2}\mu_1(u_1w_1) + \sum_{w_2u_2\in E_2}\sigma_1(v_1)$$

$$+\sum_{w_1v_1\in E_1, w_1\neq u_1}\mu_1(w_1v_1)+\sum_{w_1v_1\in E_1, w_2u_2\in E_2}\mu_1(w_1v_1)$$

$$=2c_1 d_{G_2^*}(u_2)+d_{G_1}(u_1v_1)+d_{G_2^*}(u_2)d_{G_1}(u_1)+d_{G_2^*}(u_2)d_{G_1}(v_1)$$

$$\therefore d_{G_1\bullet G_2}((u_1,u_2)(v_1,u_2))=d_{G_1}(u_1v_1)+d_{G_2^*}(u_2)(d_{G_1}(u_1)+d_{G_1}(v_1)+2c_1).$$

(3). From (4.1.3), for any $(u_1,u_2)(v_1,v_2)\in E$,

$$d_{G_1\bullet G_2}((u_1,u_2)(v_1,v_2))=\sum_{u_2w_2\in E_2}\sigma_1(u_1)+\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+\sum_{u_1w_1\in E_1, u_2w_2\in E_2}\mu_1(u_1w_1)+\sum_{w_2v_2\in E_2}\sigma_1(v_1)$$

$$+\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)+\sum_{w_1v_1\in E_1, w_2v_2\in E_2}\mu_1(w_1v_1)-2\mu_1(u_1v_1)$$

$$=c_1(d_{G_2^*}(u_2)+d_{G_2^*}(v_2))+d_{G_1}(u_1)+d_{G_1}(v_1)-2\mu_1(u_1v_1)+d_{G_2^*}(u_2)d_{G_1}(u_1)+d_{G_2^*}(v_2)d_{G_1}(v_1)$$

$$\therefore d_{G_1\bullet G_2}((u_1,u_2)(v_1,v_2))=d_{G_1}(u_1v_1)+d_{G_2^*}(u_2)(d_{G_1}(u_1)+c_1)+d_{G_2^*}(v_2)(d_{G_1}(v_1)+c_1).$$

4.2. Edge regular property of normal product of two fuzzy graphs

Remark 4.2.1. If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two edge regular fuzzy graphs, then $G_1\bullet G_2$ need not be edge regular fuzzy graph. For example, in figure 4.2.1, G_1 and G_2 are edge regular fuzzy graphs, but $G_1\bullet G_2$ is not an edge regular fuzzy graph.

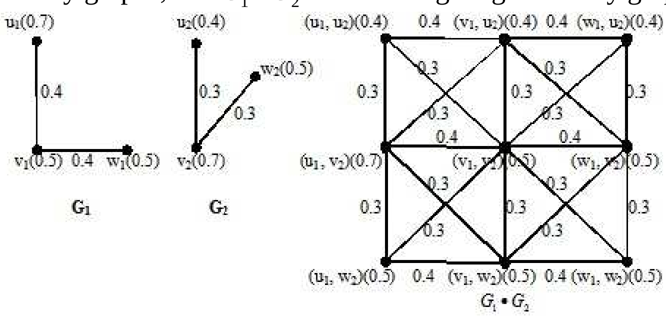

Figure 4.2.1:

Remark 4.2.1. If $G_1\bullet G_2$ is an edge regular fuzzy graph, then $G_1:(\sigma_1,\mu_1)$ or $G_2:(\sigma_2,\mu_2)$ need not be edge regular fuzzy graph. For example, in figure 4.2.2, $G_1\bullet G_2$ is 1.8 – edge regular fuzzy graph, but G_1 is not an edge regular fuzzy graph.

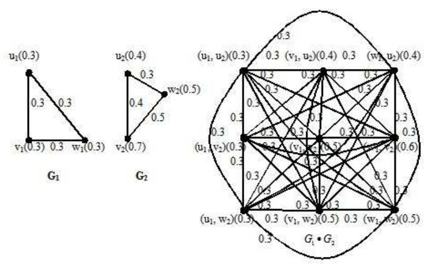

Figure 4.2.2:

Theorem 4.2.1. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two regular fuzzy graphs of same degree with $\sigma_1\geq\mu_2,\sigma_2\geq\mu_1$ and $\mu_1\geq\mu_2$. Let $G_1^*:(V_1,E_1)$ be regular graph. Then

$G_1 \bullet G_2$ is an edge regular fuzzy graphif and only if G_2 and G_2 are edge regular fuzzy graphs of same degree.

Proof: Let $d_{G_1}(u_1) = d_{G_2}(u_2) = m$ *and* $d_{G_1^*}(u_1) = n, \forall u_1 \in V_1$ *and* $u_2 \in V_2$, where m *and* n are constants.

Assume that G_1 and G_2 are k - edge regular fuzzy graphs, where k is a constant.

By theorem 4.1.1, for any $(u_1, u_2)(v_1, v_2) \in E$, when $u_1 = v_1, u_2v_2 \in E_2$,

$d_{G_1 \bullet G_2}((u_1, u_2)(u_1, v_2)) = d_{G_2}(u_2v_2) + 2d_{G_1}(u_1) + d_{G_1^*}(u_1)(d_{G_2}(u_2) + d_{G_2}(v_2))$

$= d_{G_2}(u_2v_2) + 2d_{G_1}(u_1) + d_{G_1^*}(u_1)(d_{G_2}(u_2) + d_{G_2}(v_2)) = k + 2m + n(m + m) = k + 2m(n+1)$

Similarly, when $u_1v_1 \in E_1, u_2 = v_2$ and $u_1v_1 \in E_1, u_2v_2 \in E_2$,

$d_{G_1 \bullet G_2}((u_1, u_2)(v_1, u_2)) = d_{G_1 \bullet G_2}((u_1, u_2)(v_1, v_2)) = k + 2m(n+1)$

Hence $G_1 \bullet G_2$ is an edge regular fuzzy graph.

Conversely, assume that $G_1 \bullet G_2$ is an edge regular fuzzy graph. To prove that G_1 and G_2 are edge regular fuzzy graphs of same degree.

Let $u_1v_1, w_1x_1 \in E_1$ be any two edges of G_1. Fix $u \in V_2$.

Then $(u_1, u)(v_1, u)$ *and* $(w_1, u)(x_1, u) \in E$, $d_{G_1 \bullet G_2}((u_1, u)(v_1, u)) = d_{G_1 \bullet G_2}((w_1, u)(x_1, u))$.

$d_{G_1}(u_1v_1) + d_{G_2}(u)(d_{G_1^*}(u_1) + d_{G_1^*}(v_1) + 2) = d_{G_1}(w_1x_1) + d_{G_2}(u)(d_{G_1^*}(w_1) + d_{G_1^*}(x_1) + 2)$

$d_{G_1}(u_1v_1) + m(n + n + 2) = d_{G_1}(w_1x_1) + m(n + n + 2)$

$d_{G_1}(u_1v_1) = d_{G_1}(w_1x_1)$, $\forall$ u_1v_1 *and* $w_1x_1 \in E_1$.

$\therefore G_1$ is an edge regular fuzzy graph.

Similarly, G_2 is an edge regular fuzzy graph.

Now, to prove that G_1 and G_2 are edge regular fuzzy graphs of same degree.

Suppose that G_1 is k_1 – edge regular fuzzy graph and G_2 is k_2 – edge regular fuzzy graph with $k_1 \neq k_2$.

$\therefore d_{G_1 \bullet G_2}((u_1, u_2)(u_1, v_2)) = d_{G_2}(u_2v_2) + 2d_{G_1}(u_1) + d_{G_1^*}(u_1)(d_{G_2}(u_2) + d_{G_2}(v_2))$

$= k_2 + 2m + n(m + m) = k_2 + 2m(n+1)$ (4.2.1)

$\therefore d_{G_1 \bullet G_2}((u_1, u_2)(v_1, u_2)) = d_{G_1}(u_1v_1) + d_{G_2}(u_2)(d_{G_1^*}(u_1) + d_{G_1^*}(v_1) + 2)$

$= k_1 + m(n + n + 2) = k_1 + 2m(n+1)$ (4.2.2)

From (4.2.1) and (4.2.2), $d_{G_1 \bullet G_2}((u_1, u_2)(u_1, v_2)) \neq d_{G_1 \bullet G_2}((u_1, u_2)(v_1, u_2))$, since $k_1 \neq k_2$.

This is a contradiction to our assumption that $G_1 \bullet G_2$ is an edge regular fuzzy graph.

$\therefore G_1$ and G_2 are edge regular fuzzy graphs of same degree.

Theorem 4.2.2. [8] Let μ = c be a constant function in G: (σ, μ) on G^*: (V, E). If G is regular, then G is edge regular.

Theorem 4.2.3. Let $G_1:(\sigma_1,\mu_1)$ be a strong and regular fuzzy graph with $\sigma_1 \le \mu_2$, σ_1 be a constant function with $\sigma_1(u)=c_1$ for all $u\in V_1$ and let $G_2:(\sigma_2,\mu_2)$ be a fuzzy graph on regular crisp graph $G_2^*:(V_2,E_2)$. Then $G_1\bullet G_2$ is an edge regular fuzzy graph.

Proof: Let $d_{G_1}(u_1)=m$, $d_{G_2^*}(u_2)=n$, $\forall u_1\in V_1$ and $u_2\in V_2$, where $m\,\&\,n$ are constants.

Since G_1 is strong fuzzy graph and σ_1 is a constant function, μ_1 is a constant function.

Using theorem 4.2.2, $G_1:(\sigma_1,\mu_1)$ is edge regular.

Let $d_{G_1}(u_1v_1)=k, \forall u_1v_1\in E_1$, where k is a constant.

By theorem 3.1.1.2, for any $(u_1,u_2)(v_1,v_2)\in E$, when $u_1=v_1, u_2v_2\in E_2$,

$$d_{G_1\bullet G_2}((u_1,u_2)(u_1,v_2))=(d_{G_2^*}(u_2)+d_{G_2^*}(v_2)+2)(c_1+d_{G_1}(u_1))-4c_1=(n+n+2)(c_1+m)-4c_1$$

$$=2(n+1)(c_1+m)-4c_1=2nc_1+2nm+2c_1+2m-4c_1=2n(m+c_1)+2(m-c_1)$$

Similarly, when $u_1v_1\in E_1, u_2=v_2$ and $u_1v_1\in E_1, u_2v_2\in E_2$,

$$d_{G_1\bullet G_2}((u_1,u_2)(v_1,u_2))=d_{G_1\bullet G_2}((u_1,u_2)(v_1,v_2))=2n(m+c_1)+2(m-c_1)$$

Hence $G_1\bullet G_2$ is an edge regular fuzzy graph.

7. Conclusion

In this paper, we have found the degree of edges in $G_1\wedge G_2$ and $G_1\bullet G_2$ in terms of the degree of vertices and edges in G_1 and G_2 and also in terms of the degree of vertices in G_1^* and G_2^* under some conditions. They will we more helpful especially when the graphs are very large. Also they will be useful in studying various conditions, properties of tensor product and normal product of two fuzzy graphs.

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