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Generalized Fuzzy Right H-Ideals of Hemirings

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Abstract. In this paper, we introduce the notions of $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right and right h-ideals. We establish necessary and sufficient conditions for a fuzzy set to be a fuzzy $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-right and right h-ideal. We introduce the notion of generalized fuzzy h-closure. We characterize fuzzy $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-right ideals using generalized fuzzy h-closure.

Keywords: Fuzzy algebra, hemirings, right h-ideals, h-closure, fuzzy right h-ideals

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1. Introduction

In 1965, Zadeh [20] introduced the concepts of fuzzy set. Since then fuzzy sets have been applied to many branches of mathematics. The fuzzification of algebraic structures was initiated by Rosenfeld [18], he introduced the notion of fuzzy subgroups. The fuzzy algebraic structure play an important role in mathematics with wide applications in theoretical physics, computer science, control engineering, information science, coding theory and topological spaces [7,19]. Hemirings, appears in a natural manner, in some applications to the theory of automata, the theory of formal languages and in computer sciences [19]. In [9], La Torre initiated the study of h-ideals and k-ideals of hemirings. The notions of “belongingness” and “quasi-coincidence” of fuzzy points and fuzzy sets proposed and discussed in [15,17]. Generalizing the concept of quasi-coincident of a fuzzy point with a fuzzy set, in [8], Jun defined $(\in,\in\vee q)$-fuzzy sub algebras in BCK/BCI- algebras. Mohanraj et al. characterized semiregular semirings using intuitionistic fuzzy k-ideals in [4]. Mohanraj et al. generalized redefined fuzzy prime ideals of ordered semigroups in [13]. Mohanraj and Prabu generalized fuzzy prime ideal of hemirings [11] and redefined generalized fuzzy right h-ideals of hemirings [12]. In this paper, we introduce the notion of $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right and right h-ideals. We establish necessary and sufficient conditions for a fuzzy set to be a fuzzy $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-right and right h-ideal. We introduce the notion of generalized fuzzy h-closure. We characterize fuzzy $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-right ideals using generalized fuzzy h-closure.

2. Preliminaries

By a hemiring, we mean a structure $(H,+,\cdot)$ in which the following conditions are satisfied:

(1) $(H,+)$ is a commutative semigroup.

(2) $(H,\cdot)$ is a semigroup.

(3) $a\cdot(b+c)=a\cdot b+a\cdot c$ and $(a+b)\cdot c=a\cdot c+b\cdot c$

(4) There is $0\in H$ such that $0\cdot x=x\cdot 0$ and $0+x=x+0=x$ for all $x\in H.$

A non-empty subset A of a hemiring H is called a sub-hemiring of H if it contains zero and is closed with respect to the addition and multiplication. A non-empty subset I of hemiring H is called a right (left) ideal in H if I is closed under addition and $IH \subseteq I\,(HI \subseteq I)$. Furthermore I is called an ideal of H if it is both a right ideal and left ideal in H. A right [left] ideal I of a hemiring H is called a right [left] h-ideal if $a,b\in I$ and $x+a+z=b+z,$ for $x,z\in H$ imply $x\in I.$ The h-closure $\overline{A}$ of a non-empty subset A of a hemiring H is defined as: $\overline{A}=\{x\in H \mid x+a+z=b+z$ for some $a,b\in A, z\in H\}$

Definition 2.1. A mapping $\mu : X \rightarrow [0,1]$ is called a fuzzy set of X .

Definition 2.2. Let μ be any fuzzy set of H and let $t\in[0,1]$. The set $\mu_t=\{x\in H \mid \mu(x)\geq t\}$ is called a level set of $\mu.$

Definition 2.3. A fuzzy set μ of H of the form

$$\mu(y)=\begin{cases} t\in(0,1] & \text{if} \quad y=x \\ 0 & \text{if} \quad y\neq x \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t.

Definition 2.4. A fuzzy point x_t is said to belong to a fuzzy set μ if $\mu(x)\geq t$ and it is denoted by $x_t \in \mu$.

Definition 2.5. A fuzzy point x_t is said to quasi-coincident with a fuzzy set μ if $\mu(x)+t>1$ and it is denoted by $x_t q\mu.$

Definition 2.6. A fuzzy point x_t is said to not belong to a fuzzy set μ if $\mu(x)<t$ and it is denoted by $x_t\overline{\in}\mu.$

Definition 2.7. A fuzzy set μ of H is said to be an $(\overline{\in},\overline{\in}\vee\overline{q})$-fuzzy right [left] ideal of H if (i) $(x+y)_t\overline{\in}\mu$ implies $x_t\overline{\in}\vee\overline{q}\mu$ or $y_t\overline{\in}\vee\overline{q}\mu.$

(ii) $(xy)_t\overline{\in}\mu$ implies $x_t\overline{\in}\vee\overline{q}\mu$ $[y_t\overline{\in}\vee\overline{q}\mu]$ for all $t\in(0,1]$ and for $x,y\in H$.

Definition 2.8. An $(\overline{\in},\overline{\in}\vee\overline{q})$-fuzzy right ideal is said to be $(\overline{\in},\overline{\in}\vee\overline{q})$-fuzzy right h-ideal of a hemiring H if $x+a+z=b+z$ and $x_t\overline{\in}\mu$ imply $a_t\overline{\in}\vee\overline{q}\mu$ or $b_t\overline{\in}\vee\overline{q}\mu$ for $a,b,x,z\in H$.

Definition 2.9. Let $0\le k<1$. Then,

(i) We mean $x_t q_k \mu$ if $\mu(x)+t+k>1$.

(ii) We mean $x_t\in\vee q_k\mu$ if $x_t\in\mu$ or $x_t q_k\mu$.

(iii) We mean $x_t\in\wedge q_k\mu$ if $x_t\in\mu$ and $x_t q_k\mu$.

(iv) We mean $x_t\overline{q_k}\mu$ if $\mu(x)+t+k\le 1$.

Definition 2.10. Let μ and ν be fuzzy sets in a hemiring H. Then the fuzzy product of μ and ν denoted by $\mu\circ\nu$ is defined as follows:

$$(\mu\circ\nu)(x)=\begin{cases}\bigvee_{x=yz}\{\mu(y)\wedge\nu(z)\} & \text{if } x=yz \text{ for some } y,z\in H\\ 0 & \text{if 'x' cannot be expressible as } x=yz\end{cases}$$

Definition 2.11. A fuzzy set μ of a hemiring H is said to be fuzzy right [left] ideal of H if it satisfies (i) $\mu(x+y)\ge\min\{\mu(x),\mu(y)\}$ for $x,y\in H$,
(ii) $\mu(xy)\ge\mu(x)\ [\mu(xy)\ge\mu(y)]$ for $x,y\in H$.

Definition 2.12. A fuzzy right ideal μ is said to be fuzzy right h-ideal of a hemiring H if $x+a+z=b+z$ implies $\mu(x)\ge\min\{\mu(a),\mu(b)\}$ for $a,b,x,z\in H$.

3. $(\overline{\in},\overline{\in}\vee\overline{q_k})$-fuzzy right h-Ideals

Throughout this paper, H denotes hemiring unless otherwise specified. Let $0\le k<1$.

Lemma 3.1. Let μ be a fuzzy set of a hemiring H. Then the following statements are equivalent (i) $(x+y)_t\overline{\in}\mu$ implies $x_t\overline{\in}\vee\overline{q}_k\mu$ or $y_t\overline{\in}\vee\overline{q}_k\mu$ for all $t\in(0,1]$ and for $x,y\in H$.
(ii) $\max\{\mu(x+y),\frac{1-k}{2}\}\ge\min\{\mu(x),\mu(y)\}$ for $x,y\in H$.

Proof: $(i)\Rightarrow(ii)$ If there exist $x,y\in H$ such that $\max\{\mu(x+y),\frac{1-k}{2}\}<\min\{\mu(x),\mu(y)\}$, then choose $t\in(\frac{1-k}{2},1)$ such that $\max\{\mu(x+y),\frac{1-k}{2}\}<t<\min\{\mu(x),\mu(y)\}$. Thus $(x+y)_t\overline{\in}\mu$, $x_t\in\mu$ and $y_t\in\mu$. Now, $t>\frac{1-k}{2}$ and $\mu(x)>t>\frac{1-k}{2}$ and $\mu(y)>t>\frac{1-k}{2}$ imply $x_t q_k\mu$ and $y_t q_k\mu$. Therefore $(x+y)_t\overline{\in}\mu$ but $x_t\in\wedge q_k\mu$ and $y_t\in\wedge q_k\mu$, which is a contradiction.

Conversely, let $(x+y)_t\overline{\in}\mu$. If $t>\frac{1-k}{2}$, then
$\min\{\mu(x),\mu(y)\}\le\max\{\mu(x+y),\frac{1-k}{2}\}<\max\{t,\frac{1-k}{2}\}=t$. Thus $\min\{\mu(x),\mu(y)\}<t$.
Therefore $x_t\overline{\in}\mu$ or $y_t\overline{\in}\mu$. If $t\le\frac{1-k}{2}$, then
$\min\{\mu(x),\mu(y)\}\le\max\{\mu(x+y),\frac{1-k}{2}\}<\max\{t,\frac{1-k}{2}\}=\frac{1-k}{2}$. Therefore $\mu(x)+t+k\le 1$

or $\mu(y)+t+k\leq 1$. Thus $x_t\overline{q}_k\mu$ or $y_t\overline{q}_k\mu$. Hence $(x+y)_t\overline{\in}\mu$ implies $x_t\overline{\in}\vee\overline{q}_k\mu$ or $y_t\overline{\in}\vee\overline{q}_k\mu$.

Lemma 3.2. Let μ be fuzzy set of a hemiring H. Then the following statements are equivalent(i) $(xy)_t\overline{\in}\mu$ implies $x_t\overline{\in}\vee\overline{q}_k\mu$ or $y_t\overline{\in}\vee\overline{q}_k\mu$ for all $t\in(0,1]$ and for $x,y\in H$.
(ii) $\max\{\mu(xy),\frac{1-k}{2}\}\geq\mu(x)$ for $x,y\in H$.
Proof: The proof is similar to the proof of Lemma 3.1.

Definition 3.1. A fuzzy set μ of H is said to be an $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right [left] ideal of H if (i) $(x+y)_t\overline{\in}\mu$ implies $x_t\overline{\in}\vee\overline{q}_k\mu$ or $y_t\overline{\in}\vee\overline{q}_k\mu$.
(ii) $(xy)_t\overline{\in}\mu$ implies $x_t\overline{\in}\vee\overline{q}_k\mu$ $[y_t\overline{\in}\vee\overline{q}_k\mu]$ for all $t\in(0,1]$ and for $x,y\in H$.

Theorem 3.1. Let μ be a fuzzy set of H. Then μ is a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$ -fuzzy right ideal if and only if(i) $\max\{\mu(x+y),\frac{1-k}{2}\}\geq\min\{\mu(x),\mu(y)\}$
(ii) $\max\{\mu(xy),\frac{1-k}{2}\}\geq\mu(x)$
Proof: The proof follows from the Lemma 3.1 and 3.2.

Definition 3.2. An $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right ideal is said to be $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal of a hemiring H if $x+a+z=b+z$ and $x_t\overline{\in}\mu$ imply $a_t\overline{\in}\vee\overline{q}_k\mu$ or $b_t\overline{\in}\vee\overline{q}_k\mu$ for $a,b,x,z\in H$.

Remark 3.1. If $k=0$, then $x_t\overline{\in}\vee\overline{q}_k\mu$ coincides $x_t\overline{\in}\vee\overline{q}\mu$.

Lemma 3.3. Let μ be fuzzy set of a hemiring H. Then the following statements are equivalent.
(i) $x+a+z=b+z$ and $x_t\overline{\in}\mu$ imply $a_t\overline{\in}\vee\overline{q}_k\mu$ or $b_t\overline{\in}\vee\overline{q}_k\mu$ for $a,b,x,z\in H$.
(ii) $x+a+z=b+z$ implies $\max\{\mu(x),\frac{1-k}{2}\}\geq\min\{\mu(a),\mu(b)\}$ for $a,b,x,z\in H$.
Proof: $(i)\Rightarrow(ii)$ If there exist $a,b,x,z\in H$ such that
$\max\{\mu(x+y),\frac{1-k}{2}\}<\min\{\mu(a),\mu(b)\}$ and $x+a+z=b+z$, then choose $t\in(\frac{1-k}{2},1)$ such that $\max\{\mu(x),\frac{1-k}{2}\}<t<\min\{\mu(a),\mu(b)\}$. Thus $x_t\overline{\in}\mu$ implies $a_t\in\mu$ and $b_t\in\mu$. Moreover, $t>\frac{1-k}{2}$ implies $a_tq_k\mu$ and $b_tq_k\mu$. Therefore $x_t\overline{\in}\mu$ but $a_t\in\wedge q_k\mu$ and $b_t\in\wedge q_k\mu$, which is a contradiction.
Conversely, let $x_t\overline{\in}\mu$. If $t>\frac{1-k}{2}$, then
$\min\{\mu(a),\mu(b)\}\leq\max\{\mu(x),\frac{1-k}{2}\}<t=\max\{t,\frac{1-k}{2}\}$. Thus $a_t\overline{\in}\mu$ or $b_t\overline{\in}\mu$. If $t\leq\frac{1-k}{2}$, then $\min\{\mu(a),\mu(b)\}\leq\max\{\mu(x),\frac{1-k}{2}\}\leq\max\{t,\frac{1-k}{2}\}=\frac{1-k}{2}$. Thus $a_t\overline{q}_k\mu$ or $b_t\overline{q}_k\mu$. Hence $x_t\overline{\in}\mu$ implies $a_t\overline{\in}\vee\overline{q}_k\mu$ or $b_t\overline{\in}\vee\overline{q}_k\mu$.

Corollary 3.1. Let μ be a fuzzy set of H. Then μ is a $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy right ideal of H if and only if (i) $\max\{\mu(x+y), 0.5\} \geq \min\{\mu(x), \mu(y)\}$
(ii) $\max\{\mu(xy), 0.5\} \geq \mu(x)$
Proof: By taking $k = 0$ in Theorem 3.1, we get the result.

Theorem 3.2. Let μ be a fuzzy set of H. Then μ is a $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy right h-ideal if and only if (i) $\max\{\mu(x+y), \frac{1-k}{2}\} \geq \min\{\mu(x), \mu(y)\}$
(ii) $\max\{\mu(xy), \frac{1-k}{2}\} \geq \mu(x)$
(iii) $\max\{\mu(x), \frac{1-k}{2}\} \geq \min\{\mu(a), \mu(b)\}$ for $a, b, x, z \in H$.
Proof: The proof follows from the Lemma 3.1, 3.2 and 3.3.

Corollary 3.2. A $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy right ideal is a $(\overline{\in}, \overline{\in} \vee \overline{q}_k)$-fuzzy right h-ideal of H if and only if $x + a + z = b + z$ implies $\max\{\mu(x), \frac{1-k}{2}\} \geq \min\{\mu(a), \mu(b)\}$ for $a, b, x, z \in H$.
Proof: The Proof follows from the Lemma 3.3 and Theorem 3.2.

Corollary 3.3. Let μ be a fuzzy set of H. Then μ is a $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy right h-ideal if and only if(i) $\max\{\mu(x+y), 0.5\} \geq \min\{\mu(x), \mu(y)\}$
(ii) $\max\{\mu(xy), 0.5\} \geq \mu(x)$
(iii) $\max\{\mu(x), 0.5\} \geq \min\{\mu(a), \mu(b)\}$ for $a, b, x, z \in H$.
Proof: By taking $k = 0$, in Theorem 3.2, we get the result.

3.1. Generalized fuzzy right h-ideals

Definition 3.1.1. Let μ and λ be the fuzzy sets of H. We mean that $\lambda \supseteq \vee \overline{q}_k \mu$ if $x_t \overline{\in} \lambda$ implies $x_t \overline{\in} \vee \overline{q}_k \mu$.

Lemma 3.1.1. Let μ and λ be the fuzzy sets of H. Then $\lambda \supseteq \vee \overline{q}_k \mu$ if and only if $\max\{\lambda(x), \frac{1-k}{2}\} \geq \mu(x)$.

Proof: Let $\lambda \supseteq \vee \overline{q}_k \mu$. If there exists $x \in H$ such that $\max\{\lambda(x), \frac{1-k}{2}\} < t < \mu(x)$, then $x_t \overline{\in} \lambda$, $x_t \in \mu$ and $t > \frac{1-k}{2}$. Thus $t > \frac{1-k}{2}$ implies $x_t q_k \mu$. Therefore $x_t \in \wedge q_k \mu$, which is a contradiction.

Conversely, let $x_t \overline{\in} \lambda$. If $t \leq \frac{1-k}{2}$, then $\frac{1-k}{2} = \max\{t, \frac{1-k}{2}\} > \max\{\lambda(x), \frac{1-k}{2}\} \geq \mu(x)$. Now, $\frac{1-k}{2} > \mu(x)$ and $\frac{1-k}{2} \geq t$ imply $\mu(x) + t < 1 - k$. Thus $x_t \overline{q}_k \mu$. If $t > \frac{1-k}{2}$, then $t = \max\{t, \frac{1-k}{2}\} > \max\{\lambda(x), \frac{1-k}{2}\} \geq \mu(x)$. Thus $x_t \overline{\in} \mu$. Therefore $x_t \overline{\in} \vee \overline{q}_k \mu$. Hence $\lambda \supseteq \vee \overline{q}_k \mu$.

Definition 3.1.2. Let μ be the fuzzy set of H. Then fuzzy h-closure of μ denoted by $\overline{\mu}$ is defined as: $\overline{\mu}(x)=\vee_{x+a+z=b+z}\min\{\mu(a),\mu(b)\}$. The fuzzy set μ is generalized fuzzy h-closed if $\mu \supseteq \vee \overline{q}_k \overline{\mu}$.

Theorem 3.1.1. Every $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right ideal μ is a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal if and only if $\mu \supseteq \vee \overline{q}_k \overline{\mu}$.

Proof: Let μ be a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right ideal of H. Let $\mu \supseteq \vee \overline{q}_k \overline{\mu}$. By Lemma 3.1.1, $\max\{\mu(x),\frac{1-k}{2}\}\geq \overline{\mu}(x)$. Now $x+a+z=b+z$ implies $\max\{\mu(x),\frac{1-k}{2}\}\geq \overline{\mu}(x)=\vee_{x+a+z=b+z}\min\{\mu(a),\mu(b)\}$. Therefore μ is a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal of H.

Conversely, $x+a+z=b+z$ implies $\max\{\mu(x),\frac{1-k}{2}\}\geq\min\{\mu(a),\mu(b)\}$. Thus $\max\{\mu(x),\frac{1-k}{2}\}\geq \vee_{x+a+z=b+z}\min\{\mu(a),\mu(b)\}=\overline{\mu}(x)$. By Lemma 3.1.1, $\mu \supseteq \vee \overline{q}_k \overline{\mu}$.

Theorem 3.1.2. Let μ be a fuzzy set of H. Then μ is a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right ideal of H if and only if μ_t is a fuzzy right [left] ideal of H for all $t\in(\frac{1-k}{2},1]$ whenever non-empty.

Proof: Let μ be an $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right ideal of H. Let $x,y\in\mu_t$ for $t\in(\frac{1-k}{2},1]$. By Theorem 3.1, $\max\{\mu(x+y),\frac{1-k}{2}\}\geq\min\{\mu(x),\mu(y)\}\geq t>\frac{1-k}{2}$. Thus $x+y\in\mu_t$. By Theorem3.1, $\max\{\mu(xz),\frac{1-k}{2}\}\geq\mu(x)\geq t$. Thus $xz\in\mu_t$ for all $x\in\mu_t$ and $z\in H$. Therefore μ_t is a fuzzy right ideal in H for all $t\in(\frac{1-k}{2},1]$ whenever non-empty.

Conversely, if there exists $x,y\in H$ such that $\max\{\mu(x+y),\frac{1-k}{2}\}<\min\{\mu(x),\mu(y)\}$ then choose $t\in(\frac{1-k}{2},1)$ such that $\max\{\mu(x+y),\frac{1-k}{2}\}<t<\min\{\mu(x),\mu(y)\}$. Thus $x,y\in\mu_t$ and $t\in(\frac{1-k}{2},1]$ but $x+y\notin\mu_t$, which is a contradiction. If there exists $x,z\in H$ such that $\max\{\mu(xz),\frac{1-k}{2}\}<\mu(x)$ then choose $s\in(\frac{1-k}{2},1)$ such that $\max\{\mu(xz),\frac{1-k}{2}\}<s<\mu(x)$. Thus $x\in\mu_s$ and $s\in(\frac{1-k}{2},1]$ but $xz\notin\mu_s$, which is a contradiction.

Theorem 3.1.3. Let μ be a fuzzy set of H. Then μ is a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal of H if and only if μ_t is a fuzzy right h-ideal of H for all $t\in(\frac{1-k}{2},1]$ whenever non-empty.

Proof: Let μ be a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal of H. By Theorem 3.1.2, μ_t is a fuzzy right ideal of H for $t\in(\frac{1-k}{2},1]$ whenever non-empty. Let $x+a+z=b+z$ and $a,b\in\mu_t$, $t\in(\frac{1-k}{2},1]$. Thus $\max\{\mu(x),\frac{1-k}{2}\}\geq\min\{\mu(a),\mu(b)\}\geq t>\frac{1-k}{2}$. Therefore $x\in\mu_t$. Hence μ_t is a fuzzy right h-ideal of H for all $t\in(\frac{1-k}{2},1]$ whenever non-empty.

Conversely, by Theorem 3.1.2, μ is a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right ideal of H. If there exists $a,b,x,z\in H$ such that $\max\{\mu(x),\frac{1-k}{2}\}<\min\{\mu(a),\mu(b)\}$, then choose $t\in(\frac{1-k}{2},1)$ such that $\max\{\mu(x),\frac{1-k}{2}\}<t<\min\{\mu(a),\mu(b)\}$. Thus $a,b\in\mu_t$ but $x\notin\mu_t$, which is a contradiction. Hence μ is a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal of H.

Theorem 3.1.4. Every fuzzy right h-ideal of H is a $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal of H.
Proof: The proof is straight forward.

Theorem 3.1.5. Every $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal of H is a $(\overline{\in},\overline{\in}\vee\overline{q}_r)$-fuzzy right h-ideal of H for $r<k$.
Proof: Now, $r<k$ implies $\frac{1-r}{2}>\frac{1-k}{2}$. Thus
$\max\{\mu(x+y),\frac{1-r}{2}\}\geq\max\{\mu(x+y),\frac{1-k}{2}\}\geq\min\{\mu(x),\mu(y)\}$ and
$\max\{\mu(xy),\frac{1-r}{2}\}\geq\max\{\mu(xy),\frac{1-k}{2}\}\geq\mu(x)$. Then $x+a+z=b+z$ implies
$\max\{\mu(x),\frac{1-r}{2}\}\geq\max\{\mu(x),\frac{1-k}{2}\}\geq\min\{\mu(a),\mu(b)\}$.

Corollary 3.1.1. Every $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal of H is a $(\overline{\in},\overline{\in}\vee\overline{q})$-fuzzy right h-ideal of H.
Proof: By taking $k=0$ in Theorem 4.8, we get the result.

Remark 3.1.1.
(1) Every $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal need not be afuzzy right h-ideal.
(2) Every $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal need not be a $(\in,\in\vee q_k)$-fuzzy right h-ideal.
(3) Every $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy right h-ideal need not be a $(\overline{\in},\overline{\in}\vee\overline{q}_r)$-fuzzy right h-ideal for $r>k$.

Example 3.1.1. Let $H=\{0,a,b,c\}$ be a hemiring with the Cayley table as follows:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	a	b	c
c	0	a	b	c

Now, we define a fuzzy set μ and level sets of μ as follows:

$$\mu(x)=\begin{cases}0.8 & \text{if } x=0\\ 0.6 & \text{if } x=a\\ 0.4 & \text{if } x=b\\ 0.2 & \text{if } x=c\end{cases}$$

By Theorem 4.6, μ is a $(\overline{\in},\overline{\in} \vee \overline{q}_k)$-fuzzy right h-ideal of H for $k = 0.2$. Now, $\mu(a+b) = \mu(c) = 0.2 \ngeq 0.4 = \mu(b) = \mu(a) \wedge \mu(b)$ implies μ is not fuzzy right h-ideal of H. Now, $\mu(b \cdot c) = \mu(c) = 0.2 \ngeq 0.4 = \mu(b) \wedge 0.4$. Then μ is not a $(\in,\in \vee q_k)$-fuzzy right h-ideal for $k = 0.2$.

By taking $r = 0.3$, $\mu(a+b) \vee \frac{1-r}{2} = \mu(a+b) \vee 0.35 = \mu(c) \vee 0.35 = 0.35 \ngeq 0.4 = \mu(b) = \mu(a) \wedge \mu(b)$. Thus μ is not $(\overline{\in},\overline{\in} \vee \overline{q}_r)$ -fuzzy right h-ideal for $r = 0.3$.

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