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Diagnosis of Viral Hepatitis Using New Distance Measure of Intuitionistic Fuzzy Sets

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Abstract. Viral hepatitis is a deadly disease especially in the Africa continent. Most people living with it do not know that they are infected. This places them at greater risk for severe, even fatal complications and increases the likelihood of spreading the viruses to others, and so testing or diagnosis for the disease is of great important to curb the rate of spread. In this paper, we proposed the numerical diagnosis of viral hepatitis using intuitionistic fuzzy sets (IFSs) via a new distance measure of intuitionistic fuzzy sets.

Keywords: Diagnosis, distance measure, fuzzy sets, intuitionistic fuzzy sets, viral hepatitis.

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

Hepatitis (hepatitides for plural) is a medical condition defined by the inflammation of the liver and characterized by the presence of inflammatory cells in the tissue of the organ. Hepatitis is considered globally as a deadly disease especially in Africa. This disease is largely regarded as viral infection although liver inflammation can also happen due to autoimmune, medications, drugs, toxin, and alcohol [2, 3, 7]. This infection is common in Africa due to poor sanitation culture and unprotected sex, careless blood transfusion among others.

Many methods of diagnosing viral hepatitis have been researched. Some of these methods are liver biopsy, physical examination, liver function tests, ultrasound, blood tests, and viral antibody testing[7, 8]. In this article, we propose numerical diagnosis of viral hepatitis types using intuitionistic fuzzy sets (IFSs) method via a new distance measure of IFSs.

The concept of intuitionistic fuzzy sets (IFSs) proposed by Atanassov [9-11] as the generalization of fuzzy sets in [17] attracted much attention due to its significant in tackling vagueness or the representation of imperfect knowledge in decision making. Many applications of IFSs in decision making have been proposed and researched such as in career determination, electoral system, pattern recognition, medical diagnosis, medical imaging etc. [12-16].

For the application analysis, we assume there is a database of hepatitis types and symptoms in IFSs values, and samples of suspect hepatitis patients are collected and analysed with respect to symptoms in IFSs values. We use the new distance measure to calculate the distance between each of the patients and each of the hepatitis types to find which type of viral hepatitis each patient is infected with before proceeding on treatment.

2. Meaning of intuitionistic fuzzy sets

Definition 1. Crisp set A of a universal set X is defined as the characteristic function of A and is denoted by $f_A(x)$, mathematically, $f_A(x): X \to \{0,1\}$

where, $f_A(x) = \begin{Bmatrix} 1, if\ x \in A \\ 0, if\ x \notin A \end{Bmatrix}$

Definition 2[17]. Fuzzy set A of a non-empty set X is defined by the membership function of the set $s.t.\ \mu_A(x): X \to [0,1]$,

where, $\mu_A(x) = \begin{Bmatrix} 1,\ if\ x\ is\ totally\ in\ A \\ 0, if\ x\ is\ not\ in\ A \\ (0,1), if\ x\ is\ partly\ in\ A \end{Bmatrix}$

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, where the grades 1 and 0 represents full membership and full non-membership.

Definition 3[9]. Let X be nonempty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form;

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x),\ \nu_A(x): X \longrightarrow [0, 1]$define the degree of membership and degree of non-membership of the element $x \in X$ to the set A. For every $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Furthermore, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the intuitionistic fuzzy set index or hesitation margin and is the degree of indeterminacy concerning the membership of x in A, then $0 \leq \mu_A(x) + \nu_A(x) + \pi_A(x) \leq 1$. Whenever $\pi_A(x) = 0$, IFS reduces automatically to fuzzy set.

3. Some distance measures of intuitionistic fuzzy sets

Definition 4[18]. Let X be nonempty such that IFSs $A, B, C \in X$. Then the distance measure d between IFSs A and B is a mapping $d: X \times X \to [O, 1]$; if $d(A, B)$ satisfies the following axioms:

$A1$ $0 \leq d(A, B) \leq 1$(boundedness),
$A2$ $d(A, B) = 0$ if and only if $A = B$,
$A3$ $d(A, B) = d(B, A)$ (symmetric),
$A4$ $d(A, C) + d(B, C) \geq d(A, B)$,
$A5$ if $A \subseteq B \subseteq C$, then $d(A, C) \geq d(A, B)$ and $d(A, C) \geq d(B, C)$.

Distance measure is a term that describes the difference between intuitionistic fuzzy sets and can be considered as a dual concept of similarity measure. We make use of the four distance measures proposed in [18-20]between intuitionistic fuzzy sets, which were partly based on the geometric interpretation of intuitionistic fuzzy sets, and have some good geometric properties.

Let $A = \{\langle x, \mu_A(x_i), \nu_A(x_i), \pi_A(x_i)\rangle : x \epsilon X\}$and $B = \{\langle x, \mu_B(x_i), \nu_B(x_i), \pi_B(x_i)\rangle : x \epsilon X\}$ be two IFSs in $X = \{x_1, x_2, \dots, x_n\}$, for $i = 1, 2, \ .., n$. Based on the geometric

interpretation of IFSs, Szmidt and Kacprzyk [18, 19] proposed the following four distance measures between A and B:
The Hamming distance:

$$d_H(A,B) = \frac{1}{2}\sum_{i=1}^{n}\left(\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right| + \left|\pi_A(x_i) - \pi_B(x_i)\right| \right)$$

The Euclidean distance:

$$d_E(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}$$

The normalized Hamming distance:

$$d_{n-H}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right| + \left|\pi_A(x_i) - \pi_B(x_i)\right| \right)$$

The normalized Euclidean distance:

$$d_{n-E}(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}$$

The new distance measure is given as thus:

$$d(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\Big[\left|\mu_A(x_i) - \mu_B(x_i)\right| + \Big| \left|\mu_A(x_i) - \nu_A(x_i)\right| - \left|\mu_B(x_i) - \nu_B(x_i)\right| \Big| + \Big| \left|\mu_A(x_i) - \pi_A(x_i)\right| - \left|\mu_B(x_i) - \pi_B(x_i)\right| \Big| \Big]$$

Example 3.1. Let$A = \{\langle 0.6,0.2,0.2\rangle, \langle 0.5,0.3,0.2\rangle\}$ and $B = \{\langle 0.5,0.4,0.1\rangle, \langle 0.4,0.1,0.5\rangle\}$ be IFSs in X such that $X = \{x_1, x_2\}$. We use the above distance measures to calculate the distance between A and B.
$d_H(A,B) = 0.25$, $d_E(A,B) = 0.2189$, $d_{n-H}(A,B) = 0.125$, $d_{n-E}(A,B) = 0.1548$ and$d(A,B) = 0.1$ i.e. the new distance measure of A and B. From these results, we observe that the new distance measure is more accurate because it produces the shortest distance measure.

4. Application of intuitionistic fuzzy sets in diagnosis of viral hepatitis

Hepatitis is simply the disease that causes inflammation of the liver and hence affects the liver's functions. The liver is the largest and most important organs in the body. It is about the size of a football and is located on the upper right side of the abdomen. The liver performs important functions such as: (i) filtering of blood (ii) building of proteins and other important chemicals important for digestion and healing (iii) storing of vitamins, sugars, fats, and other nutrients (iv) transforming nutrients into materials the body uses etc. [3, 4].Hepatitis disease is commonly regarded as viral infection caused byone of the five hepatitis viruses, referred to as types A, B, C, D and E. While all of these viruses cause liver disease, they vary significantly in terms of epidemiology, natural history, prevention, diagnosis and treatment.

Viral hepatitis could be acute or chronic. Many people who are infected with chronic viral hepatitis often have no symptoms. In fact, people can be infected with chronic hepatitis and not feel sick or show symptoms for 20 to 30 years. When and if symptoms do appear, they are similar to acute infection, but can be a sign of serious liver damage.

Types of viral hepatitis [4]

Hepatitis A virus (HAV) is usually transmitted by the faecal-oral route, either through person-to-person contact or ingestion of contaminated food or water. Certain sex practices can also spread HAV. Infections are in many cases mild, with most people making a full recovery and remaining immune from further HAV infections. However, HAV infections can also be severe and life threatening. Most people in areas of the world with poor sanitation have been infected with this virus. Safe and effective vaccines are available to prevent HAV infection.

Hepatitis B virus (HBV) is transmitted through exposure to infectious blood, semen, and other body fluids. HBV can be transmitted from infected mothers to infants at the time of birth, or from family members to infants in early childhood. Transmission may also occur through unsafe sexual intercourse, transfusions of HBV-infected blood and blood products, contaminated injections during medical procedures, and sharing of needles and syringes among injecting drug users. HBV also poses a risk to healthcare workers who sustain accidental needle-stick injuries while caring for HBV-infected people. A safe and effective vaccine is available to prevent HBV infection. HBV is the major cause of chronic hepatitis, cirrhosis, and hepatocellular carcinoma. The global epidemiological scenario of HBV infection has been changing rapidly over the last two decades due to an effective immunization programme initiated by the World Health Organization [1].

Hepatitis C virus (HCV) is mostly transmitted through exposure to infectious blood. This may happen through transfusions of HCV-infected blood and blood products, contaminated injections during medical procedures, and sharing of needles and syringes among injecting drug users. Sexual or interfamilial transmission is also possible, but is much less common. There is no vaccine against HCV. Both HBV and HCV can cause cancer to humans.

Antiviral agents against HBV and HCV exist. Treatment of HBV infection has been shown to reduce the risk of developing liver cancer and death.HCV is generally considered to be a curable disease but for many people this is not the reality. Access to treatment remains a constraint in many parts of the world.

Hepatitis D virus (HDV) infections occur exclusively in persons infected with HBV. The dual infection of HDV and HBV can result in more serious disease and worse outcomes. The hepatitis B vaccine provides protection from HDV infection.

Hepatitis E virus (HEV), like HAV, is transmitted through consumption of contaminated water or food. HEV is a common cause of hepatitis outbreaks in the developing world and is increasingly recognized as an important cause of disease in developed countries. HEV infection is associated with increased morbidity and mortality in pregnant women and new-borns. A safe and effective vaccine against HEV was licenced in January 2012 but is not yet widely available.

Common symptoms of viral hepatitis

Chronic viral hepatitis (HBV and HCV) last more than six months and may not have symptoms in the beginning i.e. too subtle to notice, but the acute types that last for six months only develop symptoms quickly and hence easily to tackle. Symptoms of all types of viral hepatitis can include one or more of the following: fever, fatigue, loss of appetite, nausea, vomiting, abdominal pain, joint pain, jaundice [5, 6].

Many people with viral hepatitis recover with a lifelong immunity to the disease, but some people with viral hepatitis die in the acute phase. Hepatitis Band C may progress to chronic hepatitis, in which the liver remains inflamed for more than six months. This can leads to cirrhosis, liver cancer, liver failure, and sometimes death. Hepatitis can be medically diagnosis by physical examination, liver biopsy, liver function tests, ultrasound, blood tests (HBV and HCV), and viral antibody testing [8].

5. Numerical experiment using the new distance measure of IFSs

Let $P = \{p_1, p_2, p_3, p_4, p_5\}$ be the set of patients to be diagnosed for various types of viral hepatitis, let $S =${fever, fatigue, loss of appetite, nausea, vomiting, abdominal pain, joint pain, jaundice} be the set of symptoms of viral hepatitis, and let $H = \{$A, B, C, D, E$\}$ be the set of types of viral hepatitis. From the common symptoms of viral hepatitis, we assumed the symptoms of each of the viral hepatitis in IFS values as shown below.

	fatigue	fever	Vomiting	joint pain	abdominal pain	loss of appetite	nausea	jaundice
A	(0.6,0.3)	(0.5,0.4)	(0.7,0.2)	(0.6,0.3)	(0.5,0.3)	(0.5,0.4)	(0.6,0.2)	(0.7,0.3)
B	(0.7,0.2)	(0.8,0.1)	(0.7,0.1)	(0.6,0.3)	(0.8,0.1)	(0.8,0.0)	(0.7,0.1)	(0.9,0.0)
C	(0.8,0.1)	(0.7,0.2)	(0.8,0.1)	(0.7,0.1)	(0.8,0.1)	(0.8,0.1)	(0.6,0.2)	(0.8,0.2)
D	(0.6,0.3)	(0.5,0.3)	(0.6,0.2)	(0.4,0.4)	(0.6,0.1)	(0.5,0.4)	(0.5,0.3)	(0.6,0.3)
E	(0.4,0.5)	(0.5,0.2)	(0.5,0.3)	(0.6,0.4)	(0.6,0.3)	(0.5,0.3)	(0.6,0.3)	(0.6,0.1)

Table 1: Hepatitis types and symptoms in IFS values

The first entry is membership function μ , the second entry is non-membership function $\nu$and the third parameter called hesitation margin π is gotten by$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.We assumed hypothetically that, after samples are collected from the patients and analysed, we get the results below.

	fatigue	fever	vomiting	joint pain	abdominal pain	loss of appetite	nausea	jaundice
p_1	(0.5,0.4)	(0.8,0.2)	(0.4,0.3)	(0.4,0.5)	(0.6,0.2)	(0.7,0.1)	(0.8,0.1)	(0.7,0.2)
p_2	(0.8,0.2)	(0.6,0.3)	(0.6,0.1)	(0.3,0.5)	(0.8,0.1)	(0.8,0.1)	(0.9,0.0)	(0.8,0.1)
p_3	(0.6,0.3)	(0.7,0.3)	(0.7,0.2)	(0.2,0.6)	(0.9,0.1)	(0.8,0.2)	(0.9,0.1)	(0.7,0.2)
p_4	(0.7,0.1)	(0.8,0.1)	(0.9,0.0)	(0.5,0.4)	(0.5,0.3)	(0.8,0.1)	(0.9,0.0)	(0.7,0.1)
p_5	(0.8,0.1)	(0.5,0.4)	(0.8,0.1)	(0.8,0.2)	(0.6,0.3)	(0.5,0.3)	(0.6,0.3)	(0.6,0.2)

Table 2: Patients and symptoms in IFS values

Using the new distance measure to calculate the distance between the hepatitis types and the patients with respect to the symptoms, we get the result.

	A	B	C	D	E
p_1	0.0438	0.0406	0.0516	0.0406	0.0359
p_2	0.0219	0.0297	0.0313	0.0484	0.0547
p_3	0.0469	0.0344	0.0375	0.0484	0.0578
p_4	0.0438	0.0344	0.0375	0.0531	0.0578
p_5	0.0250	0.0500	0.0320	0.0359	0.0313

Table 3: Mathematical computation of the diagnosis

For the diagnosis, if the distance of a patient is the closest to any of the hepatitis types, the patient is infected with the hepatitis type. The numerical diagnosis of hepatitis A, B, C, D, and E of the patientsis thus; p_1 is infected with hepatitis E, p_2 is infected with hepatitis A, p_3 is infected with hepatitis B, p_4 is infected with hepatitis B, and p_5 is infected with hepatitis A. We observe that none of the patient is infected with hepatitis C and D.

6. Conclusion

We conclude that the concept of intuitionistic fuzzy sets is very applicable in decision making. This novel application of intuitionistic fuzzy sets in the diagnosis of viral hepatitis is of great significance because it provides accurate and reliable diagnosis. In the proposed application, we used a new distance measure to calculate the distance of each patient from each hepatitis type in respect to the symptoms, and obtained results.

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