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Structural Core Graph of Double Layered Fuzzy Graph

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Abstract. The Double Layered Fuzzy Graph (DLFG) gives the 3–D structure to fuzzy graph. In this paper, we constructed the structural core graph for the given DLFG using a new algorithm and also the structural core graph for the union of two DLFG is also constructed using the same algorithm. Some of its diagrammatic properties are studied.

Keywords: Fuzzy graph, double layered fuzzy graph, face value, structural core graph.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

Fuzzy graph theory was introduced by Rosenfeld in 1975 [5]. Though introduced recently, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness with fuzzy graphs [8]. Mordeson and Peng introduced the concept of operations on fuzzy graphs, Sunitha and Vijayakumar discussed about the operations of union, join, Cartesian product and composition on two fuzzy graphs [14]. The degree of a vertex in some fuzzy graphs was discussed by Nagoorgani and Radha [7]. Nagoorgani and Malarvizhi have defined different types of fuzzy graphs and discussed its relationships with isomerism in fuzzy graphs [3]. The double layered fuzzy graph was introduced by Pathinathan and Jesintha Rosline, they have examined some of the properties of DLFG [4]. The matrix representation of DLFG and its properties was given by the same authors [5]. The vertex degree of cartesian product of intuitionistic fuzzy graph is given by Pathinathan and Jesintha Rosline [13].

In this paper, the core graph for the DLFG is constructed using the new algorithm. First we will see some of the basic concepts in fuzzy graph.

2. Preliminaries

Definition 2.1. A fuzzy graph G is a pair of functions G:(σ,μ) where σ is a fuzzy subset of a non empty set S and μ is a symmetric fuzzy relation on σ . The underlying crisp graph of G:(σ,μ) is denoted by $G^* : (\sigma^*, \mu^*)$

Definition 2.2. Let $G:(\sigma,\mu)$ be a fuzzy graph, the order of G is defined as $O(G)=\sum_{u\in V}\sigma(u)$

Definition 2.3. Let $G:(\sigma,\mu)$ be a fuzzy graph with the underlying crisp graph $G^*:(\sigma^*,\mu^*)$. The pair $DL(G):(\sigma_{DL},\mu_{DL})$ is defined as follows. The node set of $DL(G)$ be $\sigma^*\cup\mu^*$. The fuzzy subset σ_{DL} is defined as $\sigma_{DL}=\begin{cases}\sigma(u) \text{ if } u\in\sigma^* \\ \mu(uv) \text{ if } uv\in\mu^*\end{cases}$

The fuzzy relation μ_{DL} on $\sigma^*\cup\mu^*$ is defined as

$$\mu_{DL}=\begin{cases}\mu(uv) \text{ if } u,v\in\sigma^* \\ \mu(e_i)\wedge\mu(e_j) \text{ if the edge } e_i \text{ and } e_j \textit{ have} \text{ a node in common between them} \\ \sigma(u_i)\wedge\mu(e_i) \text{ if } u_i\in\sigma^* \text{ and } e_i\in\mu^* \text{ and each } e_i \text{ is incident with single } u_i \\ \qquad\qquad \text{either clockwise or anticlockwise.} \\ 0 \text{ otherwise}\end{cases}$$

By definition $\mu_{DL}(u,v)\le\sigma_{DL}(u)\wedge\sigma_{DL}(v)$ for all u,v in $\sigma^*\cup\mu^*$. Here μ_{DL} is a fuzzy relation on the fuzzy subset σ_{DL}. Hence the pair $DL(G):(\sigma_{DL},\mu_{DL})$ is defined as **Double Layered Fuzzy Graph (DLFG)**.

Definition 2.4. The union of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G=G_1\cup G_2:(\sigma_1\cup\sigma_2,\mu_1\cup\mu_2)$ on $G^*:(\mathrm{V},\mathrm{E})$ where $V=\mathrm{V}_1\cup\mathrm{V}_2$ and $\mathrm{E}=\mathrm{E}_1\cup E_2$ with

$$(\sigma_1\cup\sigma_2)(u)=\begin{cases}\sigma_1(u) & \text{if } u\in \mathrm{V}_1-\mathrm{V}_2 \\ \sigma_2(u) & \text{if } u\in \mathrm{V}_2-\mathrm{V}_1 \\ \sigma_1(u)\vee\sigma_2(\mathrm{u}) & \text{if } u\in \mathrm{V}_1\cap\mathrm{V}_2\end{cases}$$

and

$$(\mu_1\cup\mu_2)(uv)=\begin{cases}\mu_1(uv) & \text{if } uv\in \mathrm{E}_1-\mathrm{E}_2 \\ \mu_2(uv) & \text{if } uv\in \mathrm{E}_2-\mathrm{E}_1 \\ \mu_1(uv)\vee\mu_2(uv) & \text{if } uv\in \mathrm{E}_1\cap\mathrm{E}_2\end{cases}$$

3. Structural core graph

ALGORITHM

1. Construct a DLFG with 2n vertices and 3n edges where n is the number of vertex in the base graph whose crisp graph is a cycle.

2. Calculate all possible face values using the formulae $\min\left\{\frac{\mu(a,b)}{\sigma(a)\wedge\sigma(b)}\right\}$ where $\mu(a,b)$ is the weight of the edge (a,b) and $\sigma(a)\,\&\,\sigma(b)$ are membership value of vertices a and b.
3. Select a face with least value. If two or more faces are there with same value, choose a face with least order value.
4. Choose a vertex with least value in the selected face.
5. Select the smallest fuzzy distance edge from the selected vertex and include that in T. If two or more edge are there with the same value choose an edge with least adjacent vertex value.
6. If 2 or more vertex are there with same value then choose the edge with least intersecting face value.
7. Repeat steps from 5, stop when T becomes a spanning tree which covers all the vertices.

Example 1.

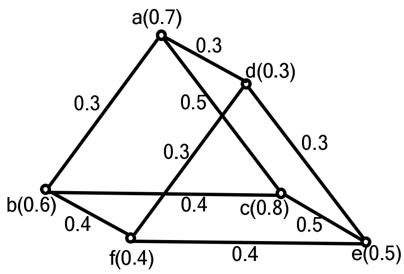

Figure 1: DLFG of n=3 vertices

Face value calculation:

$F_1(abc) \to \min\{0.5, 0.67, 0.71\} = 0.5$

$F_2(acdf) \to \min\{1,1,1, 0.71\} = 0.71$

$F_3(def) \to \min\{1,1,1\} = 1$

$F_4(abed) \to \min\{0.5,1,1,1\} = 0.5$

$F_5(bcfe) \to \min\{0.67,1,1,1\} = 0.67$

Reached node	Edge	Membership value	Iteration
d	Df	0.3	1
df	fe	0.4	2
dfe	ed(introduces cycle)	-	NO
dfe	ec	0.5	3
dfec	cb	0.4	4
dfecb	ba	0.3	5→ n-1

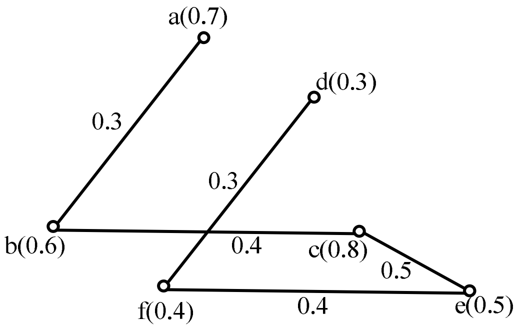

Figure 2: Structural Core graph of DLFG of n=3 vertices

Example 2.

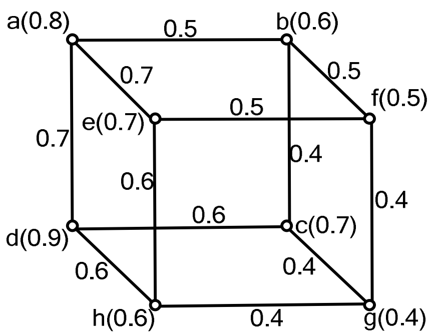

Figure 3: DLFG of n=4 vertices

Face value calculation:

$$F_1(abcd) \rightarrow \min\{0.83, 0.67, 0.86, 0.875\} = 0.67$$
$$F_2(bcgf) \rightarrow \min\{0.67,1,1,1\} = 0.67$$
$$F_3(efgh) \rightarrow \min\{1,1,1,1\} = 1$$
$$F_4(adeh) \rightarrow \min\{0.875,1,1,1\} = 0.875$$
$$F_5(abfe) \rightarrow \min\{0.83,1,1,1\} = 0.83$$
$$F_6(abfe) \rightarrow \min\{0.86,1,1,1\} = 0.86$$

Reached node	edge	Membership value	Iteration
g	gf	0.4	1
gf	fb	0.5	2
gfb	bc	0.4	3
gfbc	cg(introduces cycle)	-	NO
gfbc	cd	0.6	4

gfbcd	dh	0.6	5
gfbcdh	hg(introduces cycle)	-	NO
gfbcdh	he	0.6	6
gfbcdhe	ef(introduces cycle)	-	NO
gfbcdhe	ea	0.7	7→ n-1

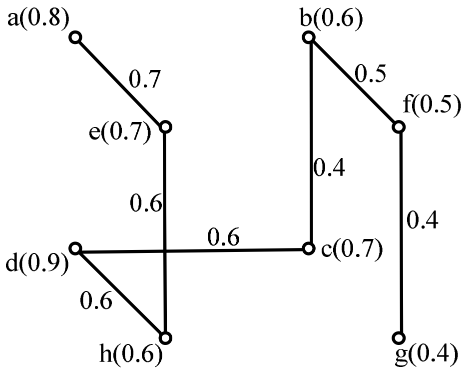

Figure 4: Structural Core graph of DLFG of n=4 vertices

Example 3.

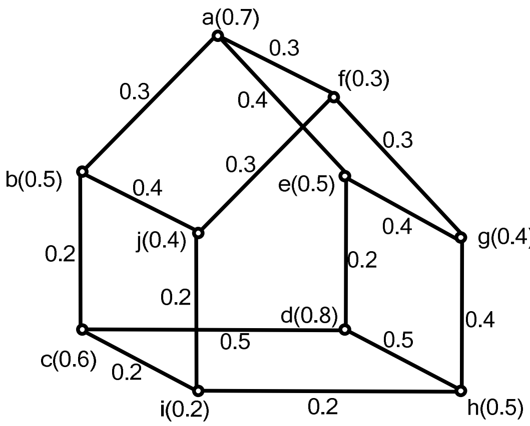

Figure 5: DLFG of n=4 vertices

Face value calculation:

$$F_1(abcde) \to \min\{0.6, 0.8, 0.83, 0.4, 0.8\} = 0.4$$

$$F_2(\deg h) \to \min\{0.4,1,1,1\} = 0.4$$

$$F_3(efghij) \to \min\{1,1,1,1,1\} = 1$$

$$F_4(bcij) \to \min\{0.8,1,1,1\} = 0.8$$

$$F_5(abfj) \to \min\{0.6,1,1,1\} = 0.6$$

$F_6(aegf) \to \min\{0.8,1,1,1\} = 0.8$

$F_7(cdhi) \to \min\{0.83,1,1,1\} = 0.83$

Reached node	Edge	Membership value	Iteration
g	gf	0.3	1
gf	fj	0.3	2
gfj	ji	0.2	3
gfji	ih	0.2	4
gfjih	hg(introduces cycle)	-	NO
gfjih	hd	0.5	5
gfjihd	de	0.2	6
gfjihde	eg(introduces cycle)	-	NO
gfjihde	ea	0.4	7
gfjihdea	af(introduces cycle)	-	NO
gfjihdea	ab	0.3	8
gfjihdeab	bc	0.2	9→ n-1

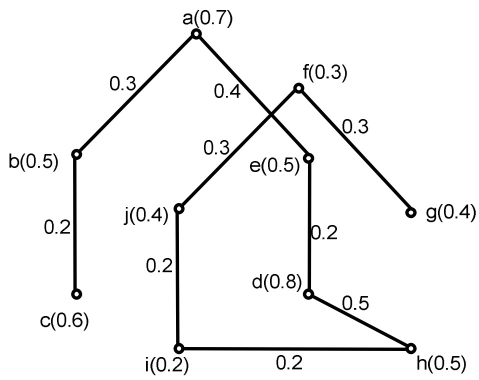

Figure 6: Structural Core graph of DLFG of n=3 vertices

For different values of n we will get different DLFG and when we apply the algorithm we will get different structures for each graph.

4. Theoretical concepts

Consider the DLFG in example 1 with different labeling

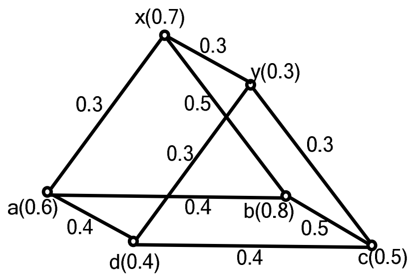

Figure 7: $\mathrm{DL}(G_1)$

Consider the DLFG in example 2 with different labeling

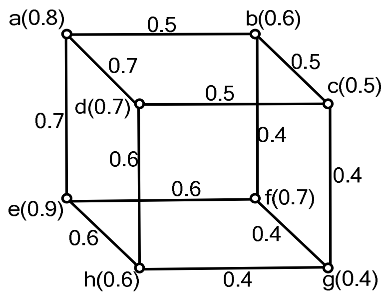

Figure 8: $\mathrm{DL}(G_2)$

Then the union of the above two graphs is given by

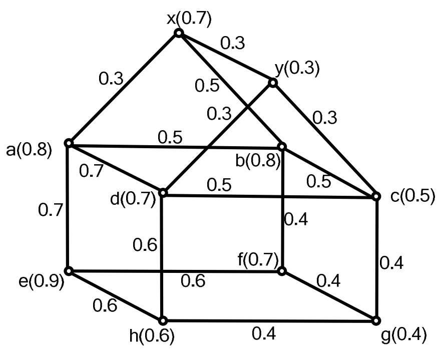

Figure 9: $\mathrm{DL}(G_1) \cup \mathrm{DL}(G_2)$

Then the core graph is given by

Face value calculation:

$F_1(abfe) \to \min\{0.625, 0.571, 0.857, 0.875\} = 0.571$

$F_2(bcg\,f) \to \min\{0.571,1,1,1\} = 0.571$

$F_3(cdgh) \to \min\{1,1,1,1\} = 1$

$F_4(adeh) \to \min\{0.875,1,1,1\} = 0.875$

$F_5(xab) \to \min\{0.714, 0.625, 0.429\} = 0.429$

$F_6(xybc) \to \min\{0.714,1,1,1\} = 0.714$

$F_7(ycd) \to \min\{1,1,1\} = 1$

$F_8(xyad) \to \min\{0.429,1,1,1\} = 0.429$

$F_9(abcd) \to \min\{0.625,1,1,1\} = 0.625$

$F_{10}(efgh) \to \min\{0.8571,1,1,1\} = 0.8571$

Reached node	Edge	Membership value	Iteration
x	xy	0.3	1
xy	yc	0.3	2
xyc	cg	0.4	3
xycg	gh	0.4	4
xycgh	hd	0.6	5
xycghd	dy(introduces cycle)	-	NO
xycghd	dc(introduces cycle)	-	NO
xycghd	da	0.7	6
xycghda	ax(introduces cycle)	-	NO
xycghda	ab	0.8	7
xycghdab	bf	0.4	8
xycghdabf	fg(introduces cycle)	-	NO
xycghdabf	fe	0.6	9→ n-1

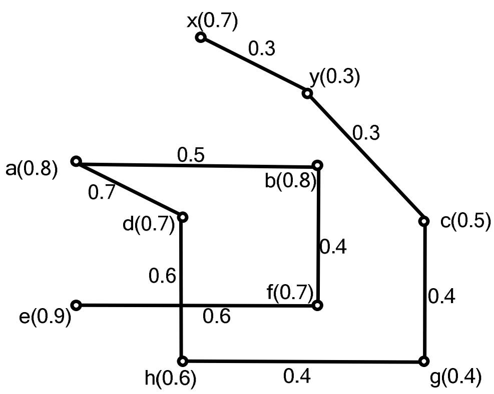

Figure 10: Structural Core graph of $\mathrm{DL}(G_1) \cup \mathrm{DL}(G_2)$

5. Conclusion

The structural core graph for both the DLFG and the union of two DLFG is constructed in this paper. This graph can be used in different networks to minimize the time for any particular problem whose graphical representation is a double layered. Further work can be done to apply this concept of Structural core graph in real life situations.

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