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Intuitionistic L-Fuzzy Strong β-Filters on β-Algebras

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Abstract. In this paper, we define the notion of an intuitionistic L- fuzzy strong β-filter on β-algebras and investigate some of their properties and results.

Keywords: β- filter, Intuitionistic L- Fuzzy β- filter, Intuitionistic L- Fuzzy Strong β-Filter

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1. Introduction

In 2002, Neggers and Kim [3], introduced a new notion of algebra: namely β- algebra. The theory of fuzzy sets proposed by Zadeh [6] in 1965 is generalized in 1986 by Atanassov [1] an Intuitionistic fuzzy sets. Then many researchers have been engaged in extending the concepts and results of abstract algebra. The notion of filters was introduced by Cartan in 1937. In 1991, Hoo [2] introduced the concept of the filters in BCI-algebras. Also in our earlier papers [4,5] we introduced the notions of fuzzy β-filter and Intuitionistic Fuzzy β-filter in β-algebras. In this paper, we discuss the concept of Intuitionistic L- fuzzy β-filters in β-algebras and prove some of their properties and theorems.

2. Preliminaries

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1. A β-algebra is a non-empty set X with a constant 0 and two binary operations + and - satisfying the following axioms:
(1) $x - 0 = x$
(2) $(0 - x) + x = 0$
(3) $(x - y) - z = x - (z + y)$ for all $x, y, z \in X$.

Definition 2.2. Let X and Y be two β-algebras. A mapping f : X → Y is said to be a β-homomorphism, if f(x+y) = f(x)+f(y) and f(x-y) = f(x)-f(y) for all x, y ϵ X.

Definition 2.4. Let X be a β-algebra and A be β-subalgebra. Then A is said to be a β-filter on X, if $x \Delta y = x + (x + y)$ and $x \nabla y = x - (x - y) \in A$ for all x, y ∈ A.

Definition 2.5. Let X be a β-algebra and A be fuzzy β-subalgebra. A is said to be fuzzy β- filter on X, if it satisfies the following conditions. For all x, y ∈ A,
1) $\mu_A(x \Delta y) \geq \min\{\mu_A(x),\ \mu_A(x+y)\}$ 2) $\mu_A(x \nabla y) \geq \min\{\mu_A(x),\ \mu_A(x-y)\}$
3) $\mu_A(y) \geq \mu_A(x)$, if $x \leq y$.

Definition 2.6. Let X be a β-algebra and A be an Intuitionistic L-fuzzy β-subalgebra. A is said to be an Intuitionistic L-fuzzy β- filter on X, if it satisfies the following conditions.
For all x, y ∈ A,
1) $\mu_A(x \Delta y) \geq \{\mu_A(x) \wedge \mu_A(x+y)\}$ and $\mu_A(x \nabla y) \geq \{\mu_A(x) \wedge \mu_A(x-y)\}$
2) $\vartheta_A(x \Delta y) \leq \{\vartheta_A(x) \vee \vartheta_A(x+y)\}$ and $\vartheta_A(x \nabla y) \leq \{\vartheta_A(x) \vee \vartheta_A(x-y)\}$
3) $\mu_A(y) \geq \mu_A(x)$ and $\vartheta_A(y) \leq \vartheta_A(x)$, if $x \leq y$.

3. Intuitionistic L- Fuzzy Strong β-Filter

In this section, we introduce the notion of Intuitionistic L- fuzzy strong β-filter on a β-algebra. We begin with the definition and examples.

Definition 3.1. Let X be a β-algebra and A be an Intuitionistic L-fuzzy β-subalgebra. Then A is said to be an Intuitionistic L-fuzzy strong β- filter on X, if it satisfies the following conditions.
For all x, y ∈ A,
1) $\mu_A(x \Delta y) = \mu_A(x \nabla y)$
2) $\vartheta_A(x \Delta y) = \vartheta_A(x \nabla y)$
3) $\mu_A(y) \geq \mu_A(x)$ and $\vartheta_A(y) \leq \vartheta_A(x)$, if $x \leq y$.

Example 3.2. Let X= {0,1,2,3} be a β-algebra and let $t_0, t_1 \in L$ where $t_0 \geq t_1$.

+	0	1	2	3
0	0	0	0	0
1	1	2	1	1
2	0	3	2	2
3	3	1	3	3

-	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

Now, A = {2,3} is β-filter on X.
A is an Intuitionistic L-fuzzy β-subalgebra, defined by the membership function and non membership function, $\mu_A(x) = \begin{cases} 0, if\ x = 2 \\ t_0, if\ x = 3 \end{cases}$ and $\vartheta_A(x) = \begin{cases} 1.0, if\ x = 2 \\ t_1\ , if\ x = 3 \end{cases}$
Then we can observe that, A is an Intuitionistic L-fuzzy strong β-filter on X.

Lemma 3.3. If A and B be any two Intuitionistic L-fuzzy strong β-filters on X, then A ∩ B is also an Intuitionistic L- fuzzy strong β-filter of X.

Theorem 3.4. Every Intuitionistic L- fuzzy strong β-filter is also an Intuitionistic L-fuzzy β-subalgebra.
Proof directly follows from our definition of Intuitionistic L- fuzzy strong β- filter.

The following example shows that the converse part of the above theorem need not be true.

Example 3.5. Let X= {0,1,2,3} be a β-algebra and let $t_0, t_1 \in$ L where $t_0 \geq t_1$.

+	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	1	1	2	3
3	0	1	3	3

-	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

Now, A = {2,3} is β-filter on X. Let $t_0, t_1 \in$ L where $t_0 \geq t_1$.
A is fuzzy β-subalgebra, defined by the membership function and non membership function, $\mu_A(x) = \begin{cases} t_0, if\ x = 2 \\ 1, if\ x = 3 \end{cases}$ and $\vartheta_A(x) = \begin{cases} t_1\ , if\ x = 2 \\ 0, if\ x = 3 \end{cases}$
Then we can observe that, A is not an Intuitionistic L- fuzzy strong β-filter on X.
Since $\mu_A(2\ \Delta\ 3) \neq \mu_A(2\ \nabla\ 3)$

Theorem 3.6. If A is an Intuitionistic L-fuzzy strong β- filter of X, then $\mu_A(x\ \Delta\ y) \geq \mu_A(x)$ and $\vartheta_A(x\ \nabla\ y) \leq \vartheta_A(y)$, where x ≤ y.
Proof: Assume that μ is an Intuitionistic L-fuzzy strong β-filter of X.
Let x, y ∈ X. Then we get,

$$\begin{aligned}\mu_A(x\ \Delta\ y) &= \mu_A(x+(x+y)) \\ &\geq \{\mu_A(x) \wedge \mu_A(x+y)\} \\ &\geq \{\mu_A(x) \wedge \{\mu_A(x) \wedge \mu_A(y)\}\} \\ &= \{\mu_A(x) \wedge \mu_A(x)\} \\ &= \mu_A(x)\end{aligned}$$

Similarly, we can prove that, $\vartheta_A(x\ \nabla\ y) \leq \vartheta_A(y)$, where x ≤ y.

Definition 3.7. Let μ be an Intuitionistic L- fuzzy strong β-filter in a β-subalgebra X. For s ∈ [0,1], the set $\mu_s = \{x \in X/\mu(x) \geq s\}$ is called a level set of filter μ in X.

Theorem 3.8. An intuitionistic L-fuzzy subset A of β-algebra X is an Intuitionistic L-fuzzy strong β-filter iff for any t ∈ [0,1] the t - level subset $A_t = \{x \in X |\ A(x) \geq t\}$ is either a β-filter or $A_t \neq \emptyset$.

Proof: Assume that the level subset of A in X, $A_t \neq \emptyset$.
Then x, y ∈ A_t, A(x) ≥ t, A(y) ≥ t
Now, μ_A(x Δ y) = μ_A(x+$(x + y)$

$\geq \{ \mu_A(x) \wedge \mu_A(x+y)\}$

$\geq \{\mu_A(x) \wedge \mu_A(x)\}$

$= t$

which implies x Δ y ∈ $\mu_{A_{t.}}$
Also we can prove that, the non membership function, x Δ y ∈ $\vartheta_{A_{t.}}$
Hence A_t is a β-filter of X .
Conversely, assume that A_t is a β-filter of X .
For all x, y ∈ X, x Δ y and x ∇ y ∈ A_t
⇒ μ_A (x Δ y) ≥ t and μ_A (x ∇ y) ≥ t.
μ_A (x Δ y) = μ_A (x+(x+y)) ≥ t

= {μ_A (x) ∧ μ_A (x+y)}.

Also we can prove that, the non membership function, ϑ_A (x Δ y) ≤ t
Thus proving that A is an Intuitionistic L- fuzzy strong β-filter.

Theorem 3.9. Let f be an onto β-algebra homomorphism from X to Y. If B is an intuitionistic L- fuzzy strong β-filter of Y, then f^{-1}(B) is also an Intuitionistic L-fuzzy strong β-filter on X.
Proof: Let B be an Intuitionistic L- fuzzy strong β-filter of Y.
For x, y ∈ X, then

$f^{-1}(\mu_B$ (x Δ y)) = $f^{-1}(\mu_B$ (x + $(x +$ y)))

$= \mu_B(f(x + (x + y))$

$= \mu_B(f(x) + f(x + y))$

$\geq \{\mu_B(f(x)) \wedge \mu_B(f(x + y))\}$

$= \{f^{-1}(\mu_B$ (x) $\wedge f^{-1}(\mu_B$ $(x +$ y))}

Let x, y ∈ X, such that x ≥ y. Since B is an Intuitionistic L- fuzzy strong β-filter, we have $\mu_B(f(y)) \geq \mu_B(f(x)) = f^{-1}(\mu_B$ (x)) such that $f^{-1}(\mu_B$ (y)) $\geq f^{-1}(\mu_B$ (y)).
Also we can prove that, the non membership function
Thus we can conclude that f^{-1}(B) is an intuitionistic L- fuzzy strong β-filter on X.

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