Intern. J. Fuzzy Mathematical Archive
Vol. 9, No. 1, 2015, 1-9
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 8 October 2015
www.researchmathsci.org

International Journal of
Fuzzy Mathematical Archive

Properties on Total and Middle Intuitionistic Fuzzy Graph

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Received 1 September 2015; accepted 21 September 2015

Abstract. In this paper, some properties of total intuitionistic fuzzy graph and middle intuitionistic fuzzy graph are discussed.

Keywords: Total IFG, Middle IFG.

AMS Mathematics Subject Classification (2010): 03E72, 03F55

1. Introduction

In 1965, Zadeh [9] introduced the notion of a fuzzy subset of a set as a method for representing uncertainity. In 1975, Rosenfeld [8] introduced the concept of fuzzy graphs. Atanassov [1] introduced the concept of intuitionistic fuzzy relation and intuitionistic fuzzy graph. Karunambigai and Parvathi [3] introduced intuitionistic fuzzy graph as a special case of Atanassov's intuitionistic fuzzy graph. NagoorGani and Shajitha Begum [7] introduced busy Nodes and free Nodes in intuitionistic fuzzy graph. NagoorGani and Shajitha Begum [5] introduced the concept of degree, order and size in intuitionistic fuzzy graph.

A study on total and middle intuitionistic fuzzy graph was introduced by NagoorGani and Rahman [6] .We study some properties of total and middle intuitionistic fuzzy graph and relations between them are discussed.

2. Preliminaries

Definition 2.1. An Intuitionistic fuzzy graph is of the form G = < V, E > wherei) V={v_1,v_2,….,v_n} such that μ_1: V –> [0,1] and γ_1: V –> [0,1] denote the degree of membership and non-membership of the element $v_i \in V$ respectively and
$0 \leq \mu_1 (v_i) +\gamma_1(v_i) \leq 1$, for every $v_i \in V$ (i = 1,2, ……. n).
ii) $E \subseteq V \times V$ where μ_2: V × V –> [0,1] and γ_2: V × V –> [0,1] such that
$\mu_2 (v_i ,v_j) \leq \min [\mu_1 (v_i), \mu_1 (v_j)]$, $\gamma_2 (v_i ,v_j) \leq \max [\gamma_1 (v_i), \gamma_1 (v_j)]$ and$0 \leq \mu_2 (v_i, v_j) + \gamma_2 (v_i,v_j) \leq 1$ for every $(v_i,v_j) \in E$ (i,j = 1,2, …n)

Definition 2.2. An IFG, G = <V, E> is said to be a Strong IFG if $\mu_{2ij}= \min(\mu_{1i},\mu_{1j})$and $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$, for all $(v_i ,v_j)\in E$.

Definition 2.3. Let G = < V,E> be an IFG.Then the degree of a vertex v is defined by d(v) = (dμ(v), dγ(v)) where $d\mu(v) = \Sigma_{u\neq v} \mu_2(v,u)$ and $d\gamma(v) = \Sigma_{u\neq v} \gamma_2(v,u)$.

Definition 2.4. Let G = <V,E> be an IFG.Then the order of G is defined to be$O(G) = (O\mu(G),O\gamma(G))$ where $O\mu(G) = \Sigma_{v \in V}\mu_1(v)$ and $O\gamma(G) = \Sigma_{v \in V}\gamma_1(v)$.

Definition 2.5. Let G = <V,E> be an IFG. Then the size of G is defined to be$S(G) = (S\mu(G),S\gamma(G))$ where $S\mu(G) = \Sigma u\neq v\ \mu_2(u,v)$ and $S\gamma(G) = \Sigma\ u\neq v\ \gamma_2(u, v)$.

Definition 2.6:.Let G :(σ, μ) be a fuzzy graph with the underlying set V and crisp graph G*: (σ*, μ*). The pair T(G) : (σ_T ,μ_T) of G is defined as follows. Let the node set of T(G) be VUE. The fuzzy subset σ_T is defined on VUE as,

$\sigma_T(u) = \sigma(u)$ if $u\in V$

$= \mu(e)$ if $e\in E$.

The fuzzy relation μ_T is defined as,

$\mu_T(u, v) = \mu(u, v)$ if $u, v \in V$.

$\mu_T(u, e) = \sigma(u) \wedge \mu(e)$ if $u\in V$, $e\in E$ and the node 'u' lies on the edge 'e',

$= 0$ otherwise.

$\mu_T(e_i, e_j) = \mu(e_i) \wedge \mu(e_j)$ if the edges e_i and e_j have a node in common between them,

$= 0$ otherwise.

By the definition, $\mu_T(u, v) \leq \sigma_T(u) \Lambda \sigma_T(v)$ for all u, v in VUE. Hence μ_T is a fuzzy relation on the fuzzy subset σ_T. Hence the pair T(G) : (σ_T ,μ_T) is a fuzzy graph, and is termed as **Total fuzzy graph** of G.

Definition 2.7. A fuzzy graph G : (σ, μ) with the underlying crisp graph G* : (σ*, μ*) be given. Let G* be (V, E). The nodes and edges of G are taken together as node set of the pair M(G) : (σ_M, μ_M) where

$\sigma_M(u) = \sigma(u)$ if $u\in \sigma^*$

$= \mu(u)$ if $u\in \mu^*$

$= 0$ otherwise.

$\mu_M(e_i, e_j) = \mu(e_i) \wedge \mu(e_j)$ if $e_i, e_j\in \mu^*$ and are adjacent in G*

$= 0$ otherwise.

$\mu_M(v_i, v_j) = 0$ if v_i , v_j are in σ*

$\mu_M(v_i, e_j) = \mu(e_j)$ if v_i in σ* lies on the edge $e_j\in \mu^*$

$= 0$ otherwise.

As σ_M is defined only through the values of σ and μ, $\sigma_M : VUE \rightarrow [0,1]$ is a well-defined fuzzy subset on VUE. Also μ_M is a fuzzy relation on σ_M and $\mu_M(u, v) \leq \sigma_M(u) \Lambda \sigma_M(v)$ for all u, v in VUE. Hence the pair M(G) : (σ_M, μ_M) is a fuzzy graph called **Middle fuzzy graph** of G.

Definition 2.8. Let G :(σ, μ) be a intuitionistic fuzzy graph with its underlying set V and crisp graph G*: (σ*, μ*).Total intuitionistic fuzzy graph T(G): (σ_T ,μ_T) of G is defined as follows: Let the node set of T(G) be VUE. The intuitionistic fuzzy subset σ_{1T} and σ_{2T} are defined on VUE as,

$\sigma_{1T}(u) = \sigma_1(u)$ if $u\in V$

$\sigma_{2T}(u) = \sigma_2(u)$ if $u\in V$ and

$\sigma_{1T}(u) = \mu_1(e)$ if $e \in E$
$\sigma_{2T}(u) = \mu_2(e)$ if $e \in E$.

The intuitionistic fuzzy relation μ_{1T} and μ_{2T} on the intuitionistic fuzzy subset σ_{1T} and σ_{2T} are defined as

$\mu_{1T}(u, v) = \mu_1(u, v)$ if $u, v \in V$
$\mu_{2T}(u, v) = \mu_2(u, v)$ if $u, v \in V$
$\mu_{1T}(u, e) = \sigma_1(u) \wedge \mu_1(e)$ if $u \in V$, $e \in E$ and the node 'u' lies on the edge 'e'.
$\mu_{1T}(u, e) = 0$, otherwise.
$\mu_{2T}(u, e) = \sigma_2(u) \vee \mu_2(e)$ if $u \in V$, $e \in E$ and the node 'u' lies on the edge 'e'.
$\mu_{2T}(u, e) = 0$, otherwise.
$\mu_{1T}(e_i, e_j) = \mu_1(e_i) \wedge \mu_1(e_j)$ if the edges e_i and e_j have a node in common between them.

$\mu_{1T}(e_i, e_j) = 0$, otherwise.
$\mu_{2T}(e_i, e_j) = \mu_2(e_i) \vee \mu_2(e_j)$ if the edges e_i and e_j have a node in common between them.
$\mu_{2T}(e_i, e_j) = 0$, otherwise.

By the definition, $\mu_{1T}(u, v) \leq \sigma_{1T}(u) \wedge \sigma_{1T}(v)$, $\mu_{2T}(u, v) \leq \sigma_{2T}(u) \vee \sigma_{2T}(v), \forall u,v \in VUE$. Hence μ_{1T}, μ_{2T} are the intuitionistic fuzzy relation on the intuitionistic fuzzy subset $(\sigma_{1T}, \sigma_{2T})$. Hence the pair $T(G) : (\sigma_T, \mu_T)$ is a intuitionistic fuzzy graph, and is termed as**Total intuitionistic fuzzy graph** of G.

Definition 2.9. If $G :(\sigma, \mu)$ be a intuitionistic fuzzy graph with the underlying crisp graph $G^*: (\sigma^*, \mu^*)$ be given. Let G^* be (V, E). Middle intuitionistic fuzzy graph is $M(G): (\sigma_M, \mu_M)$ of G is defined as follows. Let the node set of M(G) be VUE.

$\sigma_{1M}(u) = \sigma_1(u)$ if $u \in \sigma^*$
$\sigma_{2M}(u) = \sigma_2(u)$ if $u \in \sigma^*$
$\sigma_{1M}(u) = \mu_1(u)$ if $u \in \mu^*$
$\sigma_{1M}(u) = 0$, otherwise.
$\sigma_{2M}(u) = \mu_2(u)$ if $u \in \mu^*$
$\sigma_{2M}(u) = 0$, otherwise.
$\mu_{1M}(e_i, e_j) = \mu_1(e_i) \wedge \mu_1(e_j)$ if $e_i, e_j \in \mu^*$ and are adjacent in G^*
$\mu_{1M}(e_i, e_j) = 0$ otherwise.
$\mu_{2M}(e_i, e_j) = \mu_2(e_i) \vee \mu_2(e_j)$ if $e_i, e_j \in \mu^*$ and are adjacent in G^*
$\mu_{2M}(e_i, e_j) = 0$ otherwise.
$\mu_{1M}(v_i, v_j) = 0$ if $v_i , v_j \in \sigma^*$
$\mu_{2M}(v_i, v_j) = 0$ if $v_i , v_j \in \sigma^*$
$\mu_{1M}(v_i, e_j) = \mu_1(e_j)$, if v_i in σ^* lies on the edge $e_j \in \mu^*$
$\mu_{1M}(v_i, e_j) = 0$ otherwise.
$\mu_{2M}(v_i, e_j) = \mu_2(e_j)$, if v_i in σ^* lies on the edge $e_j \in \mu^*$
$\mu_{2M}(v_i, e_j) = 0$ otherwise.

As σ_{iM} is defined as only through the values of σ_i and μ_i, $\sigma_{iM} : VUE \rightarrow [0,1]$ is a well-defined intuitionistic fuzzy subset on VUE. Also μ_{iM} is a intuitionistic fuzzy relation on σ_{iM} (where i= 1,2) and $\mu_{1M}(u, v) \leq \sigma_{1M}(u) \wedge \sigma_{1M}(v)$ $\forall$ u, v in VUE, $\mu_{2M}(u, v) \leq \sigma_{2M}(u) \vee \sigma_{2M}(v)$ $\forall$ u,v in VUE. Hence the pair $M(G) :(\sigma_M, \mu_M)$ is a intuitionistic fuzzy graph and is called **Middle intuitionistic fuzzy graph** of G.

Definition 2.10. The busy value of a node v of an IFG G = <V,E> is $(D_\mu(v),D_\nu(v))$ where$D_\mu(v) = \sum \mu_1(v) \wedge \mu_1(v_i)$ and$D_\nu(v) = \sum \nu_1(v) \vee \nu_1(v_i)$where vi are neighbours of v. The busy value of an IFG G is defined to be the sum of the busy values of an IFG G is defined to be the sum of the busy values of all nodes of G. (i.e.)$D(G) = (\sum_i D_\mu(v_i) , \sum_i D_\nu(v_i))$ where vi are nodes of G.

3. Main results

Theorem 3.1. If G is a strong IFG, then $Size(T(G)) = 3Size(G) + (\sum_{ei,ej \in E} \mu_1(e_i) \wedge \mu_1(e_j), \sum_{ei,ej \in E} \mu_2(e_i) \vee \mu_2(e_j))$

Proof: $Size(T(G)) = (S\mu_1(T(G) , S\mu_2(T(G))$

$= (\sum_{u,v \in V \cup E} \mu_{1T}(u,v) , \sum_{u,v \in V \cup E} \mu_{2T}(u,v))$

$= (\sum_{u,v \in V} \mu_{1T}(u,v) , \sum_{u,v \in V} \mu_{2T}(u,v))+(\sum_{u \in V,e \in E} \mu_{1T}(u,e), \sum_{u \in V,e \in E} \mu_{2T}(u,e))+(\sum_{ei,ej \in E} \mu_{1T}(e_i,e_j), \sum_{ei,ej \in E} \mu_{2T}(e_i,e_j))$

{if $(u,v) \in E(G)$ in I summation, u lies on e in G in II summation, there is a common node between e_i and e_j in III summation}

$= (\sum_{u,v \in V} \mu_1(u,v) , \sum_{u,v \in V} \mu_2(u,v))+(\sum_{u \in V,e \in E} \sigma_1(u) \wedge \mu_1(e), \sum_{u \in V,e \in E} \sigma_2(u) \vee \mu_2(e))+(\sum_{ei,ej \in E} \mu_1(e_i) \wedge \mu_1(e_j), \sum_{ei,ej \in E} \mu_2(e_i) \vee \mu_2(e_j))$

$= Size(G)+ 2(\sum_{e \in E} \mu_1(e),\sum_{e \in E} \mu_2(e)) + (\sum_{ei,ej \in E} \mu_1(e_i) \wedge \mu_1(e_j), \sum_{ei,ej \in E} \mu_2(e_i) \vee \mu_2(e_j))$

$= Size(G)+2Size(G)+(\sum_{ei,ej \in E} \mu_1(e_i) \wedge \mu_1(e_j), \sum_{ei,ej \in E} \mu_2(e_i) \vee \mu_2(e_j))$

$Size(T(G)) = 3Size(G)+ (\sum_{ei,ej \in E} \mu_1(e_i) \wedge \mu_1(e_j), \sum_{ei,ej \in E} \mu_2(e_i) \vee \mu_2(e_j))$.

Example 3.2.

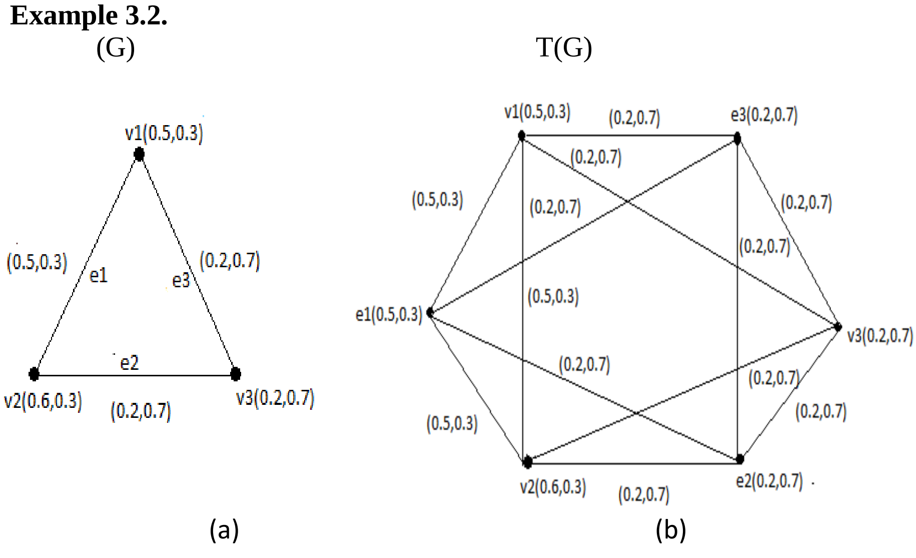


Figure 1:

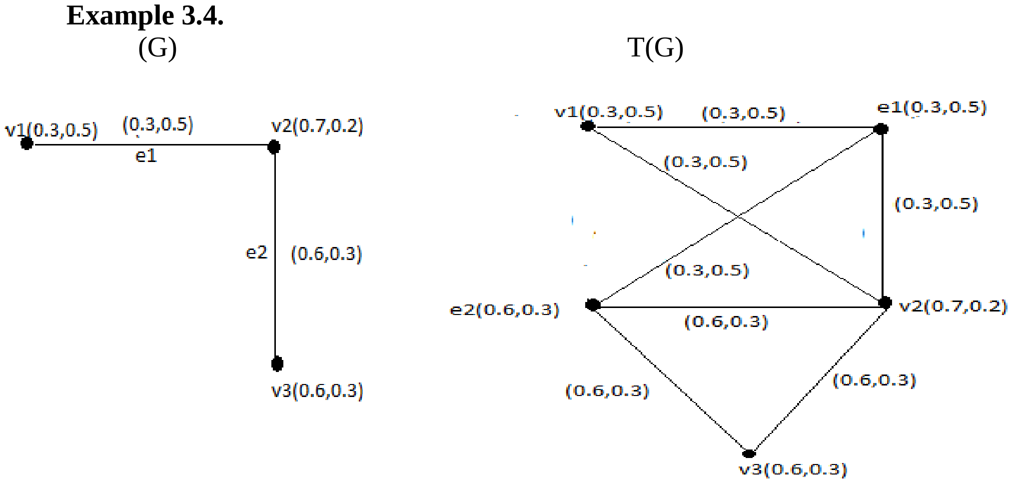
$3size(G)+(\sum_{ei,ej \in E} \mu_1(e_i) \wedge \mu_1(e_j),\sum_{ei,ej \in E} \mu_2(e_i) \vee \mu_2(e_j))$

$= 3(0.9,1.7)+(0.6,2.1)$ (i,j=1,2,3)

$= (2.7,5.1) + (0.6,2.1) = (3.3,7.2) = Size(T(G))$

Theorem 3.3. $d_{T(G)}(u) = 2d_G(u)$, if u€V
$d_{T(G)}(e_i)$ = busy value of e_i in T(G), if e_i€E.
Proof: **Case (i) :** Let u€V,
$d_{T(G)}(u)= (\sum_{v€V}\mu_{1T}(u,v) , \sum_{v€V}\mu_{2T}(u,v))+(\sum_{e€E}\mu_{1T}(u,e),\sum_{e€E}\mu_{2T}(u,e))$
$= (\sum_{u,v€V}\mu_1(u,v),\sum_{u,v€V}\mu_2(u,v))+(\sum_{u€V,e€E}\sigma_1(u) \wedge \mu_1(e), \sum_{u€V,e€E}\sigma_2(u) V \mu_2(e))$
$= (\sum_{e€E\&v€V}\mu_1(e), \sum\mu_2(e)) + (\sum_{e€E}\mu_1(e), \sum\mu_2(e)) = 2(\sum_{e€E}\mu_1(e), \sum\mu_2(e)) = 2d_G(u)$
Case (ii): If e_i€E,
$d_{T(G)}(e_i) =(\sum_{u€V,ei€E}\mu_{1T}(u,e_i), \sum_{u€V,ei€E}\mu_{2T}(u,e_i))+(\sum_{ei,ej€E}\mu_{1T}(e_i,e_j), \sum_{ei,ej€E}\mu_{2T}(e_i,e_j))$
[where u lies on e_i€E(G) in I summation and e_i,e_j has a common node in G in II summation]
$= (\sum_{u€V,ei€E}\sigma_1(u) \wedge \mu_1(e_i), \sum_{u€V,ei€E}\sigma_2(u) V \mu_2(e_i))+(\sum_{ei,ej€E}\mu_1(e_i) \wedge \mu_1(e_j), \sum_{ei,ej€E}\mu_2(e_i) V \mu_2(e_j))$
Where u,$e_j$are neighbours of e_i in T(G).
$d_{T(G)}(e_i)$ = busy value of e_i in T(G).

Example 3.4.



(a) (b)

Figure 2:

Here, $d_{T(G)}(v_1) = (0.6,1.0) = 2(0.3,0.5)= 2d_G(v_1)$
$d_{T(G)}(v_2) = (1.8,1.6) = 2(0.9,0.8)= 2d_G(v_2)$
$d_{T(G)}(v_3) = (1.2,0.6) = 2(0.6,0.3)= 2d_G(v_3)$, if u€V
If e_i€E, $d_{T(G)}(e_1) = (0.9,1.5)$ = busy value of e_1 in T(G).
$d_{T(G)}(e_2) = (1.5,1.1)$ = busy value of e_2 in T(G).

4. More results

Theorem 4.1. If G is a strongIFG, then $Size(M(G)) = 2Size(G)+(\sum_{ei,ej€\mu*} \mu_1(e_i) \wedge \mu_1(e_j),\sum_{ei,ej€\mu*}\mu_2(e_i) V \mu_2(e_j))$.
Proof: By definition of Size of IFG,
$Size(M(G)) = (\sum_{u,v€VUE}\mu_{1M}(u,v) , \sum_{u,v€VUE} \mu_{2M}(u,v))$
$= (\sum_{u,v€V}\mu_{1M}(u,v),\sum_{u,v€V}\mu_{2M}(u,v))+(\sum_{u€V,e€E}\mu_{1M}(u,v) , \sum_{u€V,e€E}\mu_{2M}(u,v))+(\sum_{u,v€E}\mu_{1M}(u,v), \sum_{u,v€E}\mu_{2M}(u,v))$

$= 0+(\sum_{u,v\in E}\mu_1(u) \wedge \mu_1(e),\sum_{u,v\in E}\mu_2(u) \vee \mu_2(e))+(\sum_{ej\in E\&u \text{ lies on } ej}\mu_1(e_j), \sum\mu_2(e_j))$

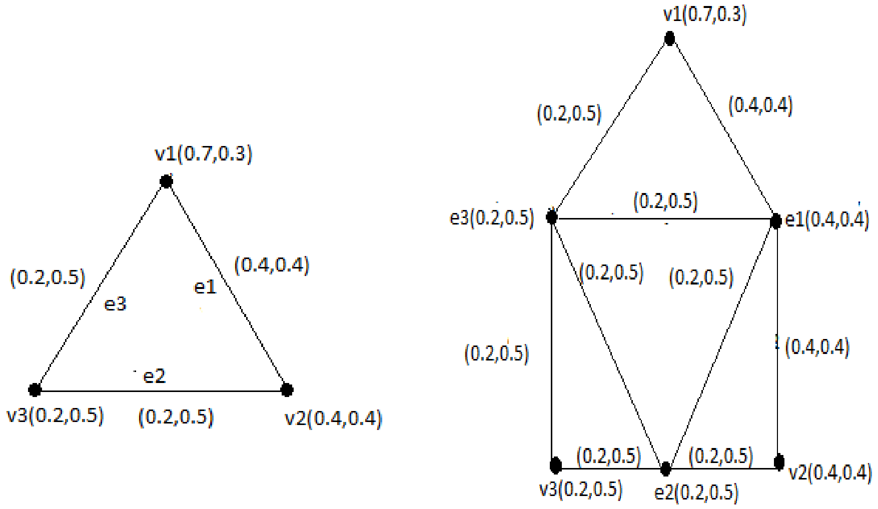
$= (\sum_{ei,ej\in\mu^*}\mu_1(e_i)\wedge\mu_1(e_j),\sum_{ei,ej\in\mu^*}\mu_2(e_i)\vee\mu_2(e_j))+2(\sum_{ej\in E}\mu_1(e_j), \sum\mu_2(e_j))$

As each edge in E(G) is incident with exactly 2 nodes in G.

Hence, Size(M(G)) = 2Size(G)+$(\sum_{ei,ej\in\mu^*} \mu_1(e_i) \wedge \mu_1(e_j),\sum_{ei,ej\in\mu^*}\mu_2(e_i) \vee\mu_2(e_j))$.

Example 4.2.

(G) M(G)



(a) (b)

Figure 3:

Size(M(G)) = (2.2,4.3) = (0.6,1.5)+2(0.8,1.4)

=2Size(G)+$(\sum_{ei,ej\in\mu^*} \mu_1(e_i) \wedge \mu_1(e_j),\sum_{ei,ej\in\mu^*}\mu_2(e_i) \vee \mu_2(e_j))$.

Theorem 4.3. M(G) is a strong intuitionistic fuzzy graph.

Proof: Consider an edge (u,v) in M(G).

Then, $\mu_{1M}(u,v) = \mu_1(e_j)$ [if u in σ^* lies on the edge v= $e_j\in \mu^*$]

$= \mu_1(e_i) \wedge \mu_1(e_j)$ [if u = e_i, v =$e_j\in \mu^*$ and are adjacent in G*]

And $\mu_{2M}(u,v) = \mu_2(e_j)$ [if u in σ^* lies on the edge v= $e_j\in \mu^*$]

$= \mu_2(e_i) \vee \mu_2(e_j)$ [if u = e_i, v =$e_j\in \mu^*$ and are adjacent in G*]

Case (i): If , $\mu_{1M}(u,v) = \mu_1(e_j)$ [if u in σ^* lies on the edge v= $e_j\in \mu^*$] then

= $\sigma_{1M}(e_j)$ as v = $e_j\in \mu^*$

$= \sigma_{1M}(e_j)\wedge\sigma_{1M}(u)$

Hence, $\mu_{1M}(u,v) = \sigma_{1M}(u)\wedge\sigma_{1M}(v)$

And $\mu_{2M}(u,v) = \mu_2(e_j)$

= $\sigma_{2M}(e_j)$

$= \sigma_{2M}(e_j)\vee\sigma_{2M}(u)$

Hence, $\mu_{2M}(u,v) = \sigma_{2M}(u)\vee \sigma_{2M}(v)$

Case (ii): If $\mu_{1M}(u,v) = \mu_1(e_i) \wedge \mu_1(e_j)$ [if u = e_i, v =$e_j\in \mu^*$ and are adjacent in G*]

Then, $\mu_{1M}(u,v) = \sigma_{1M}(e_i) \wedge\sigma_{1M}(e_j)$

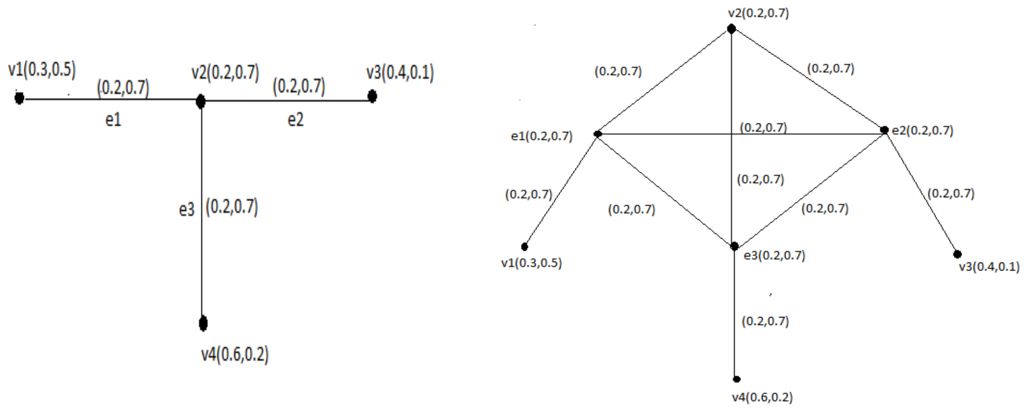
And, $\mu_{2M}(u,v) = \mu_2(e_i) \vee \mu_2(e_j)$

$= \sigma_{2M}(e_i) \vee \sigma_{2M}(e_j)$

Hence by case(i) and case(ii), if (u,v) is in μ_{1M} and μ_{2M} then
$\mu_{1M}(u,v) = \sigma_{1M}(e_i) \wedge \sigma_{1M}(e_j)$ and $\mu_{2M}(u,v) = \sigma_{2M}(e_i) \vee \sigma_{2M}(e_j)$
Therefore, M(G) is a strong IFG.

Example 4.4.

(G) M(G)



(a) (b)

Figure 4:

Theorem 4.5. $d_{M(G)}(u) = d(u)$, if $u \in V$ and

$= 2[\mu_1(e_i), \mu_2(e_i)] + [\sum_{e_j \in E} \mu_1(e_i) \wedge \mu_1(e_j), \sum_{e_j \in E} \mu_2(e_i) \vee \mu_2(e_j)]$

{If $u = e_i$ and $e_i, e_j \in \mu^*$ are adjacent in G^*}

Proof: Case (i): Let $u \in V$, $d_{M(G)}(u) = [\sum_{e_j \in E} \mu_{1M}(u,e_j), \sum \mu_{2M}(u,e_j)]$

Where u lies on the edge $e_j \in \mu^*$, then

$= [\sum_{e_j \in E} \mu_{1M}(e_j), \sum \mu_{2M}(e_j)]$

$= d(u)$ in G.

Case (ii): Let $u \in E$, if $u = e_i$ then

$d_{M(G)}(u) = d_{M(G)}(e_i) = [\sum_{v \in V \cup E} \mu_{1M}(e_i,v), \sum_{v \in V \cup E} \mu_{2M}(e_i,v)]$

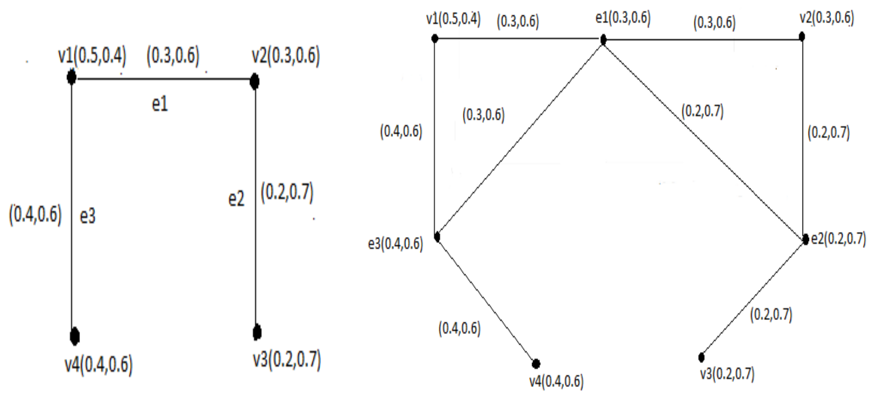
$= [\sum_{v \in V} \mu_{1M}(e_i,v), \sum_{v \in V} \mu_{2M}(e_i,v)] + [\sum_{v=e_j \in E} \mu_{1M}(e_i,e_j), \sum_{v=e_j \in E} \mu_{2M}(e_i,e_j)]$

$= 2[\mu_1(e_i), \mu_2(e_i)] + [\sum_{e_j \in E} \mu_1(e_i) \wedge \mu_1(e_j), \sum_{e_j \in E} \mu_2(e_i) \vee \mu_2(e_j)]$

(As exactly 2 nodes lies on e_i and by definition of μ_{1M} and μ_{2M} where $e_i, e_j \in \mu^*$ are adjacent).

Example 4.6.

(G) M(G)



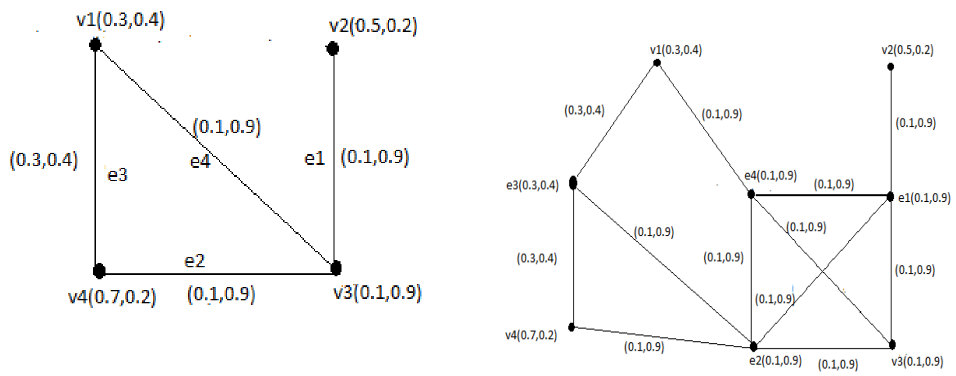
(a) (b)

Figure 5:

Here, $d_{M(G)}(v_1) = (0.7,1.2)= d(u)$ in G.
$d_{M(G)}(e_1) = 2(0.3,0.6)+[(0.3,0.6)+(0.2,0.7)]=(0.6,1.2)+(0.5,1.3)=(1.1,2.5)$.

Theorem 4.7.
Busy value of M(G) $=4Size(G)+2(\sum_{ei,ej\in E}\mu_1(e_i)\wedge\mu_1(e_j),\sum_{ei,ej\in E}\mu_2(e_i)\vee \mu_2(e_j))$
Proof: Busy value of M(G)$=\sum_{v\in VUE}D(v)$
$= \sum_{v\in VUE}d(v)$ { M(G) being strong D(v) = d(v)}
$= 2Size(M(G))$
$= 2[2Size(G)+(\sum_{ei,ej\in\mu*}\mu_1(e_i)\wedge \mu_1(e_j),\sum_{ei,ej\in\mu*}\mu_2(e_i) \vee \mu_2(e_j))]$
Busy value of M(G) $= 4Size(G)+2(\sum_{ei,ej\in E}\mu_1(e_i)\wedge\mu_1(e_j),\sum_{ei,ej\in E}\mu_2(e_i)\vee \mu_2(e_j))$
{where$e_i,e_j\in E$ and e_i,e_j are adjacent in G}.

Example 4.8.



6(a) (G) (b) M(G)

Figure 6:

Busy value of M(G) = $D(v_1) + D(e_1) + D(v_2) + D(v_3) + D(e_2) + D(v_4) + D(e_3) + D(e_4)$
= (0.4,1.3)+(0.4,3.6)+(0.1,0.9)+(0.3,2.7)+(0.5,4.5)+(0.4,1.3)+(0.7,1.7)+(0.4,3.6)
= (3.2,19.6) (i)
4(0.6,3.1) + 2[(0.1,0.9)+(0.1,0.9)+(0.1,0.9)+(0.1,0.9)]
=(2.4,12.4)+2(0.4,3.6)=(3.2,19.6) (ii)
From (i) and (ii)
Busy value of M(G) =4Size(G)+2($\sum_{ei,ej \in E} \mu_1(e_i) \wedge \mu_1(e_j), \sum_{ei,ej \in E} \mu_2(e_i) \vee \mu_2(e_j)$).

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