

Smarandache – R-Module and BF-Algebras

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Abstract. In this paper we introduced Smarandache – 2 – algebraic structure of R-Module namely Smarandache – R-Module. A Smarandache – 2 – algebraic structure on a set N means a weak algebraic structure A_0 on N such that there exist a proper subset M of N , which is embedded with a stronger algebraic structure A_1 , stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, from the unit element if any, from the whole set. We define Smarandache-R-Module and obtain some of its characterization through S-Algebra and BF Algebras.

Keyword: R-Module, S-algebra, Smarandache – R-Module, BF-Algebras

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1. Introduction

New notions are introduced in algebra to study more about the congruence in number theory by Florentinsmarandache [2]. By <proper subset> of a set A , We consider a set P included in A and different from A , different from the empty set, and from the unit element in A – if any they rank the algebraic structures using an order relationship.

The algebraic structures $S_1 \ll S_2$ if :both are defined on the same set :: all S_1 laws are also S_2 laws; all axioms of S_1 law are accomplished by the corresponding S_2 law; S_2 law strictly accomplishes more axioms than S_1 laws, or in other words S_2 laws has more laws than S_1 .

For example : semi group $\ll$ monoid $\ll$ group $\ll$ ring $\ll$ field, or Semi group $\ll$ commutative semi group, ring $\ll$ unitary ring, etc. they define a General special structure to be a structure SM on a set A , different from a structure SN , such that a proper subset of A is an SN structure, where $SM \ll SN$.

Definition 1. Let R be a module, called R-module. If R is said to be smarandache – R – module. Then there exist a proper subset A of R which is an S- algebra with respect to the same induced operations of R .

Definition 2. The *B-algebra* is an algebra $(A; *, 0)$ of type $(2, 0)$ (i.e., a nonempty set A with a binary operation $*$ and a constant 0) satisfying the following axioms:

- (B1) $x * x = 0$,
- (B2) $x * 0 = x$,
- (B) $(x * y) * z = x * [z * (0 * y)]$.

In *BH-algebras*, which are a generalization of *BCK/BCI/B-algebras*. An algebra $(A; *, 0)$ of type $(2, 0)$ is a *BH-algebra* if it obeys (B1), (B2), and (BH) $x * y = 0$ and $y * x = 0$ imply $x = y$. In a *BG-algebra* is an algebra $(A; *, 0)$ of type $(2, 0)$ satisfying (B1), (B2), and (BG) $x = (x * y) * (0 * y)$.

Definition 3. A *BF-algebra* is an algebra $(A; *, 0)$ of type $(2, 0)$ satisfying (B1), (B2), and the following axiom:

- (BF) $0 * (x * y) = y * x$.

Theorem 1. Let R be a smarandache- R -module, if there exists a proper subset A of R in which satisfies (B1) to (B6) then the following axioms are true.

- (a) $0 * (0 * x) = x$ for all $x \in A$;
- (b) if $0 * x = 0 * y$, then $x = y$ for any $x, y \in A$;
- (c) if $x * y = 0$, then $y * x = 0$ for any $x, y \in A$.

Proof. Let R be a smarandache- R -module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is *BF-algebras*.

Let A be a *BF-algebra* and $x \in A$. By (BF) and (B2) we obtain $0 * (0 * x) = x * 0 = x$, that is, (a) holds.

Also (b) follows from (a). Let now $x, y \in A$ and $x * y = 0$. Then $0 - 0 * 0 = 0 * (x * y) \rightarrow y * x$. This gives (c).

Definition 4. A *BF-algebra* is called a *BF₁-algebra* (resp. a *BF₂-algebra*) if it obeys (BG) (resp. (BH)).

Theorem 2. Let R be a smarandache- R -module, if there exists a proper subset A of R in which satisfies (B1) to (B6). Then the algebra $A = (A; *, 0)$ of type $(2, 0)$ is a *BF₁-algebra* if and only if it obeys the laws (B1), (BF), and (BG).

Proof. Let R be a smarandache- R -module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is *BF-algebras*. Suppose that (B1), (BF), and (BG) are valid in A . Let $x \in A$. Substituting $y = x$, (BG) becomes $x = (x * x) * (0 * x)$. Hence applying (B1) and (BF) we conclude that $x = 0 * (0 * x) = x * 0$. Consequently, (B2) holds. Therefore A is a *BF₁-algebra*. The converse is obvious.

Theorem 3. Let R be a smarandache- R -module, if there exists a proper subset A of R in which satisfies (B1) to (B6). Then A is a *BF₂-algebra* if and only if A satisfies (B2), (BF); and the following axiom:
(BH') $x * y = 0 \Leftrightarrow x = y$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras. Let A be a BF_2 -algebra. By definition, (B2) and (BF) are valid in A . Suppose that $x * y = 0$ for $x, y \in A$. theorem 1(c) yields $y * x = 0$. From (BH) we see that $x = y$. If $x = y$, then $x * y = 0$ by (B1). Thus (BH') holds in A .

Let now A satisfies (B2), (BF), and (BH'). (BH') implies (B1) and (BH). Therefore A is BF_2 algebra.

Theorem 4. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) then the following statements are equivalent:

- (a) A is a BF_1 -algebra;
- (b) $x = [x * (0 * y)] * y$ for all $x, y \in A$;
- (c) $x = y * [(0 * x) * (0 * y)]$ for all $x, y \in A$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras (a) $\Rightarrow$ (b): Let A be a BF_1 -algebra and $x, y \in A$. To obtain (b), substitute $0 * y$ for y in (BG) and then use Theorem 1(a).

(b) $\Rightarrow$ (c): We conclude from (b) that $0 * x = [(0 * x) * (0 * y)] * y$.

Hence $0 * (0 * x) = y * [(0 * x) * (0 * y)]$ by (BF). But $0 * (0 * x) = x$, and we have (c).

(c) $\Rightarrow$ (a): Let (c) hold. (BF) clearly forces

$0 * x = [(0 * x) * (0 * y)] * y$. (1) Using (1) with $x = 0 * a$ and $y = 0 * b$

we have $0 * (0 * a) = [(0 * (0 * a)) * (0 * (0 * b))] * (0 * b)$.

Hence applying Theorem 1(a), we deduce that $a = (a * b) * (0 * b)$. Consequently, A is a BF_1 algebra.

Theorem 5. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) then the following statements are true:

- (a) A is a BG-algebra;
- (b) For $x, y \in A$, $x * y = 0$ implies $x = y$;
- (c) The right cancellation law holds in A . i.e., If $x * y = z * y$, then $x = z$ for any $x, y, z \in A$;
- (d) The left cancellation law holds in A . i.e., if $y * x = y * z$, then $x = z$ for any $x, y, z \in A$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras (a) is a direct consequence of the definitions.

(b): Let $x, y \in A$ and $x * y = 0$. By (BG), $x = (x * y) * (0 * y) = 0 * (0 * y)$. From Theorem 1(a) we conclude that $x = y$.

(c) is obvious, since the right cancellation law holds in every BG-algebra.

(d) Follows from (c) and (BF).

Definition 5. A subset I of A is called an ideal of A if it satisfies:

- (I₁) $0 \in I$,
- (I₂) $x * y \in I$ and $y \in I$ imply $x \in I$ for any $x, y \in A$.

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We say that an ideal I of A is *normal* if for any $x, y, z \in A$, $x * y \in I$ implies $(z * x) * (z * y) \in I$.

An ideal I of A is said to be *proper* if $I \neq A$.

Theorem 6. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) and let I be a normal ideal of a BF-algebra A . then the following statements are true:

(a) $x \in I \Rightarrow 0 * x \in I$,

(b) $x * y \in I \Rightarrow y * x \in I$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras

(a) Let $x \in I$. Then $x = x * 0 \in I$. Since I is normal, $(0 * x) * (0 * 0) \in I$.

Hence $0 * x \in I$.

(b) Let $x * y \in I$. By (a), $0 * (x * y) \in I$. Applying (BF) we have $y * x \in I$.

Definition 6. A nonempty subset N of A is called a *subalgebra* of A if $x * y \in N$ for any $x, y \in N$. It is easy to see that if N is a subalgebra of A , then $0 \in N$.

Theorem 7. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) and let N be a subalgebra of A . If it satisfies $x * y \in N$, then $y * x \in N$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras. Let $x * y \in N$. By (BF), $y * x = 0 * (x * y)$. Since $0 \in N$ and $x * y \in N$, we see that $0 * (x * y) \in N$. Consequently, $y * x \in N$.

Theorem 8. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) then I is a subalgebra of A satisfying the following condition:

(NI) if $x \in A$ and $y \in I$, then $x * (x * y) \in I$.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras. Let $x \in A$ and $y \in I$. Theorem 3(a) shows that $0 * y \in I$. Since I is normal, we conclude that $(x * 0) * (x * y) \in I$, i.e., $x * (x * y) \in I$. Thus (NI) holds. Let now $x, y \in I$. Therefore $x * (x * y) \in I$. By Theorem 3(b), $(x * y) * x \in I$. From the definition of ideal we have $x * y \in I$. Thus I is a subalgebra satisfying (NI).

Theorem 9. Let R be a smarandache-R-module, if there exists a proper subset A of R in which satisfies (B1) to (B6) then N is a normal subalgebra of A if and only if N is a normal ideal.

Proof. Let R be a smarandache-R-module. Then by definition there exists a proper subset A of R which is an algebra. By hypothesis A holds for (B1) to (B6) then A is BF-algebras. Let N be a normal subalgebra of A . Clearly, $0 \in N$. Suppose that $x * y \in N$ and $y \in N$. Then $0 * y \in N$. Since N is a subalgebra, we have $(x * y) * (0 * y) \in N$. But $(x * y)$

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$0 * y = x$, because every B-algebra satisfies (BG). Therefore $x \in N$, and thus N is an ideal. Let now $x, y, z \in A$ and $x * y \in N$. By (NS), $(z * x) * (z * y) \in N$. Consequently, N is normal. The converse follows from Theorem 8. Hence the Proof.

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