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Smarandache Soft-Neutrosophic-Near Ring and Soft-Neutrosophic Bi-Ideal

***N.Kannappa*[1] and *B.Fairosekani*[2]**

[1]Department of Mathematics, T.B.M.L.College
Porayar-609307, Tamil Nadu, India, Email : sivaguru91@yahoo.com
[2]Department of Mathematics, T.B.M.L.College
Porayar-609307, Tamil Nadu, India, Email: fairosemaths@gamil.com

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Abstract. In this paper, we introduced Samarandache-2-algebraic structure of Soft Neutrosophic Near-ring namely Smarandache–Soft Neutrosophic Near-ring. A Samarandache-2-algebraic structure on a set N means a weak algebraic structure S_1 on N such that there exist a proper subset M of N, Which is embedded with a stronger algebraic structure S_2, stronger algebraic structure means satisfying more axioms, that is $S_1 << S_2$,by proper subset one can understand a subset different from the empty set, from the unit element if any , from the Whole set. We define Smarandache-Soft Neutrosophic Near-ring and obtain the some of its characterization through bi-ideals.

Keywords: Soft neutrosophic near-ring, soft neutrosophic near-field, smarandache -soft neutrosophic near- ring, soft neutrosophic bi-ideals

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1. Introduction

In order that, New notions are introduced in algebra to better study the congruence in number theory by Florentinsmarandache [2]. By <proper subset> of a set A we consider a set P included in A, and different from A, different from empty set, and from the unit element in A-if any they rank the algebraic structures using an order relationship:
They say that the algebraic structures $S_1 << S_2$ if: both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms that S_1 laws, or S_2 has more laws than S_1.
For example: Semi group <<Monoid<<group<< ring<<field, or Semi group << to commutative semi group, ring << unitary ring etc. They define a general special structure to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is a structure, where SM <<SN. In addition we have published [6,7,8].

2. Preliminaries

Definition 2.1. Let $\langle NUI\rangle$ be a neutrosophic near-ring and (F, A) be a soft set over $\langle NUI\rangle$.Then (F, A) is called soft neutrosophic near-ring if and only if $F(a)$ is a neutrosophic sub near-ring of $\langle NUI\rangle$ for all $a \in A$.

Definition 2.2. Let $K(I) = \langle KUI \rangle$ be a neutrosophic near-field and let (F, A) be a soft set over $K(I)$. Then (F, A) is said to be soft neutrosophic near-field if and only if $F(a)$ is a neutrosophic sub near-field of $K(I)$ for all $a \in A$.

Definition 2.3. Let (F,A) be a soft neutrosophic zero symmetric near-ring over $\langle N \cup I \rangle$, which contains a distributive element $F(a_1) \neq 0$ .Then (F,A) is a near-field if and only if for each
$F(a) \neq 0$ in (F,A), (F,A)F(a) = (F,A) .

Now we have introduced our basic concept, called smarandache–soft neutrosophic–near ring.

Definition 2.4. A Soft neutrosophic –near ring is said to be Smarandache –soft neutrosophic –near ring, if a proper subset of it is a soft neutrosophic –near field with respect to the same induced operations.

Definition 2.5. Let (F, A) be a Smarandache - soft Neutrosophic near –ring over $\langle NUI \rangle$. The two subsets (H,A) and (G,A) of (F,A) the product is defined as
(H,A)(G,A) = { $H(a_1)$ G(a) / $H(a_1)$ in (H,A), G(a) in (G,A) }.
Also we define another operation " *" on the class of subsets of (F,A) given by
(H,A) * (G,A) = { $H(a_1)$ ($H(a_2)$ + G(a)) - $H(a_1)$ $H(a_2)$ / $H(a_1)$, $H(a_2)$ in (H,A) , G(a) in (G,A)}, where (H,A) is a proper subset of (F,A) ,which is a Soft Neutrosophic near-field.

Definition 2.6. Let (F,A) be a Smarandache - soft Neutrosophic near –ring over $\langle NUI \rangle$,then a subgroup (L_B,A) of ((F,A),+) is said to be a Soft Neutrosophic Bi –ideal of (F,A) if (L_B,A)(F,A)(L_B,A) ∩((L_B,A)(F,A)) * (L_B,A)⊆ (L_B,A).

3. Preliminary results on soft Neutrosophic bi-ideals

Here we obtain certain results for our future use.

Proposotion 3.1. If (L_B,A) be a Soft Neutrosophic bi-ideal of a Smarandache-soft Neutrosophic-near ring (F,A) over $\langle NUI \rangle$ and (F_1,A) is a Smarandache-soft Neutrosophic sub near ring of (F,A) , then (L_B,A)∩ (F_1,A) is a Soft Neutrosophic bi-ideal of (F_1,A).

Proof: Since (F,A) be a Smarandache -soft Neutrosophic near –ring over $\langle NUI \rangle$, then by definition, there exists a proper subset (H,A), which is Soft neutrosophic near field.
Since (L_B,A) is a Soft Neutrosophic bi-ideal of (F,A),
(L_B,A)(F,A)(L_B,A)∩ ((L_B,A)(F,A)) * (L_B,A) ⊆ (L_B,A) ,
let (L_{B_1},A) = (L_B,A)∩ (F_1,A),
now (L_{B_1},A)(F_1,A)(L_{B_1},A) ∩ ((L_{B_1},A)(F_1,A) * (L_{B_1},A) = ((L_B,A)∩(F_1,A)) (F_1,A) ((L_B,A) ∩ (F_1,A)) ∩ (((L_B,A)∩ (F_1,A))(F_1,A)) * ((L_B,A)∩(F_1,A))
⊆ (L_B,A)(F_1,A)(L_B,A)∩(F_1,A)∩((L_B,A)(F_1,A)) * (L_B,A)
⊆ (L_B,A)∩(F_1,A) = (L_{B_1},A).Hence (L_{B_1},A) is a Soft neutrosophic bi-ideal of (F_1,A).

Proposition 3.2. Let (F,A) be a Smarandache- soft Neutrosophic near –ring over $\langle NUI\rangle$,which is zerosymmetric.A subgroup (L_B,A) of (F,A) is a Soft Neutrosophic bi-ideal if and only if (L_B,A)(F,A)(L_B,A) ⊆ (L_B,A).

Proof: Since (F,A) be a Smarandache - soft Neutrosophic near –ring over $\langle NUI\rangle$, then by definition, there exists a proper subset (H,A),which is Soft neutrosophic near field.

For a subgroup (L_B,A) of ((F,A),+) , if (L_B,A)(F,A)(L_B,A) ⊆ (L_B,A), then (L_B,A) is a Soft Neutrosophic bi-ideal of (F,A).

Conversly, if (L_B,A) is a Soft Neutrosophic bi-ideal, we have

(L_B,A)(F,A)(L_B,A)∩((L_B,A)(F,A)) * (L_B,A)) ⊆ (L_B,A),

since (F,A) is Soft Neutrosophic zero symmetric near ring, (F,A) (L_B,A) ⊆ (F,A) * (L_B,A), we get

(L_B,A)(F,A)(L_B,A) = (L_B,A)(F,A)(L_B,A)∩ (L_B,A)(F,A)(L_B,A)

⊆ (L_B,A)(F,A)(L_B,A) ∩ ((L_B,A) (F,A)) * (L_B,A) ⊆ (L_B,A).

Proposition 3.3. Let (F,A) be a Smarandache-soft Neutrosophic near–ring over $\langle NUI\rangle$, which is zero symmetric. If (L_B,A) is a Soft neutrosophic bi-ideal of (F,A), then (L_B,A)H(n_1) and H(n_2)(L_B,A) are Soft Neutrosophic bi-ideals of (F,A) , where H(n_1), H(n_2) in (H,A) and H(n_2) is distributive element in (H,A), where (H,A) is a proper subset of (F,A), which is a Soft Neutrosophic near-field.

Proof: Clearly (L_B,A)H(n_1) is a subgroup of ((F,A),+) and

(L_B,A)H(n_1) (H,A) (L_B,A)H(n_1) ⊆ (L_B,A) (H,A) (L_B,A)H(n_1) ⊆ (L_B,A)H(n_1),

we get (L_B,A) H(n_1) is a Soft Neutrosophic bi-ideal of (F,A).

Again H(n_2) (L_B,A) is a subgroup since H(n_2)is distributive in (H,A) and

H(n_2) (L_B,A) (H,A) H(n_2) (L_B,A) ⊆ H(n_2) (L_B,A)(H,A)(L_B,A) ⊆ H(n_1) (L_B,A).

Thus H(n_2)(L_B,A) is a Soft Neutrosophic bi-ideal of (F,A).

Corollary 3.1. If (L_B,A) is a Soft Neutrosophic bi-ideal of a Smarandache-soft neutrosophic-near ring (F,A) over$\langle NUI\rangle$ and L_B(a) is a distributive element in (F,A), then L_B(a) (L_B,A) H(a) is a Soft Neutrosophic bi-ideal of (F,A),where H(a) in (H,A), where (H,A) is a proper subset of (F,A), which is a Soft Neutrosophic near-field.

4. Minimal soft neutrosophic bi-ideals and soft neutrosophic near field

Definition 4.1. A Smarandache - soft Neutrosophic near –ring (F,A) over $\langle NUI\rangle$ is said to be L_B-simple if it has no proper Soft Neutrosophic bi-ideals.

In this section we obtain a characterization of Smarandache- soft neutrosophic near-ring using Soft Neutrosophic bi-ideals.

Lemma 4.1. Let (F,A) be a Smarandache- soft Neutrosophic near –ring over $\langle NUI\rangle$ with more than one element .Then the following conditions are equivalent:

(i) (H,A) is a Soft Neutrosophic near-field,

(ii) (H,A) is L_B - simple, H(d) ≠ {0} and for 0 ≠H(n_1) in (H,A) there exists an element H(n_2) of (H,A) such that H(n_2)H(n_1) ≠0.

where (H,A) is a proper subset of (F,A), which is a Soft Neutrosophic near-field.

Proof: (i)⇒(ii) If (H,A) is a Soft Neutrosophic near -field, then {0} and (H,A) are the only Soft Neutrosophic bi-ideals of (H,A).For if 0≠ (L_B,A) is a Soft neutrosophic bi-ideal of (H,A) , then , for $0\neq L_B(a)$ in (L_B,A) we get (H,A) = $(H,A)L_B(a)$ and (H,A) = $L_B(a)(H,A)$. Now

$(H,A) = (H,A)^2 = (L_B(a)(H,A))\ (H,A)L_B(a)) \subseteq L_B(a)(H,A)L_B(a) \subseteq (L_B,A)$,since (L_B,A) is a Soft Neutrosophic bi-ideal of (H,A). ie.(H,A) = (L_B,A).Hence (H,A) is L_B - simple and the identity element in (H,A) satisfies the required condition.

(ii)⇒(i)

Since $H(d)\neq\{0\}$ we get (H,A) is not constant. We know that H(0) is a Soft neutrosophic bi-ideal of (H,A) and since (H,A) is L_B - simple we get (H,A) = H(0). Let $0\neq H(n_1)$ in (H,A), by proposition 3, $(H,A)H(n_1)$ is a Soft Neutrosophic bi-ideal of (H,A) and $0\neq H(n_2)\ H(n_1)$ in $(H,A)H(n_1)$ for some $H(n_2)$ in (H,A). Hence $(H,A)H(n_1) = (H,A)$.

Therefore we have (H,A) is a Soft Neutrosophic near field.

Theorem 4.1. If a minimal (F,A)-subgroup (H_{min},A) of a Smarandache-Soft Neutrosophic zero symmetric near ring (F,A) over $\langle NUI\rangle$ which is zerosymmetric has a non-zero distributive idempotent element H(e), then H(e) (H_{min},A) is a Multiplicative subgroup of (F,A). Moreover it is a minimal Soft Neutrosophic bi-ideal of (F,A).

Proof: Since (F,A) be a Smarandache -soft Neutrosophic near –ring over $\langle NUI\rangle$, then by definition, there exists a proper subset (H,A), which is Soft neutrosophic near field.

By Proposition 3, H(e) (H_{min},A) is a Soft Neutrosophic bi-ideal of (F,A).

Clearly H(e) is a left identity for $H(e)(H_{min},A)$. If $H(e)H_{min}(a)\neq 0$, for some $H_{min}(a)$ in (H_{min},A), then the non-zero $(H_{min},A)\ (H(e)H_{min}(a))$ is a (F,A) - subgroup of (F,A) and also $(H_{min},A)\ (H(e)\ H_{min}(a)) \subseteq (H_{min},A)$. Thus we get $(H_{min},A)\ (H(e)H_{min}(a)) = (H_{min}, A)$, which implies that $(H(e)(H_{min},A))\ (H(e)H_{min}(a)) = H(e)(H_{min},A)$, i.e. the non-zero element $H(e)H_{min}(a)$ has a left inverse H(e)H(t) such that $(H(e)H(t))\ (H(e)H_{min}(a))$ = H(e).Hence the non – zero elements of $H(e)(H_{min},A)$ from a multiplicative subgroup of (H,A). We have that $H(e)(H_{min},A)$ is a Soft Neutrosophic near field.

Now $H(e)(H_{min},A) \subseteq H(0)$. If (L_{B_1},A) is a Soft Neutrosophic bi-ideal of (F,A) such that $\{0\}\neq(L_{B_1},A) \subseteq H(e)(H_{min},A)$, then $(L_{B_1},A)\ (H(e)(H_{min},A))\ (L_{B_1},A) \subseteq (L_{B_1},A)(F,A)(L_{B_1},A)\subset (L_{B_1},A)$, which implies that (L_{B_1},A) is Soft Neutrosophic bi-ideal of H(e) (H_{min},A).

But H(e) (H_{min},A) is a Soft neutrosophic near field and so by lemma 1, we get $H(e)(H_{min},A)$ is L_B-simple. Hence, $H(e)(H_{min},A)=(L_{B_1},A)$. i.e. $H(e)(H_{min},A)$ is a minimal Soft Neutrosophic bi-ideal of (F,A).

Lemma 4.2. If a Minimal Soft Neutrosophic bi-ideal (L_B,A) of Smarandache -soft neutrosophic near ring (F,A) over $\langle NUI\rangle$ which is zero symmetric contains a distributive element $L_B(a)$ such that $L_B(a)$ is neither a left zero divisor nor right zero divisor ,then (F,A) must have a two-sided identity.

Proof: Since (F,A) be a Smarandache -soft Neutrosophic near –ring over $\langle NUI\rangle$.

Then by definition (H,A) is a proper subset of (F,A), which is a Soft Neutrosophic near-field.

Clearly $(L_B(a))^3\neq 0$ and $(L_B(a))^3$ in $L_B(a)(F,A)L_B(a) \subseteq (L_B,A)$. By corollary 4, $L_B(a)(F,A)L_B(a)$ is a Soft Neutrosophicbi-ideal of (F,A) and $L_B(a)(F,A)L_B(a)$ = (L_B,A), since (L_B,A) is minimal.Therefore $L_B(a) = L_B(a)F(a)L_B(a)$ for some F(a) in (F,A).

For F(x),F(y) in (F,A) ,we have $F(x) = F(x)L_B(a)F(a)$ and $F(y) = F(a)L_B(a)F(y)$,

Since $L_B(a)$ is neither a left zero divisor nor a right zero divisor.
i.e. $F(a)L_B(a)$ and $L_B(a)F(a)$ are left and right identities for (F,A) respectively and hence $F(a) L_B(a) = L_B(a)F(a)$ is the required identity element in (F,A)

Now we prove the main theorem of this paper.

Theorem 4.2. Let (F,A) be a Smarandache-soft neutrosophic near ring over $\langle NUI \rangle$,which is zerosymmetric. Then (H,A) is a Soft Neutrosophicnear field if and only if (H,A) has a distributive element which is neither a left zero divisor nor a right zero divisor and which is contained in a minimal Soft Neutrosophicbi-ideal (L_B,A) of (F,A).
Proof: Since (F,A) be a Smarandache -soft Neutrosophic near –ring over $\langle NUI \rangle$.
Then by definition (H,A) is a proper subset of (F,A), which is a Soft Neutrosophic near-field. If (H,A) is a Soft Neutrosophic near field ,then (H,A) itself is a minimal Soft Neutrosophic bi-ideal satisfying the required conditions.

Conversely, let (L_B,A) be a minimal Soft Neutrosophicbi-ideal of (F,A) containing a distributive element H(d) which is neither a left nor a right zero divisor .
By lemma 3, (H,A) contains a identity H(e).
Again by corollary 4, $H(d)^2 (H,A) H(d)^2$ is a Soft Neutrosophicbi–ideal and $0 \neq H(d)^2 (H,A) H(d)^2 \subseteq (L_B,A)(H,A)(L_B,A) \subseteq (L_B,A)$.
Since (L_B,A) is a minimal we get $(L_B,A) = H(d)^2 (H,A) H(d)^2$.
Now H(d) in $(L_B,A) = H(d)^2 (H,A) H(d)^2$ implies that $H(d) = H(n)H(d)^2$ for some H(n) in (H,A).

But $H(d) = H(e)H(d)$ and so $H(e) = H(n)H(d)$
i.e. H(e) in (H,A)H(d). Similarly we get H(e) in H(d)(H,A).
Therefore $H(e) = H(e)^2$ in $(H(d)(H,A))$ $((H,A)H(d)) \subseteq H(d)\,(H,A)\,H(d) \subseteq (L_B,A)$, whence $(H,A) = H(e)(H,A)H(e) \subseteq (L_B,A)\,(H,A)(L_B,A) \subseteq (L_B,A)$, that is $(H,A) = (L_B,A)$.

This relation and minimality of (L_B,A) implies that (H,A) is L_B - simple and so (H,A) is a Soft Neutrosophic near –field by lemma 1.

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