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Operations on Intuitionistic Trapezoidal Fuzzy Numbers using Interval Arithmetic

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Abstract. In this paper operations based on (α, β) −cut for intuitionistic trapezoidal fuzzy numbers are defined as addition, multiplication, scalar multiplication, subtraction etc., including exponentiation, extracting n^{th} root, and taking logarithm using the degree of acceptance and rejection.

1 Introduction

Transportation problem is a special kind of linear programming problem in which goods are transported from a set of sources to destinations with respect to the supply and demand respectively, such that the total cost of transportation is minimized. Intuitionistic fuzzy set (IFS) is one of the components of fuzzy set theory [10] and also introduced by Atanassov [1,2]. IFS is exaggerated by a degree of acceptance and a degree of rejection function so that the sum of both values is less than one [1]. Mahapatra and Roy [6] defined the triangular intuitionistic fuzzy number (TrIFN) and trapezoidal intuitionistic fuzzy number (TrIFN) and their arithmetic operations based on intuitionistic fuzzy extension principle using (α, β) −cuts method. Li [4] developed the ratio ranking method for ranking trapezoidal intuitionistic fuzzy numbers.

This paper is organized as follows, in section 2, the representation of trapezoidal intuitionistic fuzzy numbers and (α, β) −cut sets. In section 3, operations including exponentiation, logarithms and n^{th} root of trapezoidal intuitionistic fuzzy number.

2 Preliminaries

Definition 2.1. Let X be the universal set. An intuitionistic fuzzy set (IFS) $\tilde{A}^i$ in X is given by
$\tilde{A}^i = \left\{ (x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) / x \in X \right\}$ where the function $\mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)$ define the degree of acceptance and degree of rejection of the element $x \in X$ to the set $\tilde{A}^I$, for every $x \in X$, $0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1$.

Definition 2.2. Let $\tilde{A}^i$ be the Trapezoidal intuitionistic fuzzy number with parameters $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ which is denoted

by $\widetilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$.The degree of acceptance and degree of rejection functions are defined as

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & , \ a_1 \le x \le a_2 \\ 1 & , \ a_2 \le x \le a_3 \\ \frac{x-a_4}{a_3-a_4} & , \ a_3 \le x \le a_4 \\ 0 & , \ x > a_4 \end{cases} \text{and} \gamma_{\tilde{A}^i}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1} & , \ b_1 \le x \le b_2 \\ 0 & , \ b_2 \le x \le b_3 \\ \frac{x-b_4}{b_3-b_4} & , \ b_3 \le x \le b_4 \\ 1 & , \ x > a_4 \end{cases}$$

Definition 2.3. The(α, β) −cut trapezoidal intuitionistic fuzzy numbers is defined as usually, by $\tilde{A}^i_{\alpha,\beta} = \left\{ [\tilde{A}^i_1(\alpha), \tilde{A}^i_2(\alpha)], [\tilde{A}^{i'}_1(\beta), \tilde{A}^{i'}_2(\beta)] \right\}$,$\alpha + \beta \le 1$, $\alpha, \beta \in [0,1]$ where $\tilde{A}^i_1(\alpha) = a_1 + \alpha(a_2 - a_1), \tilde{A}^i_2(\alpha) = a_4 - \alpha(a_4 - a_3)$ and $\tilde{A}^{i'}_1(\beta) = b_2 - \beta(b_2 - b_1), \ A'_2(\beta) = b_3 + \beta(b_4 - b_3)$.

Definition 2.4. A α −cut sets of $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ is a crisp subset of R, which is defined as $\tilde{A}^i_\alpha = \{x/\mu_{\tilde{A}^i}(x) \ge \alpha\}$. Using $\mu_{\tilde{A}^i}(x)$ and definition of α −cut it follows that $\tilde{A}^i_\alpha$ is a closed interval, denoted by$\tilde{A}^i_\alpha = [L_\alpha(\tilde{A}^i), R_\alpha(\tilde{A}^i)] = [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]$.

Definition 2.5. A β −cut sets of $\tilde{A}^i = (b'_1, a'_1, b'_2, a'_2, a'_3, b'_3, a'_4, b'_4)$ a crisp subset of R, which is defined as $\tilde{A}^i_\beta = \{x/\gamma_{\tilde{A}^i}(x) \ge \beta\}$. Using $\gamma_{\tilde{A}^i}(x)$and definition of β −cut, it follows that $\tilde{A}^i_\beta$ is a closed interval, denoted by $\tilde{A}^i_\beta = [L_\beta(A), R_\beta(A)] = [b_2 + \beta(b_2 - b_1), a_4 - \beta(b_4 - b_3)]$.

3 Operations on TrIFNs
Theorem 3.1. If $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and
$\tilde{B}^i = (b'_1, a'_1, b'_2, a'_2, a'_3, b'_3, a'_4, b'_4)$ are two TrIFNs, then $\tilde{A}^i + \tilde{B}^i = (b_1 + b'_1, a_1 + a'_1, b_2 + b'_2, a_2 + a'_2, a_3 + a'_3, b_3 + b'_3, a_4 + a'_4, b_4 + b'_4)$ is also a TrIFN.
Proof: For every$\alpha, \beta \in [0,1]$,
$\tilde{A}^i_\alpha + \tilde{B}^i_\alpha = [a_1 + a'_1 + \alpha((a_2 - a_1) + (a'_2 - a'_1)), a_4 + a'_4 - \alpha((a'_4 - a'_3) + (a_4 - a_3))]$ 　　　　　　　　　　　　　　　　　　　　　　　(3.1)
$\tilde{A}^i_\beta + \tilde{B}^i_\beta = [b_2 + b'_2 - \beta((b_2 - b_1) + (b'_2 - b'_1)), b_3 + b'_3 + \beta((b_4 - b_3) + (b'_4 - b'_3))]$ 　　　　　　　　　　　　　　　　　　　　　　　(3.2)

$x = a_1 + a'_1 + \alpha((a_2 - a_1) + (a'_2 - a'_1))$ and
$$x = a_4 + a'_4 - \alpha((a'_4 - a'_3) + (a_4 - a_3))$$

which gives, $\mu_{A+B}(x) = \begin{cases} \frac{x-(a_1+a'_1)}{(a_2+a'_2)-(a_1+a'_1)} & , a_1 + a'_1 \le x \le a_2 + a'_2 \\ 1 & , a_2 + a'_2 \le x \le a_3 + a'_3 \\ \frac{x-(a_4+a'_4)}{(a_3+a'_3)-(a_4+a'_4)} & , a_3 + a'_3 \le x \le a_4 + a'_4 \\ 0 & , \quad otherwise \end{cases}$ 　　(3.3)

$x = b_2 + b_2' - \beta\big((b_2 - b_1) + (b_2' - b_1')\big)$ and $x = b_3 + b_3' + \beta\big((b_4 - b_3) + (b_4' - b_3')\big)$
which gives,

$$\gamma_{A+B}(x) = \begin{cases} \dfrac{x - (b_2 + b_2')}{(b_1 + b_1') - (b_2 + b_2')} & , \quad b_1 + b_1' \leq x \leq b_2 + b_2' \\[2ex] 0 & , \quad b_2 + b_2' \leq x \leq b_3 + b_3' \\[2ex] \dfrac{x - (b_3 + b_3')}{(b_4 + b_4') - (b_3 + b_3')} & , \quad b_3 + b_3' \leq x \leq b_4 + b_4' \\[2ex] 1 & , \quad otherwise \end{cases} \quad (3.4)$$

So $A + B$ represented by (3.3) and (3.4) is a trapezoidal shaped IFN. It can be approximated to TrIFN. Thus we have $\tilde{A}^i + \tilde{B}^i = (b_1 + b_1',\ a_1 + a_1',\ b_2 + b_2',\ a_2 + a_2',\ a_3 + a_3',\ b_3 + b_3',\ a_4 + a_4',\ b_4 + b_4')$ is also a TrIFN.

Theorem 3.2. If $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and
$\tilde{B}^i = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4')$ are two TrIFNs, then $\tilde{A}^i - \tilde{B}^i = [b_1 - b_4',\ a_1 - a_4',\ b_2 - b_3',\ a_2 - a_3', a_3 - a_2',\ b_3 - b_2',\ a_4 - a_1', b_4 - b_1']$ is also a TrIFN.
Proof. For every $\alpha, \beta \in [0,1]$. Let the additive image of $\tilde{B}^i{}_\alpha$ and $\tilde{B}^i{}_\beta$ be $-\tilde{B}^i{}_\alpha = [-a_4' + \alpha(a_4' - a_3'), -a_1' - \alpha(a_2' - a_1')]$ and $\tilde{B}^i{}_\beta = [-b_3' - \beta(b_4' - b_3'), -b_2'\beta(b_2' - b_1')]$.

$\tilde{A}^i{}_\alpha - \tilde{B}^i{}_\alpha = (a_1 - a_4') + \alpha[(a_2 - a_1) + (a_4' - a_3')], (a_4 - a_1') - \alpha[(a_4 - a_3) + (a_2' - a_1')]$ (3.5)

$\tilde{A}^i{}_\beta - \tilde{B}^i{}_\beta = (b_2 - b_3') - \beta[(b_2 - b_3') - (b_1 - b_4')], (b_3 - b_2') + \beta[(b_4 - b_1') - (b_3 - b_2')]$ (3.6)

$x = (a_1 - a_4') + \alpha[(a_2 - a_1) + (a_4' - a_3')]$ and
$$x = (a_4 - a_1') - \alpha[(a_4 - a_3) + (a_2' - a_1')]$$

$\mu_{A-B}(x)$

$$= \begin{cases} \dfrac{x - (a_1 - a_4')}{(a_2 - a_3') - (a_1 - a_4')}, & (a_1 - a_4') \leq x \leq (a_2 - a_3') \\[2ex] 1 & , \quad (a_2 - a_3') \leq x \leq (a_3 - a_2') \\[2ex] \dfrac{(a_4 - a_1') - x}{(a_4 - a_1') + (a_3 - a_2')}, & (a_3 - a_2') \leq x \leq (a_4 - a_1') \\[2ex] 0 & , \quad otherwise \end{cases} \quad (3.7)$$

$x = (b_2 - b_3') - \beta[(b_2 - b_3') - (b_1 - b_4')]$ and
$x = (b_3 - b_2') + \beta[(b_4 - b_1') - (b_3 - b_2')]$. Now expressing β in terms of x and setting $\beta = 0$ and $\beta = 1$ in (3.6) which gives,

$\gamma_{A-B}(x)$

$$= \begin{cases} \dfrac{x - (b_2 - b_3')}{(b_1 - b_4') - (b_2 - b_3')}, & (b_1 - b_4') \le x \le (b_2 - b_3') \\ 0, & (b_2 - b_3') \le x \le (b_3 - b_2') \\ \dfrac{x - (b_3 - b_2')}{(b_4 - b_1')(b_3 - b_2')}, & (b_3 - b_2') \le x \le (b_4 - b_1') \\ 1, & otherwise \end{cases} \quad (3.8)$$

So $A - B$ represented by (3.7) and (3.8) is a trapezoidal shaped IFN. It can be approximated to TrIFN. Thus we have $\tilde{A}^i - \tilde{B}^i = [b_1 - b_4', a_1 - a_4', b_2 - b_3', a_2 - a_3', a_3 - a_2', b_3 - b_2', a_4 - a_1', b_4 - b_1']$ is also a TrIFN.

Theorem 3.3. If $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and
$\tilde{B}^i = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4')$ are two TrIFNs, then
$\tilde{A}^i * \tilde{B}^i = (b_1 b_1', a_1 a_1', b_2 b_2', a_2 a_2', a_3 a_3', b_3 b_3', a_4 a_4', b_4 b_4')$ is also a TrIFN.

Proof. For every $\alpha, \beta \in [0,1]$. To calculate the multiplication of TrIFNs $\tilde{A}^i$ and $\tilde{B}^i$, we first multiply the (α, β)−cuts of $\tilde{A}^i$ and $\tilde{B}^i$ using interval arithmetic.

$$\tilde{A}^i{}_\alpha * \tilde{B}^i{}_\alpha = [a_1 + \alpha(a_2 - a_1) * (a_1' + \alpha(a_2' - a_1')), (a_4 - \alpha(a_4 - a_3) * (a_4' - \alpha(a_4' - a_3'))] \quad (3.9)$$

$$\tilde{A}^i{}_\beta * \tilde{B}^i{}_\beta = [(b_2 - \beta(b_2 - b_1)) * (b_2' - \beta(b_2' - b_1')), (b_3 + \beta(b_4 - b_3) * b_3' + \beta(b_4' - b_3')] \quad (3.10)$$

$x = [(a_2 - a_1)(a_2' - a_1')]\alpha^2 + (a_1(a_2' - a_1') + (a_1'(a_2 - a_1))\alpha + a_1 a_1'$ and
$x = [(a_4 - a_3)(a_4' - a_3')\alpha^2 - [(a_4' - a_3') + a_4'(a_4 - a_3)]\alpha + a_4 a_4'$

$\mu_{A*B}(x)$

$$= \begin{cases} \dfrac{-[a_1(a_2' - a_1') + a_1'(a_2 - a_1) \pm \sqrt{[a_1(a_2' - a_1') + a_1'(a_2 - a_1)]^2 - 4(a_2 - a_1)(a_2' - a_1')(a_1 a_1' - x)}}{2(a_2 - a_1)(a_2' - a_1')}, \\ \qquad\qquad aa_1' \le x \le a_2 a_2' \\ 1, \quad a_2 a_2' \le x \le a_3 a_3' \\ \dfrac{[a_4(a_4' - a_3') + a_4'(a_4 - a_3) \pm \sqrt{[a_4(a_4' - a_3') + a_4'(a_4 - a_3)]^2 - 4(a_4 - a_3)(a_4' - a_3')(a_4 a_4' - x)}}{2(a_4 - a_3)(a_4' - a_3')}, \\ \qquad\qquad a_3 a_3' \le x \le a_4 a_4' \\ 0, \quad x > a_4 a_4' \qquad (3.11) \end{cases}$$

$x = [(b_2 - b_1)(b_2' - b_1')]\beta^2 - [b_2(b_2' - b_1') + b_2'(b_2 - b_1)]\beta + b_2 b_2'$
and $x = [(b_4 - b_3)(b_4' - b_3')]\beta^2 + [b_3(b_4' - b_3') + b_3'(b_4 - b_3)]\beta + b_3 b_3'$

So $A * B$ represented by (3.11) and (3.12) is a trapezoidal shaped IFN. It can be approximated to TrIFN. Thus we have
$\tilde{A}^i * \tilde{B}^i = (b_1 b_1', a_1 a_1', b_2 b_2', a_2 a_2', a_3 a_3', b_3 b_3', a_4 a_4', b_4 b_4')$ is also a TrIFN.

Theorem 3.4. Let $r \epsilon R$. If $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_2, a_3, a_4, b_4)$ is a TrIFN then $S = r\tilde{A}^i$ is also a TrIFN and is given by $D = r\tilde{A}^i = \begin{cases} rb_1, ra_1, rb_2, ra_2, ra_3, rb_3, ra_4, rb_4, & r > 0 \\ rb_4, ra_4, rb_3, ra_3, ra_2, rb_2, ra_1, rb_1, & r < 0 \end{cases}$

Proof. Let $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_2, a_3, a_4, b_4)$ be TrIFN and $r \epsilon R$

Case (i) $r > 0$

To calculate the scalar multiplication of TrIFN $\tilde{A}^i$ we first multiply r and $(\alpha.\beta)$ −cuts of $\tilde{A}^i$ gives $r\tilde{A}^i{}_\alpha = [ra_1 + \alpha(ra_2 - ra_1),\ ra_4 - \alpha(ra_4 - ra_3)]$ (3.13)

$$r\tilde{A}^i{}_\beta = [rb_2 - \beta(rb_2 - rb_1),\ rb_3 - \beta(rb_4 - rb_3)]$$ (3.14)

$x = ra_1 + \alpha(ra_2 - ra_1)$ and $x = ra_4 - \alpha(ra_4 - ra_3)$

$$\mu_S(x) = \begin{cases} \dfrac{x - ra_1}{ra_2 - ra_1} & , \quad ra_1 \le x \le ra_2 \\ 1 & , \quad ra_2 \le x \le ra_3 \\ \dfrac{x - ra_4}{ra_3 - ra_4} & , \quad ra_3 \le x \le ra_4 \\ 0 & , \quad otherwise \end{cases}$$

$x = rb_2 - \beta(rb_2 - rb_1)$ and $x = rb_3 + \beta(rb_4 - rb_3)$

$$\gamma_S(x) = \begin{cases} \dfrac{x - rb_2}{rb_1 - rb_2} & , \quad rb_1 \le x \le rb_2 \\ 0 & , \quad rb_1 \le x \le rb_2 \\ \dfrac{x - rb_3}{rb_4 - rb_3} & , \quad rb_3 \le x \le rb_4 \\ 1 & , \quad otherwise \end{cases}$$

Thus, scalar multiplication can be given as $S = [rb_1, ra_1, rb_2, ra_2, ra_3, rb_3, ra_4, rb_4]$ which is also a TrIFN when $r > 0$.

Case (ii) $r < 0$

Similarly, it can be shown that $S = r\tilde{A}^i$ if $r < 0$ where

$$\mu_S(x) = \begin{cases} \dfrac{x - ra_4}{ra_3 - ra_4} & , \quad x \in [rb_4, ra_3] \\ 1 & , \quad r \in [ra_3, ra_2] \\ \dfrac{x - ra_1}{ra_2 - ra_1} & , \quad r \in [ra_2, ra_1] \\ 0 & , \quad otherwise \end{cases} \quad \text{and} \quad \gamma_S(x) = \begin{cases} \dfrac{x - rb_3}{rb_4 - rb_3} & , \quad x \in [rb_4, rb_3] \\ 0 & , \quad r \in [rb_3, rb_2] \\ \dfrac{x - rb_2}{rb_1 - rb_2} & , \quad r \in [rb_2, rb_1] \\ 1 & , \quad otherwise \end{cases}$$

Thus, scalar multiplication can be given as $S = [rb_4, ra_4, rb_3, ra_3, ra_2, rb_2, ra_1, rb_1]$ which is also a TrIFN where $r < 0$.

Theorem 3.5. If $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and $\tilde{B}^i = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4')$ are two TrIFN, then

$D = \left(\dfrac{b_1}{b_4'}, \dfrac{a_1}{a_4'}, \dfrac{b_2}{b_3'}, \dfrac{a_2}{a_3'}, \dfrac{a_3}{a_2'}, \dfrac{b_3}{b_2'}, \dfrac{a_4}{a_1'}, \dfrac{b_4}{b_1'} \right)$.

Proof. To calculate division of TrIFNs $\tilde{A}^i$ and $\tilde{B}^i$ we first divide the (α, β) −cuts of $\tilde{A}^i$ and $\tilde{B}^i$ using interval arithmetic.

$$\frac{\tilde{A}^i{}_\alpha}{\tilde{B}^i{}_\alpha} = \frac{[a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]}{[a_1' + \alpha(a_2' - a_1'), a_4' - \alpha(a_4' - a_3')]} = \left[\frac{a_1 + \alpha(a_2 - a_1)}{a_4' - \alpha(a_4' - a_3')}, \frac{a_4 - \alpha(a_4 - a_3)}{a_1' + \alpha(a_2' - a_1')} \right]$$ (3.15)

$$\frac{\tilde{A}^i{}_\beta}{\tilde{B}^i{}_\beta} = \frac{[b_2 - \beta(b_2 - b_1), b_3 + \beta(b_4 - b_3)]}{[b_2' - \beta(b_2' - b_1'), b_3' + \beta(b_4' - b_3')]} = \left[\frac{b_2 - \beta(b_2 - b_1)}{b_3' + \beta(b_4' - b_3')}, \frac{b_3 + \beta(b_4 - b_3)}{b_2' - \beta(b_2' - b_1')} \right]$$ (3.16)

$x = \dfrac{a_1 + \alpha(a_2 - a_1)}{a_4' - \alpha(a_4' - a_3')}$ and $x = \dfrac{a_4 - \alpha(a_4 - a_3)}{a_1' + \alpha(a_2' - a_1')}$

$$\mu_{\tilde{A}^i/\tilde{B}^i}(x) = \begin{cases} \dfrac{xa_4'-a_1}{(a_2-a_1)+x(a_4'-a_3')} \ , & \dfrac{a_1}{a_4'} \leq x \leq \dfrac{a_2}{a_3'} \\ 1 & , \quad \dfrac{a_2}{a_3'} \leq x \leq \dfrac{a_3}{a_2'} \\ \dfrac{a_4-xa_1'}{x(a_2'-a_1')+(a_4-a_3)} \ , & \dfrac{a_3}{a_2'} \leq x \leq \dfrac{a_4}{a_1'} \\ 0 & , \qquad otherwise \end{cases} \qquad (3.17)$$

$x = \dfrac{b_2-\beta(b_2-b_1)}{b_3'+\beta(b_4'-b_3')}$ and $x = \dfrac{b_3+\beta(b_4-b_3)}{b_2'-\beta(b_2'-b_1')}$

$$\gamma_{\tilde{A}^i/B^i}(x) = \begin{cases} \dfrac{b_2-xb_3'}{x(b_4'-b_3')+(b_2-b_1)} \ , & \dfrac{b_1}{b_4'} \leq x \leq \dfrac{b_2}{b_3'} \\ 0 & , \quad \dfrac{b_2}{b_3'} \leq x \leq \dfrac{b_3}{b_2'} \\ \dfrac{xb_2'-b_3}{x(b_2'-b_1')+(b_4-b_3)} \ , & \dfrac{b_3}{b_2'} \leq x \leq \dfrac{b_4}{b_1'} \\ 1 & , \qquad otherwise \end{cases} \qquad (3.18)$$

So $\tilde{A}^i/\tilde{B}^i$ represented by (3.17) and (3.18) is a trapezoidal shaped IFN. It can be approximated to TrIFN, $\tilde{A}^i/\tilde{B}^i = \left(\dfrac{b_1}{b_4'} \ , \dfrac{a_1}{a_4'} \ , \dfrac{b_2}{b_3'} \ , \dfrac{a_2}{a_3'} \ , \dfrac{a_3}{a_2'} \ , \dfrac{b_3}{b_2'} \ , \dfrac{a_4}{a_1'} \ , \dfrac{b_4}{b_1'} \right)$.

Theorem 3.6.
If $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ is a positive TrIFN, then
$$\tilde{A}^{i-I} = \left(\frac{1}{b_4} \ , \frac{1}{a_4} \ , \frac{1}{b_3} \ , \frac{1}{a_3} \ , \frac{1}{a_2} \ , \frac{1}{b_2} \ , \frac{1}{a_1} \ , \frac{1}{b_1} \right)$$

Proof. Let $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ be a positive TrIFN. To calculate inverse of TrIFN $\tilde{A}^i$ we first take the inverse of the (α, β) −cuts of $\tilde{A}^i$ using interval arithmetic.

$1/_{\tilde{A}^i_\alpha} = \dfrac{1}{[a_1+\alpha(a_2-a_1), \ a_4-\alpha(a_4-a_3)]} = \left[\dfrac{1}{a_4-\alpha(a_4-a_3)} \ , \ \dfrac{1}{a_1+\alpha(a_2-a_1)} \right] \qquad (3.19)$

$1/_{\tilde{A}^i_\beta} = \dfrac{1}{[b_2-\beta(b_2-b_1), \ b_3+\beta(b_4-b_3)]} = \left[\dfrac{1}{b_3+\beta(b_4-b_3)} \ , \dfrac{1}{b_2-\beta(b_2-b_1)} \right] \qquad (3.20)$

$x = \dfrac{1}{a_4-\alpha(a_4-a_3)}$ and $x = \dfrac{1}{a_1+\alpha(a_2-a_1)}$

$$\mu_{\tilde{A}^{i-I}}(x) = \begin{cases} \dfrac{a_4x-1}{x(a_4-a_3)} \ , & \dfrac{1}{a_4} \leq x \leq \dfrac{1}{a_3} \\ 1 & , \quad \dfrac{1}{a_3} \leq x \leq \dfrac{1}{a_2} \\ \dfrac{1-xa_1}{x(a_2-a_1)} \ , & \dfrac{1}{a_2} \leq x \leq \dfrac{1}{a_1} \\ 0 & , \qquad otherwise \end{cases} \qquad (3.21)$$

$x = \dfrac{1}{b_3+\beta(b_4-b_3)}$ and $x = \dfrac{1}{b_2-\beta(b_2-b_1)}$

$$\gamma_{\tilde{A}^{i-I}}(x) = \begin{cases} \frac{1-xb_3}{x(b_4-b_3)} & , \quad \frac{1}{b_4} \leq x \leq \frac{1}{b_3} \\ 0 & , \quad \frac{1}{b_3} \leq x \leq \frac{1}{b_2} \\ \frac{xb_2-1}{x(b_2-b_1)} & , \quad \frac{1}{b_2} \leq x \leq \frac{1}{b_1} \\ 1 & , \quad otherwise \end{cases} \qquad (3.22)$$

So $\tilde{A}^{i-I}$ represented by (3.21) and (3.22) is a trapezoidal shaped IFN. It can be approximated to TRIFN $\tilde{A}^{i-I} = \left(\frac{1}{b_4}, \frac{1}{a_4}, \frac{1}{b_3}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{a_1}, \frac{1}{b_1}\right)$.

Theorem 3.7. If $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ is a trapezoidal intuitionistic fuzzy number then $e^{\tilde{A}^i} = (e^{b_1}, e^{a_1}, e^{b_2}, e^{a_2}, e^{a_3}, e^{b_3}, e^{a_4}, e^{b_4})$ is also a TrIFN.

Proof. To calculate exponential of the TrIFN $\tilde{A}^i$ we first take the exponential of the (α, β) −cut of $\tilde{A}^i$ using interval arithmetic.

$e^{\tilde{A}^i}{}_\alpha = e^{(a_1+\alpha(a_2-a_1),\ a_4-\alpha(a_4-a_3))}$, $e^{\tilde{A}^i}{}_\beta = e^{(b_2-\beta(b_2-b_1),\ b_3+\beta(b_4-b_3))}$ (3.23)& (3.24)

$x = e^{a_1+\alpha(a_2-a_1)}$ and $x = e^{a_4-\alpha(a_4-a_3)}$

$$\mu_{e^{\tilde{A}^i}}(x) = \begin{cases} \frac{lnx-a_1}{a_2-a_1} & , \quad e^{a_1} \leq x \leq e^{a_2} \\ 1 & , \quad e^{a_2} \leq x \leq e^{a_3} \\ \frac{a_4-lnx}{a_4-a_3} & , \quad e^{a_3} \leq x \leq e^{a_4} \\ 0 & , \quad otherwise \end{cases} \qquad (3.25)$$

$x = e^{b_2-\beta(b_2-b_1)}$ and $x = e^{b_3+\beta(b_4-b_3)}$

$$\gamma_{e^{\tilde{A}^i}}(x) = \begin{cases} \frac{b_2-lnx}{b_2-b_1} & , \quad e^{b_1} \leq x \leq e^{b_2} \\ 0 & , \quad e^{b_2} \leq x \leq e^{b_3} \\ \frac{lnx-b_3}{b_4-b_3} & , \quad e^{b_3} \leq x \leq e^{b_4} \\ 1 & , \quad otherwise \end{cases} \qquad (3.26)$$

So $e^{\tilde{A}^i}$ represented by (3.25) and (3.26) is a trapezoidal shaped IFN. It can be approximated to TrIFN $e^{\tilde{A}^i} = (e^{b_1}, e^{a_1}, e^{b_2}, e^{a_2}, e^{a_3}, e^{b_3}, e^{a_4}, e^{b_4})$.

Theorem 3.8. If $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ is a positive TrIFN. Then
$ln(A) = [ln(b_1), ln(a_1), ln(b_2), ln(a_2), ln(a_3), ln(b_3), ln(a_4), ln(b_4)]$ is also a TrIFN.

Proof. To calculate logarithm of the TrIFN $\tilde{A}^i$ we first take the logarithm of the (α, β) −cut of $\tilde{A}^i$ using interval arithmetic.

$$ln(\tilde{A}^i{}_\alpha) = ln[a_1 + \alpha(a_2 - a_1), \qquad a_4 - \alpha(a_4 - a_3)]$$
$$= [ln(a_1 + \alpha(a_2 - a_1)), ln(a_4 - \alpha(a_4 - a_3))] \qquad (3.27)$$

$ln(\tilde{A}^i{}_\beta) = ln[b_2 - \beta(b_2 - b_1), b_3 + \beta(b_4 - b_3)] = [ln(b_2 - \beta(b_2 - b_1)), ln(b_3 + \beta(b_4 - b_3))]$ (3.28)

$x = ln[a_1 + \alpha(a_2 - a_1)]$ and $x = ln[a_4 - \alpha(a_4 - a_3)]$.

$$\mu_{ln(\tilde{A}^i)}(x) = \begin{cases} \frac{e^x - a_1}{a_2 - a_1} \,, & ln(a_1) \leq x \leq ln(a_2) \\ 1 \quad, & ln(a_2) \leq x \leq ln(a_3) \\ \frac{a_4 - e^x}{a_4 - a_3} \,, & ln(a_3) \leq x \leq ln(a_4) \\ 0 \quad, & otherwise \end{cases} \qquad (3.29)$$

$x = ln[b_2 - \beta(b_2 - b_1)]$ and $x = ln[b_3 + \beta(b_4 - b_3)]$.

$$\gamma_{ln(\tilde{A}^i)}(x) = \begin{cases} \frac{b_2 - e^x}{b_2 - b_1} \,, & ln(b_1) \leq x \leq ln(b_2) \\ 0 \quad, & ln(b_2) \leq x \leq ln(b_3) \\ \frac{e^x - b_3}{b_4 - b_3} \,, & ln(b_3) \leq x \leq ln(b_4) \\ 1 \quad, & otherwise \end{cases} \qquad (3.30)$$

So $ln(\tilde{A}^i)$ represented by (3.29) and (3.30) is a trapezoidal shaped IFN. It can be approximated to $ln(\tilde{A}^i) = [ln(b_1), ln(a_1), ln(b_2), ln(a_2), ln(a_3), ln(b_3), ln(a_4), ln(b_4)]$

Theorem 3.9. If $\tilde{A}^i = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ is a positive TrIFN then $(\tilde{A}^i)^{1/n}$ is also a TrIFN as

$$(\tilde{A}^i)^{1/n} = \left[(b_1)^{1/n}, (a_1)^{1/n}, (b_2)^{1/n}, (a_2)^{1/n}, (a_3)^{1/n}, (b_3)^{1/n}, (a_4)^{1/n}, (b_4)^{1/n}\right].$$

Proof. To calculate the nth root of the TrIFN $\tilde{A}^i$ we first take the nth root of the (α, β) −cuts of $\tilde{A}^i$ using interval arithmetic.

$$\left(\tilde{A}^i{}_\alpha\right)^{1/n} = [a_1 + \alpha(a_2 - a_1)]^{1/n}, \ [a_4 - \alpha(a_4 - a_3)]^{1/n} \qquad (3.31)$$

$$\left(\tilde{A}^i{}_\beta\right)^{1/n} = [b_2 - \beta(b_2 - b_1)]^{1/n}, \ [b_3 + \beta(b_4 - b_3)]^{1/n} \qquad (3.32)$$

$x = [a_1 + \alpha(a_2 - a_1)]^{1/n}$ and $x = [a_4 - \alpha(a_4 - a_3)]^{1/n}$.

$$\mu_{(\tilde{A}^i)^{1/n}}(x) = \begin{cases} \frac{x^n - a_1}{a_2 - a_1} \,, & (a_1)^{1/n} \leq x \leq (a_2)^{1/n} \\ 0 \quad, & (a_2)^{1/n} \leq x \leq (a_3)^{1/n} \\ \frac{a_4 - x^n}{a_4 - a_3} \,, & (a_3)^{1/n} \leq x \leq (a_4)^{1/n} \\ 1 \quad, & otherwise \end{cases} \qquad (3.33)$$

$x = [b_2 - \beta(b_2 - b_1)]^{1/n}$ and $x = [b_3 + \beta(b_4 - b_3)]^{1/n}$.

$$\gamma_{(\tilde{A}^i)^{1/n}}(x) = \begin{cases} \frac{b_2 - x^3}{b_2 - b_1} \,, & (b_1)^{1/n} \leq x \leq (b_2)^{1/n} \\ 0 \quad, & (b_2)^{1/n} \leq x \leq (b_3)^{1/n} \\ \frac{x^3 - b_3}{b_4 - b_3} \,, & (b_3)^{1/n} \leq x \leq (b_4)^{1/n} \\ 1 \quad, & otherwise \end{cases} \qquad (3.34)$$

So $(\tilde{A}^i)^{1/n}$ represented by (3.41) and (3.42) is a trapezoidal shaped IFN by

$$(\tilde{A}^i)^{1/n} = \left[(b_1)^{1/n}, (a_1)^{1/n}, (b_2)^{1/n}, (a_2)^{1/n}, (a_3)^{1/n}, (b_3)^{1/n}, (a_4)^{1/n}, (b_4)^{1/n}\right].$$

4. Conclusion

In this paper we have deliberated many operations of TrIFN based on $(\alpha, \beta) -$cut method including n^{th} root and exponentiation.

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