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A New Notion for Fuzzy Soft Normed Linear Space

Thangaraj Beaula*[1] and *M.Merlin Priyanga

Department of Mathematics, TBML College, Porayar, Tamil Nadu, India - 609 307
e-mail: edwinbeaula@yahoo.co.in
[1]Corresponding Author

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Abstract. In this paper the concept of fuzzy soft normed linear space of set of all soft points over a scalar field K is studied in a different way. The notion of $\alpha -$norm on a fuzzy soft metric space is established. Also the concepts like boundedness, Cauchy, convergence, continuity are defined. Some theorems related to these concepts are proved.

Keywords: Fuzzy soft norm, fuzzy soft metric, Cauchy convergence and continuity

AMS Mathematics Subject Classification (2010): 47L25

1. Introduction

In real life, most of the problems are uncertain or imprecise. This is due to the lack of information required. In order to deal such situations Zadeh [7] in 1965 introduced fuzzy sets. In fuzzy set theory thereis a lack of parameterization tools which lead Molodostov [3] in 1999 to introduce soft set theory. Maji et al [2] in 2001 studied the combination of fuzzy set theory and soft set theory and gave a new concept called fuzzy soft set. This new notion widened the approach of soft set from crisp cases to fuzzy cases. Topological study on fuzzy soft set theory was initiated by Tanay and Kandemir in [6]. In 2013 Zadeh [1] coined fuzzy soft norm over a set and established the relationship between fuzzy soft norm and fuzzy norm over a set.

In this paper, we first recall the definition of soft normed linear space which has been established in [4] and the definition of fuzzy normed linear space coined by Sadaati [5] in 2005.We continue our work by coining fuzzy soft normed linear space in a different way as a fuzzy set defined on the soft normed linear space. Fuzzy soft metric is defined using fuzzy soft norm, also convergent, Cauchy sequence, completeness, fuzzy soft α-norm, bounded sequence are defined with respect to fuzzy soft norm and fuzzy soft metric. Some related theorems using these concepts are proved. Fuzzy soft open ball, closed ball and sphere are defined.

2. Preliminaries

Definition 2.1. Let X by a vector space over a field $K(K = \mathbb{R})$ and the parameter set E be the real number set $\mathbb{R}$. Let (F,E) be a soft set over X. The soft set (F,E) is said to be a soft vector and denoted by $\tilde{x}_e$ if there is exactly one $e \in \mathrm{E}$, such that $\mathrm{F}(e) = \{x\}$ for some $x \in \mathrm{X}$ and $\mathrm{F}(e') = \phi,\ \forall\ e' \in E/\{e\}$.

The set of all soft vectors over $\tilde{\mathrm{X}}$ will be denoted by $\mathrm{SV}(\tilde{\mathrm{X}})$. The set $\mathrm{SV}(\tilde{\mathrm{X}})$ is called a soft vector space.

Definition 2.2. Let $\mathrm{SV}(\tilde{\mathrm{X}})$ be a soft vector space. Then a mapping

$\|\,.\,\| : \mathrm{SV}(\tilde{\mathrm{X}}) \to \mathbb{R}^+(\mathrm{E})$ is said to be a soft norm on $\mathrm{SV}(\tilde{\mathrm{X}})$, if $\|\,.\,\|$ satisfies the following conditions:

1) $\|\tilde{x}_e\| \tilde{\geq} \tilde{0}$ for all $\mathrm{SV}(\tilde{\mathrm{X}})$ and

   $\|\tilde{x}_e\| = \tilde{0} \Leftrightarrow \tilde{x}_e = \tilde{\theta}_0$

2) $\|\tilde{r}.\tilde{x}_e\| = |\tilde{r}|\|\tilde{x}_e\|$ for all $\tilde{x}_e \tilde{\in} \mathrm{SV}(\tilde{\mathrm{X}})$ for every soft scalar $\tilde{r}$

3) $\|\tilde{x}_e + \tilde{y}_{e'}\| = \|\tilde{x}_e\| + \|\tilde{y}_{e'}\|$ for all $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} \mathrm{SV}(\tilde{\mathrm{X}})$

The soft vector space $\mathrm{SV}(\tilde{\mathrm{X}})$ with a soft norm $\|\,.\,\|$ on $\tilde{\mathrm{X}}$ is said to be a soft normed linear space and is denoted by $(\tilde{\mathrm{X}}, \|\,.\,\|)$.

Definition 2.3. Let X be a linear space over the field F (real or complex) and $*$ is a continuous t-norm. A fuzzy subset N on $\mathrm{X} \times \mathbb{R}$, $\mathbb{R}$ -set of all real numbers is called a fuzzy norm on X if and only if for $x, y \in \mathrm{X}$ and $c \in \mathrm{F}$

1) $\forall\ t \in \mathbb{R}$ with $t \le 0$, $\mathrm{N}(x,t) = 0$
2) $\forall\ t \in \mathbb{R}$ with $t > 0$, $\mathrm{N}(x,t) = 1$ if and only if $x = 0$
3) $\forall\ t \in \mathbb{R}$ with $t > 0$, $\mathrm{N}(cx,t) = \mathrm{N}\left(x, \dfrac{t}{|c|}\right)$ if $c \neq 0$
4) $\forall\ s,t \in \mathbb{R},\ x, y \in \mathrm{X};\ \mathrm{N}(x+y, t+s) \ge \mathrm{N}(x,t) * \mathrm{N}(y,s)$
5) $\mathrm{N}(x,.)$ is a continuous nondecreasing function of $\mathbb{R}$ and $\lim\limits_{x \to \infty} \mathrm{N}(x,t) = 1$

The triplet $(\mathrm{X}, \mathrm{N}, *)$ will be referred to as a fuzzy normed linear space.

3. Fuzzy soft normed linear space

Definition 3. Let $\tilde{\mathrm{X}}$ be an absolute soft linear space over the scalar field K. Suppose $*$ is a continuous t-norm, $\mathbb{R}(\mathrm{A}^*)$ is the set of all non negative soft real numbers and $\mathrm{SSP}(\tilde{\mathrm{X}})$ denote the set of all soft points on $\tilde{\mathrm{X}}$. A fuzzy subset Γ on

$SSP(\tilde{X}) \times \mathbb{R}(A^*)$ is called a fuzzy soft norm on $\tilde{X}$ if and only if for $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} SSP(\tilde{X})$ and $\tilde{k} \tilde{\in} K$ (where $\tilde{k}$ is a soft scalar) the following conditions hold

1) $\Gamma(\tilde{x}_e, \tilde{t}) = 0 \ \forall\ \tilde{t} \tilde{\in} \mathbb{R}(A^*)$ with $\tilde{t} \tilde{\leq} \tilde{0}$
2) $\Gamma(\tilde{x}_e, \tilde{t}) = 1 \ \forall\ \tilde{t} \tilde{\in} \mathbb{R}(A^*)$ with $\tilde{t} \tilde{>} \tilde{0}$ if and only if $\tilde{x}_e = \tilde{\theta}_0$
3) $\Gamma(\tilde{k} \odot \tilde{x}_e, \tilde{t}) = \Gamma\left(\tilde{x}_e, \frac{\tilde{t}}{|\tilde{k}|}\right)$ if $\tilde{k} \neq \tilde{0}\ \forall\ \tilde{t} \tilde{\in} \mathbb{R}(A^*),\ \tilde{t} \tilde{>} \tilde{0}$
4) $\Gamma(\tilde{x}_e \oplus \tilde{y}_{e'}, \tilde{t} \oplus \tilde{s}) \tilde{\geq} \Gamma(\tilde{x}_e, \tilde{t}) * \Gamma(\tilde{y}_e, \tilde{s}),\ \forall \tilde{s}, \tilde{t} \tilde{\in} \mathbb{R}(A^*),\ \tilde{x}_e, \tilde{y}_{e'} \tilde{\in} SSP(\tilde{X})$
5) $\Gamma(\tilde{x}_e, .)$ is a continuous nondecreasing function of $\mathbb{R}(A^*)$ and $\lim_{\tilde{t} \to \infty} \Gamma(\tilde{x}_e, \tilde{t}) = 1$

The triplet $(\tilde{X}, \Gamma, *)$ will be referred to as a fuzzy soft normed linear space.

Definition 3.2. Let $(\tilde{X}, \Gamma, *)$ be a fuzzy soft normed linear space and $\tilde{t} \tilde{>} \tilde{0}$ be a soft real number. We define an open ball, a closed ball and a sphere with centre at $\tilde{x}_{e_1}$ and radius α as follows

$$B(\tilde{x}_{e_1}, \alpha, \tilde{t}) = \left\{\tilde{y}_{e_2} \tilde{\in} SSP(\tilde{X}) : \Gamma(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}) \tilde{>} 1 - \alpha\right\}$$

$$\bar{B}(\tilde{x}_{e_1}, \alpha, \tilde{t}) = \left\{\tilde{y}_{e_2} \tilde{\in} SSP(\tilde{X}) : \Gamma(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}) \tilde{\geq} 1 - \alpha\right\}$$

$$S(\tilde{x}_{e_1}, \alpha, \tilde{t}) = \left\{\tilde{y}_{e_2} \tilde{\in} SSP(\tilde{X}) : \Gamma(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}) = 1 - \alpha\right\}$$

$SFS\left(B(\tilde{x}_{e_1}, \alpha, \tilde{t})\right)$, $SFS\left(\bar{B}(\tilde{x}_{e_1}, \alpha, \tilde{t})\right)$ and $SFS\left(S(\tilde{x}_{e_1}, \alpha, \tilde{t})\right)$ are called a fuzzy soft open ball, a fuzzy soft closed ball and a fuzzy soft sphere respectively with centre $\tilde{x}_{e_1}$ at and radius α.

Definition 3.3. A mapping $\Delta : SSP(\tilde{X}) \times SSP(\tilde{X}) \times \mathbb{R}(A^*) \to [0,1]$ is said to be a fuzzy soft metric on the soft set $\tilde{X}$ if Δ satisfies the following conditions

1) $\Delta(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) = 0$, for all $\tilde{t} \tilde{\leq} \tilde{0}$
2) $\Delta(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) = 1$, for all $\tilde{t} \tilde{>} \tilde{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$
3) $\Delta(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) = \Delta(\tilde{y}_{e_2}, \tilde{x}_{e_1}, \tilde{t})$

4) $\Delta\left(\tilde{x}_{e_1},\tilde{z}_{e_3},\tilde{s}\oplus\tilde{t}\right)\tilde{\geq}\Delta\left(\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{s}\right)*\Delta\left(\tilde{y}_{e_2},\tilde{z}_{e_3},\tilde{t}\right)$ for all $\tilde{t},\tilde{s}\tilde{>}\tilde{0}$

5) $\Delta\left(\tilde{x}_{e_1},\tilde{y}_{e_2},.\right):(0,\infty)\rightarrow[0,1]$ is continuous

The soft set $\tilde{X}$ with a fuzzy soft metric Δ is called a fuzzy soft metric space and denoted by $\left(\tilde{X},\Delta,*\right)$.

Definition 3.4. Let be a sequence $\left\{\tilde{x}_{e_j}^n\right\}$ of soft vectors in a fuzzy soft normed linear space$\left(\tilde{X},\Gamma,*\right)$. Then the sequence converges to $\tilde{x}_{e_j}^0$ with respect to fuzzy soft norm Γ if $\Gamma\left(\tilde{x}_{e_j}^n-\tilde{x}_{e_j}^0,\tilde{t}\right)\tilde{\geq}1-\alpha$ for every $n\geq n_0$ and $\alpha\in(0,1]$ where n_0 is a positive integer and $\tilde{t}\tilde{>}\tilde{0}$.

Or

$$\lim_{n\to\infty}\Gamma\left(\tilde{x}_{e_j}^n-\tilde{x}_{e_j}^0,\tilde{t}\right)=1,\text{ as }\tilde{t}\to\infty$$

Similarly if $\lim_{n\to\infty}\Delta\left(\tilde{x}_{e_j}^n,\tilde{x}_{e_j}^0,\tilde{t}\right)=1$, as $\tilde{t}\to\infty$ then $\left\{\tilde{x}_{e_j}^n\right\}$ is a convergent sequence in fuzzy soft metric space $\left(\tilde{X},\Delta,*\right)$.

Definition 3.5. A sequence $\left\{\tilde{x}_{e_j}^n\right\}$ in a fuzzy soft normed linear space $\left(\tilde{X},\Gamma,*\right)$ is said to be a Cauchy sequence with respect to the fuzzy soft norm Γ if $\Gamma\left(\tilde{x}_{e_j}^n-\tilde{x}_{e_j}^m,\tilde{t}\right)\tilde{\geq}1-\alpha$ for every $n,m\geq n_0$ and $\alpha\in(0,1]$ where n_0 is a positive integer and $\tilde{t}\tilde{>}\tilde{0}$.

Or

$$\lim_{n,m\to\infty}\Gamma\left(\tilde{x}_{e_j}^n-\tilde{x}_{e_j}^m,\tilde{t}\right)=1,\text{ as }\tilde{t}\to\infty$$

Similarly if $\lim_{n\to\infty}\Delta\left(\tilde{x}_{e_j}^n,\tilde{x}_{e_j}^0,\tilde{t}\right)=1$ as $\tilde{t}\to\infty$ then $\left\{\tilde{x}_{e_j}^n\right\}$ is a Cauchy sequence in fuzzy soft metric space $\left(\tilde{X},\Delta,*\right)$.

Definition 3.6. Let $\left(\tilde{X},\Gamma,*\right)$ be a fuzzy soft normed linear space. Then $\left(\tilde{X},\Gamma,*\right)$ is said to be complete if every Cauchy sequence in $\mathrm{SSP}\left(\tilde{X}\right)$ converges to a soft vector of $\mathrm{SSP}\left(\tilde{X}\right)$.

Definition 3.7. Let $\left\{\tilde{x}_{e_j}^{n}\right\}$ a sequence in a fuzzy soft metric space $\left(\tilde{X},\Delta,*\right)$. Then the sequence $\left\{\tilde{x}_{e_j}^{n}\right\}$ is said to be a bounded sequence with respect to the fuzzy soft metric Δ if $\left\|\tilde{x}_{e_j}^{n}-\tilde{x}_{e_j}^{m}\right\|_{\alpha}\leq\tilde{M}$.

By definition

$$\left\|\tilde{x}_{e_j}^{n}-\tilde{x}_{e_j}^{m}\right\|_{\alpha}=\inf\left\{\tilde{t}:\Delta\left(\tilde{x}_{e_j}^{n},\tilde{x}_{e_j}^{m},\tilde{t}\right)\tilde{\geq}\alpha,\alpha\in(0,1]\right\}$$

That is,

$\left\{\tilde{x}_{e_j}^{n}\right\}$ is said to be bounded if there exists a positive integer N depending on M such that $\Delta\left(\tilde{x}_{e_j}^{n},\tilde{x}_{e_j}^{m},\tilde{t}\right)\tilde{\geq}\alpha,\forall n,m\geq N(M)$.

4. Theorems

Theorem 4.1. Every convergent sequence is Cauchy sequence.

Proof: Let $\left\{\tilde{x}_{e_j}^{n}\right\}$ be a sequence in a fuzzy soft normed linear space $\left(\tilde{X},\Gamma,*\right)$. Consider $\left\{\tilde{x}_{e_j}^{n}\right\}$converges to $\tilde{x}_{e_j}^{0}$.

Then we have $\Gamma\left(\tilde{x}_{e_j}^{n}-\tilde{x}_{e_j}^{0},\tilde{t}\right)\tilde{\geq}1-\alpha$ for every $n\geq n_0$ and $\alpha\in(0,1]$ where $n_0\in N$ and $\tilde{t}\,\tilde{>}\,\tilde{0}$.

Therefore,

$$\begin{aligned}
\Gamma\left(\tilde{x}_{e_j}^{n}-\tilde{x}_{e_j}^{m},\tilde{t}\right) &= \Gamma\left(\tilde{x}_{e_j}^{n}-\tilde{x}_{e_j}^{m}\oplus\tilde{x}_{e_j}^{0}-\tilde{x}_{e_j}^{0},\tilde{t}\right)\\
&=\Gamma\left(\left(\tilde{x}_{e_j}^{n}-\tilde{x}_{e_j}^{0}\right)\oplus\left(\tilde{x}_{e_j}^{m}-\tilde{x}_{e_j}^{0}\right),\tilde{t}\right)\\
&\tilde{\geq}\ \Gamma\left(\tilde{x}_{e_j}^{n}-\tilde{x}_{e_j}^{0},\frac{\tilde{t}}{2}\right)*\Gamma\left(\tilde{x}_{e_j}^{m}-\tilde{x}_{e_j}^{0},\frac{\tilde{t}}{2}\right)\\
&\tilde{\geq}\ (1-\alpha)*(1-\alpha)\\
&=\min\{1-\alpha,1-\alpha\}\\
&=1-\alpha
\end{aligned}$$

$\Gamma\left(\tilde{x}_{e_j}^{n}-\tilde{x}_{e_j}^{m},\tilde{t}\right)\tilde{\geq}1-\alpha$ for every $n,m\geq n_0$ and $\alpha\in(0,1]$.

Thus $\left\{\tilde{x}_{e_j}^{n}\right\}$ is a Cauchy sequence.

Theorem 4.2. If $\left(\tilde{X},\Gamma,*\right)$ is a fuzzy soft normed linear space then

a) The function $(\tilde{x}_e, \tilde{y}_{e'}) \to \tilde{x}_e \oplus \tilde{y}_{e'}$ is continuous.

b) The function $(\tilde{c}, \tilde{x}_e) \to \tilde{c} \odot \tilde{x}_e$ is continuous.

where $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} \text{SSP}(\tilde{X})$ and $\tilde{c} \tilde{\in} K$.

Proof:

a) If $\tilde{x}_{e_n} \to \tilde{x}_e$ and $\tilde{y}_{e'_n} \to \tilde{y}_{e'}$ then as $n \to \infty$

$$\begin{aligned}\Gamma\left(\left(\tilde{x}_{e_n} \oplus \tilde{y}_{e'_n}\right) - \left(\tilde{x}_e \oplus \tilde{y}_{e'}\right), \tilde{t}\right) &= \Gamma\left(\tilde{x}_{e_n} \oplus \tilde{y}_{e'_n} - \tilde{x}_e - \tilde{y}_{e'}, \tilde{t}\right) \\ &= \Gamma\left(\left(\tilde{x}_{e_n} - \tilde{x}_e\right) \oplus \left(\tilde{y}_{e'_n} - \tilde{y}_{e'}\right), \tilde{t}\right) \\ &= \Gamma\left(\tilde{x}_{e_n} - \tilde{x}_e, \frac{\tilde{t}}{2}\right) * \Gamma\left(\tilde{y}_{e'_n} - \tilde{y}_{e'}, \frac{\tilde{t}}{2}\right) \\ &\to 1 \qquad \text{as } \tilde{t} \to \infty\end{aligned}$$

Thus the function $(\tilde{x}_e, \tilde{y}_{e'}) \to \tilde{x}_e \oplus \tilde{y}_{e'}$ is continuous.

b) If $\tilde{x}_{e_n} \to \tilde{x}_e$, $\tilde{c}_n \to \tilde{c}$ and $\tilde{c}_n \neq \tilde{0}$ then as $n \to \infty$

$$\begin{aligned}\Gamma\left(\tilde{c}_n \odot \tilde{x}_{e_n} - \tilde{c} \odot \tilde{x}_e, \tilde{t}\right) &= \Gamma\left(\tilde{c}_n \odot \tilde{x}_{e_n} \oplus \tilde{c}_n \odot \tilde{x}_e - \tilde{c}_n \odot \tilde{x}_e - \tilde{c} \odot \tilde{x}_e, \tilde{t}\right) \\ &= \Gamma\left(\tilde{c}_n \odot \left(\tilde{x}_{e_n} - \tilde{x}_e\right) \oplus \left(\tilde{c}_n - \tilde{c}\right) \odot \tilde{x}_e, \tilde{t}\right) \\ &\tilde{\geq} \Gamma\left(\tilde{c}_n \odot \left(\tilde{x}_{e_n} - \tilde{x}_e\right), \frac{\tilde{t}}{2}\right) * \Gamma\left(\left(\tilde{c}_n - \tilde{c}\right) \odot \tilde{x}_e, \frac{\tilde{t}}{2}\right) \\ &= \Gamma\left(\left(\tilde{x}_{e_n} - \tilde{x}_e\right), \frac{\tilde{t}}{2\tilde{c}_n}\right) * \Gamma\left(\tilde{x}_e, \frac{\tilde{t}}{2\left(\tilde{c}_n - \tilde{c}\right)}\right) \\ &\to 1 \qquad \text{as } \tilde{t} \to \infty\end{aligned}$$

Thus the function $(\tilde{c}, \tilde{x}_e) \to \tilde{c} \odot \tilde{x}_e$ is continuous.

Theorem 4.3. Limit of a sequence in a fuzzy soft normed linear space, if exists is unique.

Proof: Let $\left\{\tilde{x}^n_{e_j}\right\}$ be a sequence in a fuzzy soft normed linear space $(\tilde{X}, \Gamma, *)$.

Such that

$$\lim_{n\to\infty} \Gamma\left(\tilde{x}^n_{e_j} - \tilde{x}_e, \tilde{t}\right) = \tilde{1}$$

$$\lim_{n\to\infty} \Gamma\left(\tilde{x}^n_{e_j} - \tilde{x}_{e'}, \tilde{t}\right) = \tilde{1}$$

are two limits of the sequence $\left\{\tilde{x}^n_{e_j}\right\}$.

Then by definition there exists positive integers n_1, n_2 such that

$\Gamma\left(\tilde{x}^n_{e_j} - \tilde{x}_e, \tilde{t}\right) \tilde{\geq} 1-\alpha$ for every $n \geq n_1$ and $\alpha \in (0,1]$

$\Gamma\left(\tilde{x}^n_{e_j} - \tilde{x}_{e'}, \tilde{t}\right) \tilde{\geq} 1-\alpha$ for every $n \geq n_2$ and $\alpha \in (0,1]$

Choose $n \geq n_0$, $n_0 = \min\{n_1, n_2\}$

$$\Gamma\left(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}\right) = \Gamma\left(\tilde{x}_e - \tilde{x}^n_{e_j} \oplus \tilde{x}^n_{e_j} - \tilde{x}_{e'}, \tilde{t}\right)$$

$$= \Gamma\left(\left(\tilde{x}^n_{e_j} - \tilde{x}_e\right) \oplus \left(\tilde{x}^n_{e_j} - \tilde{x}_{e'}\right), \tilde{t}\right)$$

$$\tilde{\geq} \Gamma\left(\tilde{x}^n_{e_j} - \tilde{x}_e, \frac{\tilde{t}}{2}\right) * \Gamma\left(\tilde{x}^n_{e_j} - \tilde{x}_{e'}, \frac{\tilde{t}}{2}\right)$$

$$\tilde{\geq} (1-\alpha) * (1-\alpha)$$

$$= \min\{1-\alpha, 1-\alpha\}$$

$$= 1-\alpha$$

$$\Gamma\left(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}\right) \tilde{\geq} 1-\alpha$$

That implies,

$$\lim_{n\to\infty} \Gamma\left(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}\right) = 1$$

$$\Gamma\left(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}\right) = 1$$

By the definition of F.S normed linear space

$\Gamma\left(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}\right) = 1$ with $\tilde{t} \tilde{>} \tilde{0}$ if and only if $\tilde{x}_e - \tilde{x}_{e'} = \tilde{\theta}_0$

Hence $\tilde{x}_e = \tilde{x}_{e'}$

Theorem 4.4. Every fuzzy soft normed linear space is a fuzzy soft metric space.

Proof: Let $\left(\tilde{X}, \Gamma, *\right)$ be a fuzzy soft normed space.

Define the fuzzy soft metric by $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = \Gamma\left(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}\right)$ (4.4.1)

for every $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} SSP\left(\tilde{X}\right)$.

Then it is clear to show that the fuzzy soft metric axioms are satisfied.

1) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = \Gamma\left(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}\right) = 0$ if $\tilde{t} \tilde{\leq} \tilde{0}$

2) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = \Gamma\left(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}\right) = 1$ if $\tilde{t} \tilde{>} \tilde{0}$

3) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = \Gamma\left(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}\right)$

$= \Gamma\left(\tilde{y}_{e_2} - \tilde{x}_{e_1}, \tilde{t}\right)$

$= \Delta\left(\tilde{y}_{e_2}, \tilde{x}_{e_1}, \tilde{t}\right)$

$\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = \Delta\left(\tilde{y}_{e_2}, \tilde{x}_{e_1}, \tilde{t}\right)$

4) $\Delta\left(\tilde{x}_{e_1}, \tilde{z}_{e_3}, \tilde{s} \oplus \tilde{t}\right) = \Gamma\left(\tilde{x}_{e_1} - z'_{e_3}, \tilde{s} \oplus \tilde{t}\right)$

$= \Gamma\left(\tilde{x}_{e_1} - \tilde{y}_{e_2} + \tilde{y}_{e_2} - \tilde{z}_{e_3}, \tilde{s}\right)$

$\tilde{\geq} \Gamma\left(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{s}\right) * \Gamma\left(\tilde{x}_{e_1} - \tilde{z}_{e_3}, \tilde{t}\right)$

$= \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{s}\right) * \Delta\left(\tilde{x}_{e_1}, \tilde{z}_{e_3}, \tilde{t}\right)$

$\Delta\left(\tilde{x}_{e_1}, \tilde{z}_{e_3}, \tilde{s} \oplus \tilde{t}\right) \tilde{\geq} \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{s}\right) * \Delta\left(\tilde{x}_{e_1}, \tilde{z}_{e_3}, \tilde{t}\right)$

5) By the definition (4.4.1) of Δ we get Δ is continuous and $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, .\right) : (0, \infty) \to [0,1]$

Theorem 4.5. Let $\left(\tilde{X}, \Delta, *\right)$ be a fuzzy soft metric space.

Define $\left\|\tilde{x}_{e_1} - \tilde{y}_{e_2}\right\|_\alpha = \inf\left\{\tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha, \alpha \in (0,1]\right\}$.

Then $\left\{\| \ \|_\alpha : \alpha \in (0,1]\right\}$ is an ascending family of norms of fuzzy soft real numbers in $\mathbb{R}\left(A^*\right)$ on $\tilde{X}$.

Proof: Let $\left(\tilde{X}, \Delta, *\right)$ be a fuzzy soft metric space and

$\left\|\tilde{x}_{e_1} - \tilde{y}_{e_2}\right\|_\alpha = \inf\left\{\tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha, \alpha \in (0,1]\right\}$.

Then

1) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = 0$ for all $\tilde{t} \tilde{\leq} \tilde{0}$

$\Rightarrow \left\{\tilde{t} : \Delta\left(\tilde{x}_e, \tilde{y}_e, \tilde{t}\right) \tilde{\geq} \alpha, \alpha \in (0,1]\right\} = \tilde{0}$

$\inf\left\{\tilde{t} : \Delta\left(\tilde{x}_e, \tilde{y}_e, \tilde{t}\right) \tilde{\geq} \alpha, \alpha \in (0,1]\right\} = \tilde{0}$

Thus $\left\|\tilde{x}_e - \tilde{y}_e\right\|_\alpha = \tilde{0}$

2) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = 1$ for all $\tilde{t} \tilde{>} \tilde{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$

$$\Rightarrow \left\{ \tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha, \alpha \in (0,1] \right\} = \tilde{1}$$

$$\inf \left\{ \tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha, \alpha \in (0,1] \right\} = \tilde{1}$$

Thus $\left\| \tilde{x}_{e_1} - \tilde{y}_{e_2} \right\|_{\alpha} = \tilde{1}$

3) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = \Delta\left(\tilde{y}_{e_2}, \tilde{x}_{e_1}, \tilde{t}\right)$

$$\begin{aligned} \left\| \tilde{x}_{e_1} - \tilde{y}_{e_2} \right\|_{\alpha} &= \inf \left\{ \tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha, \alpha \in (0,1] \right\} \\ &= \inf \left\{ \tilde{t} : \Delta\left(\tilde{y}_{e_2}, \tilde{x}_{e_1}, \tilde{t}\right) \tilde{\geq} \alpha, \alpha \in (0,1] \right\} \\ &= \left\| \tilde{y}_{e_2} - \tilde{x}_{e_1} \right\|_{\alpha} \end{aligned}$$

$$\left\| \tilde{x}_{e_1} - \tilde{y}_{e_2} \right\|_{\alpha} = \left\| \tilde{y}_{e_2} - \tilde{x}_{e_1} \right\|_{\alpha}$$

4) Consider

$$\begin{aligned} \left\| \tilde{x}_{e_1} - \tilde{y}_{e_2} \right\|_{\alpha} + \left\| \tilde{x}_{e_1} - \tilde{z}_{e_3} \right\|_{\alpha} &= \inf \left\{ \tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha, \alpha \in (0,1] \right\} \\ &\quad + \inf \left\{ \tilde{s} : \Delta\left(\tilde{x}_{e_1}, \tilde{z}_{e_3}, \tilde{s}\right) \tilde{\geq} \alpha, \alpha \in (0,1] \right\} \\ &= \inf \left\{ \tilde{t} \oplus \tilde{s} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha, \Delta\left(\tilde{x}_{e_1}, \tilde{z}_{e_3}, \tilde{s}\right) \tilde{\geq} \alpha \right\} \end{aligned}$$

$$\begin{aligned} \left\| \tilde{x}_{e_1} - \tilde{y}_{e_2} \right\|_{\alpha} + \left\| \tilde{x}_{e_1} - \tilde{z}_{e_3} \right\|_{\alpha} &\tilde{\leq} \inf \left\{ \tilde{t} \oplus \tilde{s} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2} \oplus \tilde{z}_{e_3}, \tilde{t} \oplus \tilde{s}\right) \tilde{\geq} \alpha \right\} \\ &= \inf \left\{ \tilde{r} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2} \oplus \tilde{z}_{e_3}, \tilde{r}\right) \tilde{\geq} \alpha \right\}, \tilde{r} = \tilde{t} \oplus \tilde{s} \\ &= \left\| \tilde{x}_{e_1} - \left(\tilde{y}_{e_2} \oplus \tilde{z}_{e_3}\right) \right\|_{\alpha} \end{aligned}$$

$$\left\| \tilde{x}_{e_1} - \left(\tilde{y}_{e_2} \oplus \tilde{z}_{e_3}\right) \right\|_{\alpha} \tilde{\geq} \left\| \tilde{x}_{e_1} - \tilde{y}_{e_2} \right\|_{\alpha} + \left\| \tilde{x}_{e_1} - \tilde{z}_{e_3} \right\|_{\alpha}$$

Hence, $\left\{ \| \ \|_{\alpha} : \alpha \in (0,1] \right\}$ is an $\alpha-$norm induced by the fuzzy soft metric on $\tilde{X}$.

Let $0 < \alpha_1 < \alpha_2$.

Then

$$\left\| \tilde{x}_{e_1} - \tilde{y}_{e_2} \right\|_{\alpha_1} = \inf \left\{ \tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha_1 \right\}$$

$$\left\| \tilde{x}_{e_1} - \tilde{y}_{e_2} \right\|_{\alpha_2} = \inf \left\{ \tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha_2 \right\}$$

As $\alpha_1 < \alpha_2$,

$$\left\{\tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha_2\right\} \subset \left\{\tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha_1\right\}$$

$$\inf\left\{\tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha_1\right\} \tilde{\leq} \inf\left\{\tilde{t} : \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} \alpha_2\right\}$$

Therefore, $\left\|\tilde{x}_{e_1} - \tilde{y}_{e_2}\right\|_{\alpha_1} \tilde{\leq} \left\|\tilde{x}_{e_1} - \tilde{y}_{e_2}\right\|_{\alpha_2}$

Hence $\left\{\| \|_{\alpha} : \alpha \in (0,1]\right\}$ is an ascending family of norms of $\alpha-$fuzzy soft real numbers $\tilde{X}$ in induced by fuzzy soft metric on $\tilde{X}$.

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