# A Linear Fractional Interval Transportation Problem with and Without Budgetary Constraints 

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#### Abstract

In this paper, we consider a linear fractional interval transportation problem with and without budgetary constraints. The decision makers can make the correct decisions according to their budget by finding the number of units transported between the intervals of the total supply / total demand. Here, a solution procedure is provided and it is verified by means of an example.


Keywords: Linear Interval Fractional Transportation Problem, Budgetary constraint, Upper Bound and lower Bound.
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## 1. Introduction

The transportation problem (TP) is a special class of linear programming problem, which deals with shipping commodities from sources to destinations. Here, we consider the Transportation problems with fractional objective function, since in many real life situations where an individual or a group of community is faced with the problem of maintaining good ratios between some very important crucial parameters concerned with the transportation of commodities from certain sources to various destinations. Also,we consider the situation in which commodity can vary between source and destination so that interval method is used, and with and without budgetary constraints. Khanna et al. [4] introduced an algorithm for solving transportation flow under budgetary constraints. Weighted goal programming for unbalanced single objective transportation problem with budgetary constraint has been discussed by Kishore and Jayswal [2]. Pandian and Natarajan [3] introduced the separationmethod for finding an optimal solution to ainterval transportation problem. Lin and Cheng [5] gave a genetic algorithm for solving a transportation network under a budget Constraint.

The paper is organized as follows; The Mathematical formulation of the linear fractional interval transportation problem is given in Section 2. The Section 3 explains the definitions of the basic arithmetic operators and partial ordering on closed bounded intervals. In Section 4,an Algorithm is proposed to solve the linear fractional interval transportation problem. A numerical example is given in Section 5 and the conclusion of the paper is given in Section 6.

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## 2. Mathematical formulation

The linear fractional interval transportation problem is defined as follows
(P) $n\left[z_{1}, z_{2}\right]=\left[\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{i j}{ }^{1} x_{i j}}{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{i j}{ }^{1} x_{i j}}, \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{i j}{ }^{2} x_{i j}}{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{i j}{ }^{2} x_{i j}}\right]$

Subject to

$$
\begin{gathered}
\sum_{j=1}^{n}\left[x_{i j}, y_{i j}\right]=\left[a_{i}, p_{i}\right], i=1,2, \ldots \ldots m \\
\sum_{i=1}^{n}\left[x_{i j}, y_{i j}\right]=\left[b_{j}, q_{j}\right], j=1,2, \ldots \ldots \ldots n \\
\sum_{i=1}^{m} a_{i}=\sum_{j=1}^{n} b_{j} \text { and } \sum_{i=1}^{m} p_{i}=\sum_{j=1}^{n} q_{j} \\
x_{i j} \geq 0, y_{i j} \geq 0
\end{gathered}
$$

where $\left[a_{i}, p_{i}\right]$ is $\mathrm{i}^{\text {th }}$ source,$\left[b_{j}, q_{j}\right]$ is the $\mathrm{j}^{\text {th }}$ destination , $\mathrm{c}_{\mathrm{ij}}$ is the Total actual Transportation cost, $\mathrm{d}_{\mathrm{ij}}$ is the Total standard Transportation cost from $\mathrm{i}^{\text {th }}$ to $\mathrm{j}^{\text {th }}$ destination.

The Problem (P) can be stated as follows:
Consider the upper bound of the problem ( P ),
(UP)Min $z_{2}=\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{i j}{ }^{2} x_{i j}}{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{i j}{ }^{2} x_{i j}}$
Subject to

$$
\begin{gathered}
\sum_{j=1}^{n} x_{i j}=p_{i}, i=1,2, \ldots \ldots m \\
\sum_{i=1}^{n} x_{i j}=q_{j}, j=1,2, \ldots \ldots \ldots n \\
\text { and } x_{i j} \geq 0
\end{gathered}
$$

Now, Considering the lower bound of ( P ) as ,

$$
(\mathrm{LP}) M i n z_{1}=\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{i j}{ }^{1} x_{i j}}{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{i j}{ }^{1} x_{i j}}
$$

Subject to

$$
\begin{gathered}
\sum_{j=1}^{n} x_{i j}=a_{i}, i=1,2, \ldots \ldots m \\
\sum_{i=1}^{n} x_{i j}=b_{j}, j=1,2, \ldots \ldots \ldots n \\
\text { and } x_{i j} \geq 0
\end{gathered}
$$

Considering the set $\left\{y_{i j}\right.$, for all I and j$\}$ is an optimal solution of the upper bound transportation problem (UP ) and the set $\left\{x_{i j}\right.$, for all I and j$\}$ is an optimal solution of the lower bound transportation problem (LP) then, for the problem (P), the optimal solution is the set of intervals $\left\{\left[x_{i j}, y_{i j}\right]\right.$,foralliandj $\}$ provided $x_{i j} \leq y_{i j}$, for all i and j .

## 3. Preliminaries

Let $\mathrm{D}=\{[\mathrm{a}, \mathrm{b}], \mathrm{a} \leq \mathrm{b}$ and a and b are in R$\}$ denote the set of all closed bounded intervals on the real line R .

Definition 3.1. Let $\mathrm{A}=[\mathrm{a}, \mathrm{b}]$ and $\mathrm{B}=[\mathrm{c}, \mathrm{d}]$ be in Dd . Then,
(i) $A \oplus B=[a+c, b+d]$ and
( ii ) $\mathrm{A} \otimes \mathrm{B}=[\mathrm{p}, \mathrm{q}]$ where $\mathrm{p}=\min \{\mathrm{ac}, \mathrm{ad}, \mathrm{bc}, \mathrm{bd}\}$ and $\mathrm{q}=\max \{\mathrm{ac}, \mathrm{ad}, \mathrm{bc}, \mathrm{bd}\}$
Definition 3.2. Let $A=[a, b]$ and $B=[c, d]$ be in $D$. Then,
(i) A $\leq$ B if a $\leq$ c and $\mathrm{b} \leq \mathrm{d}$
(ii) $A \geq B$ if $a \geq c$ and $b \geq d$ and
(iii) $A=B$ if $a=c$ and $b=d$

## 4. Interval fractional transportation problem

The interval fractional-point method proceeds as follows:
Step1: Find the optimal solution for the Upper bound of the transportation problem by using [1] and let it be $\left\{y_{i j}\right.$, for all i and $\left.j\right\}$ Next, find the optimal solution of the lower bound of the transportation problem and let it be $\left\{x_{i j}\right.$, for all i and j$\}$
Step 2: Write the optimal solution of the problem as $\left\{\left[x_{i j}, y_{i j}\right]\right.$, for all i and j$\}$ and the optimal objective value of the problem ( P ) is $\left[Z_{L}, Z_{U}\right]$

Step 3: Let $Z \in\left[Z_{1}, Z_{2}\right]$ be the given budget cost for Upper bound transportation, we write Z in the form, $\mathrm{Z}=Z_{L}+\left[Z_{U}-Z_{L}\right] \mu$ for some $\mu, 0 \leq \mu \leq 1$. This implies $\mu=\frac{Z-Z_{L}}{Z_{U}-Z_{L}}$

Step 4: (i) When Budget is not given :
Compute the values of decision variables as $x_{i j}=\left[x_{i j}, y_{i j}\right]$ and
$\left[x_{i j}, y_{i j}\right]=x_{i j}+\left(y_{i j}-x_{i j}\right) \mu$, where $0 \leq \mu \leq 1$, and the unit transported cost.
(ii) When budget is given :

Find the value of $\mu$ and the decision variables using above (i) and the transportation unit cost .

## 5. Numerical example

There are two godowns (source points) from where paddy are supplied to three different stores (Demand points), $c_{i j}$ 's are the actual cost coefficient in rupee per ton, $d_{i j}$ 's are the standard cost coefficient in rupee per ton and $a_{i}, p_{i}, b_{j}, q_{j}$ are expressed in lakhs. The Transportation table is given below:

| $[1,3]$ |  | $[2,5]$ |  | $[0,3]$ |  |  |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- |
|  | $[2,5]$ |  | $[3,5]$ |  | $[4,6]$ |  |
| $[3,2], 30]$ |  |  |  |  |  |  |
|  |  | $[1,1]$ |  | $[1,2]$ |  | $[1,3]$ |
|  |  |  | $[5,7]$ | $[30,40]$ |  |  |

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| $[20,25]$ | $[20,25]$ | $[20,20]$ | $[60,70]$ |
| :---: | :--- | :--- | :--- |

Determine an optimal distribution plan to transport the items from the source points to the destination points for the budget of $75 \%$

First considering the UP of the problem


Using North West corner Rule Finding the initial basic feasible solution and then checking its optimality by finding the variables $u_{i}^{\prime}, v_{j}^{\prime}$ and $u_{i}^{\prime \prime}, v_{j}^{\prime \prime}$ associated with the numerator and denominator of objective, where $u_{i}^{\prime}$ and $u_{i}^{\prime \prime}, \mathrm{i}=1,2, \ldots \ldots \mathrm{~m}$, are corresponding to supply constraints and $v_{j}^{\prime}, v_{j}^{\prime \prime}, \mathrm{j}=1,2, \ldots . \mathrm{n}$, are corresponding to demand constraints, we get


The Minimum Transportation cost $(Z)=0.807$ units
Since the Reduced cost $\Delta_{13}$ and $\Delta_{21}$ are $\geq 0$, the current solution is optimal .
Hence, The Optimal solution to UB is
$y_{11}=25, y_{12}=5, y_{22}=20, y_{23}=20$
Now Considering the Lower Bound of the transportation problem,

A Linear Fractional Interval Transportation Problem ... Constraints


Again using the same procedure, we get,


The Minimum Transportation cost $(Z)=0.5$ units
Since the Reduced cost $\Delta_{13}$ and $\Delta_{21}$ are $\geq 0$, the current solution is optimal .
Hence, The Optimal solution to LB is
$x_{11}=20, x_{12}=10, x_{22}=10, x_{23}=20$.
So, the optimal solution to the given problem is
$\left[x_{11}, y_{11}\right]=[20,25],\left[x_{12}, y_{12}\right]=[10,5],\left[x_{22}, y_{22}\right]=[10,20]$,
$\left[x_{23}, y_{23}\right]=[20,20]$
and the minimum interval transportation cost is [ $0.5,0.807$ ].
The total transportation cost, $Z=0.5+0.307 \mu$ where $Z$ is the given budget.
This implies by $[6,7]$ that $\mu=\frac{Z-0.5}{0.307}$ and
If Budget is given as $\mathrm{Z}=75 \%$ then $\mu=0.814$ and the Optimal Solution to the UB is
$x_{11}=24.07, x_{12}=5.93, x_{22}=18.14, x_{23}=20$.
The Total Number of units transported $=68.14$ tons.
And when Budget is not given then the different values of $\mu$ is given below :
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| $\mu$ | Z | Decision Variables | Units <br> Transported |
| :---: | :---: | :---: | :---: |
| 0 | 0.5 | $x_{11}=20, x_{12}=10$, <br> $x_{22}=10, x_{23}=20$ | 60 |
| 0.1 | 0.5307 | $20.5,9.5,11,20$ | 61 |
| 0.2 | 0.5614 | $21,9,12,20$ | 62 |
| 0.3 | 0.5921 | $21.5,8.5,13,20$ | 63 |
| 0.4 | 0.6228 | $22,8,14,20$ | 64 |
| 0.5 | 0.6535 | $22.5,7.5,15,20$ | 65 |
| 0.6 | 0.6842 | $23,7,16,20$ | 66 |
| 0.7 | 0.7149 | $23.5,6.5,17,20$ | 67 |
| 0.8 | 0.7456 | $24,6,18,20$ | 68 |
| 0.9 | 0.7763 | $24.5,5.5,19,20$ | 69 |
| 1 | 0.807 | $25,5,20,20$ | 70 |

## 6. Conclusion

In this paper, we have considered the linearinterval fractional transportation problem with and without budgetary constraints. By finding the number of units transported, the decision maker make the correct decision depending on their financial position. Also, the interval transportation problem without budgetary constraint is useful for finding an optimal solution for reducing the cost and by that finding a compromise solution.

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