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Standard Basis of Intuitionistic Fuzzy Vector Spaces

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Abstract. In this paper, we represent an intuitionistic fuzzy vector space as the Cartesian product of its membership parts and its non membership parts. By using this representation, the standard basis of intuitionistic fuzzy vector space is defined and we have shown that any two basis for a finitely generated subspace over intuitionistic fuzzy algebra (IF) have the same cardinality. Also we prove that any finitely generated subspace of V_n over the intuitionistic fuzzy algebra(IF) has a unique standard basis.

Keywords: Fuzzy matrix, Intuitionistic fuzzy matrix, Intuitionistic fuzzy vector space, Standard basis.

AMS Mathematics Subject Classification (2010):

1. Introduction

Atanassov has introduced and developed the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [1]. If A = $(a_{ij}) \in (IF)_{mxn}$, then A = $(\langle a_{ij\mu}, a_{ij\nu} \rangle)$, where $a_{ij\mu}$ and $a_{ij\nu}$ are the membership values and non membership values of a_{ij} in A respectively with respect to the fuzzy sets μ and ν, maintaining the condition 0 ≤ $a_{ij\mu}$ + $a_{ij\nu}$ ≤ 1. We deal with fuzzy matrices that is, matrices over the fuzzy algebra F^M and F^N with support [0,1] and fuzzy operations {+, .} defined as a + b = max {a, b}, a.b = min{a,b} for all a,b∈ F^M and a + b = min {a, b}, a .b = max{a, b} for all a, b∈ F^N. Let F^M_{mxn} be the set of all mxn Fuzzy matrices over F. A matrix A∈ F^M_{mxn} is said to be regular if there exists X∈ F^M_{nxm} such that AXA = A, X is called a generalized inverse (g-inverse) of A. In [3], Kim and Roush have established that any finitely generated subspace of V_n over the Fuzzy algebra {0,1} has a unique basis. Cho [2] has discussed the consistency of fuzzy relational equations of the form x A=b, where A is regular. If A is regular with a g-inverse X, then b.X is a solution of x A=b. For more details on fuzzy matrices one may refer [4]. In [6], we have studied the structure of row space and column space of intuitionistic fuzzy matrices. The maximum and minimum solutions of fuzzy relational equations involving membership and non- membership matrix of the intuitionistic fuzzy matrix are determined in [7].

In this paper, we define a standard basis and prove that any finitely generated subspace of V_n over the intuitionistic fuzzy algebra (IF) has a unique standard basis and determine the unique standard linear combination of a vector in terms of the standard basis vectors, as a generalization of the results on fuzzy vector space found in [5].

2. Preliminaries

Let $(IF)_{mxn}$ be the set of all intuitionistic fuzzy matrices of order mxn. First we shall represent $A \in (IF)_{mxn}$ as Cartesian product of fuzzy matrices. The Cartesian product of any two matrices $A=(a_{ij})_{mxn}$ and $B=(b_{ij})_{mxn}$, denoted as $\langle A, B\rangle$ is defined as the matrix whose ij[th] entry is the ordered pair $\langle A,B\rangle = (\langle a_{ij}, b_{ij}\rangle)$. For $A=(a_{ij})_{mxn} = (\langle a_{ij\mu}, a_{ij\nu}\rangle)$. We define $A_\mu = (a_{ij\mu}) \in F^M_{mxn}$ as the membership part of A and $A_\nu = (a_{ij\nu}) \in F^N_{mxn}$ as the non membership part of A. Thus A is the Cartesian product of A_μ and A_ν written as $A = \langle A_\mu, A_\nu\rangle$ with $A_\mu \in F^M_{mxn}$, $A_\nu \in F^N_{mxn}$.

We shall follow the matrix operations on intuitionistic fuzzy matrices as defined in our earlier work [6].

For $A, B \in (IF)_{mxn}$, if $A = \langle A_\mu, A_\nu\rangle$ and $B = \langle B_\mu, B_\nu\rangle$, then

(2.1) $$A + B = \langle A_\mu + B_\mu, A_\nu + B_\nu\rangle$$

(2.2) $$AB = \langle A_\mu . B_\mu,\ A_\nu . B_\nu\rangle$$

$A_\mu . B_\mu$ is the max min product in F^M_{mxn},

$A_\nu . B_\nu$ is the min max product in F^N_{mxn}.

Let us define the order relation on $(IF)_{mxn}$ as ,

(2.3) $$A \le B \Leftrightarrow a_{ij\mu} \le b_{ij\mu} \ \ and \ \ a_{ij\nu} \ge b_{ij\nu}, \quad \text{for all i and j}$$

Let us consider the intuitionistic fuzzy relational equation of the form xA=b, where A is a intuitionistic fuzzy matrix of order mxn and b is the intuitionistic fuzzy vector and Ω(A,b) be the set of all solutions of the equation xA=b.

Lemma 2.1[4]. Let $x A_\mu = b_\mu$, $A_\mu \in F^M_{mxn}$, $b_\mu \in F^M_{1xn}$, if $\Omega(A_\mu, b_\mu) \neq \phi$,then it has a unique maximum solution.

Lemma 2.2[7]. Let $xA_\nu = b_\nu$, $A_\nu \in F^N_{mxn}$, $b_\nu \in F^N_{1xn}$, if $\Omega(A_\nu, b_\nu) \neq \phi$, then it has a unique minimum solution.

Lemma 2.3[7]. If xA=b, $A \in (IF)_{mxn}$,$b \in (IF)_{1xn}$ is consistent, then it has a unique maximum solution of the form $\hat{x} = < \hat{x}_\mu, \breve{x}_\nu >$, where $\hat{x}_\mu$ is the unique maximum solution of $xA_\mu = b_\mu$ and $\breve{x}_\nu$ is the unique minimum solution of $xA_\nu = b_\nu$.

Definition 2.1[4]. A basis C over the max min fuzzy algebra F^M is a standard basis iff whenever $c_i = \sum a_{ij}c_j$ for $c_i, c_j \in C$ and $a_{ij} \in F^M$ then $a_{ii}c_i = c_i$. In the same manner, A basis C over the minmax fuzzy algebra F^N is a standard basis iff whenever $c_i = \sum a_{ij}c_j$ for $c_i, c_j \in C$ and $a_{ij} \in F^N$ then $a_{ii}c_i = c_i$..

Lemma 2.4[3]. Any two basis for a finitely generated subspace of the maxmin fuzzy algebra F^M =[0,1], have the same cardinality and any finitely generated subspace over F^M has a unique standard basis.

3. Standard basis

In this section, we define a standard basis and prove that any finitely generated subspace of V_n over the intuitionistic fuzzy algebra(IF) has a unique standard basis.

Let us take any finite subspace W over an intuitionistic fuzzy vector space(V_n). W can be expressed as $W = < W^M, W^N >$, where W^M and W^N are finite subspaces over the max min fuzzy algebra F^M and the min max fuzzy algebra F^N respectively. By lemma (2.4), W^M has a unique standard basis. In the same manner, it can be proved that W^N has a unique standard basis over F^N .

Definition 3.1. A basis C over the intuitionistic fuzzy algebra (IF) is a standard basis iff whenever $c_i = \sum a_{ij}c_j$ for $c_i, c_j \in C$ then $a_{ii}c_i = c_i$. If $a_{ij} = < a_{ij\mu} , a_{ij\nu} > \in (IF)$ and $c_i = < c_{i\mu}, c_{i\nu} >$,for $c_{i\mu} \in F^M$, $c_{i\nu} \in F^N$,then $a_{ii}c_i = c_i$. This implies that $a_{ii}c_{i\mu} = c_{i\mu}$ and $a_{ii}c_{i\nu} = c_{i\nu}$.

Theorem 3.1. Any two basis for a finitely generated subspace of the intuitionistic fuzzy algebra $(IF) = < F^M, F^N >$ have the same cardinality and any finitely generated subspace over (IF) has a unique standard basis.

Proof. We first show that for any finite basis C of an intuitionistic fuzzy vector space W of (IF), there exist a standard basis having the same cardinality.

Let $C = < C_\mu , C_\nu >$ be the representation of C and S be the set of all intuitionistic fuzzy vectors each of whose entries equals some entry of a vector of C. Then S is a finite set. Suppose C is a not a standard basis, then $c_i = \sum a_{ij}c_j$ for $c_i, c_j \in C$ and $a_{ij} = < a_{ij\mu} , a_{ij\nu} > \in (IF)$ with $a_{ii}c_i \neq c_i$,then we have the following:
(i) $a_{ii}c_{i\mu} \neq c_{i\mu}$ and $a_{ii}c_{i\nu} \neq c_{i\nu}$, (ii) $a_{ii}c_{i\mu} \neq c_{i\mu}$ and $a_{ii}c_{i\nu} = c_{i\nu}$ and
(iii) $a_{ii}c_{i\mu} = c_{i\mu}$ and $a_{ii}c_{i\nu} \neq c_{i\nu}$.

Here let us take more general case(i), $c_{i\mu} \neq \min\{a_{ii,}c\}$ and $c_{i\nu} \neq \max\{a_{ii,}c\}$.Therefore $a_{ii}c_{i\mu} < c_{i\mu}$ and $a_{ii}c_{i\nu} > c_{i\nu}$.

Let $C_\mu^{'}$ be the set obtained from C_μ by replacing $c_{i\mu}$ by $a_{ii}c_{i\mu}$ and $C_\nu^{'}$ be the set obtained from C_ν by replacing $c_{i\nu}$ by $a_{ii}c_{i\nu}$. Then $|C_\mu^{'}| = |C_\mu|$ and $< C_\mu^{'} > = < C_\mu >$ and $|C_\nu^{'}| = |C_\nu|$ and $< C_\nu^{'} > = < C_\nu >$ and it can be verified that $C_\mu^{'}$ and $C_\nu^{'}$ are independent set and all the vectors of $C_\mu^{'}$ and $C_\nu^{'}$ are all in S.

Let us define an order relation on finite subsets of S as follows:

Let the weight of a finite subset be the sum of all entries of members of the subset regarded as real numbers. We define that $F_1 \leq F_2$ for finite subsets F_1 and F_2 of S then by(2.3) we get, $F_{1\mu} \leq F_{2\mu}$ and $F_{1\nu} \geq F_{2\nu}$ for finite subsets $F_{1\mu}$, $F_{2\mu}$,$F_{1\nu}$ and $F_{2\nu}$ of S if weight of $F_{1\mu} \leq$ weight of $F_{2\mu}$ and weight of $F_{1\nu} \geq$ weight of $F_{2\nu}$. Clearly this is a partial order relation on finite subsets of S. Since $a_{ii}c_{i\mu} < c_{i\mu}$ and $a_{ii}c_{i\nu} > c_{i\nu}$, $C_\mu^{'} \leq C_\mu$ and $C_\nu^{'} \geq C_\nu$ .This implies that $C^{'} \leq C$. Hence $|C^{'}| = |C|$ is finite.

If $C^{'}$ is a standard basis, then $C^{'}$ is the required standard basis with the same cardinality as C. If not then repeat the process of replacing $C^{'}$ by a basis $C^{''}$ and proceed. Therefore after replacing basis of the form C by a basis of the form $C^{'}$ the process must terminate after a finite number of steps.

This can happen only if we have obtained a standard basis with the same cardinality as C. This proves that for finite basis, there exists a standard basis with the same cardinality. Further corresponding to the standard basis $c=\{c_1,c_2,\ldots,c_n\}$ for the subspace $W=<W^M,W^N>$ we have the standard basis $c_\mu=\{c_{1\mu},c_{2\mu},\ldots,c_{n\mu}\}$ for W^M and $c_\nu=\{c_{1\nu},c_{2\nu},\ldots,c_{n\nu}\}$ for W^N. The uniqueness of C follows from the uniqueness of the standard basis c_μ and c_ν for W^M and W^N respectively[Refer lemma(2.4)].Hence the proof.

Definition 3.2. The dimension of the finitely generated subspace S of a intuitionistic fuzzy vector space V_n over the intuitionistic fuzzy algebra(IF) denoted by dim(S) is defined to be the cardinality of the standard basis of S.

Example 3.1. The set { (<1,0>,<0,1>,<0,1>), (<0,1>,<1,0>,<0,1>), (<0,1>,<0,1>,<1,0>) } forms the standard basis of V_3.

Theorem 3.2. Let S be a finitely generated subspace of V_n and let $c = \{c_1,c_2,\ldots,c_n\}$ be the standard basis for S. Then any vector $x \in S$ can be expressed uniquely as a linear combination of the standard basis.

Proof. Since $\{c_1,c_2,\ldots,c_n\}$ is the standard basis for $S = <S_\mu ,S_\nu>$,then by theorem(3.1), $c_\mu = \{c_{1\mu},c_{2\mu},\ldots,c_{n\mu}\}$ and $c_\nu =\{c_{1\nu},c_{2\nu},\ldots,c_{n\nu}\}$ are the standard basis for S_μ and S_ν respectively. x is a linear combination of the standard basis vectors.

Let $x=\sum \beta_j c_j$, where $x=<x_\mu , x_\nu>$, $c_j = <c_{j\mu}, c_{j\nu}>$. Then

(3.1) $$x_\mu = \sum_{j=1}^{n} \beta_j c_{j\mu},\ \beta_j \in F^M \text{ and}$$

(3.2) $x_\nu = \sum_{j=1}^{n} \alpha_j c_{j\nu},\ \alpha_j \in F^N$.

In this expression the coefficients β_j' s and α_j' s are not unique. If we write (3.1) in the matrix form as $x_\mu = (\beta_1, \beta_2, \ldots, \beta_n).C_\mu$ .Where C_μ is the matrix whose rows are the basis vectors $\{c_{1\mu}, c_{2\mu}, \ldots, c_{n\mu}\}$ then $x_\mu = p.C_\mu$ has a solution($\beta_1, \beta_2, \ldots, \beta_n$). Hence $\Omega(C_\mu, x_\mu) \neq \Phi$ and by lemma(2.1), it follows that this equation has a unique maximal solution ($p_1, p_2, \ldots, p_n$) (say).

In the same manner, if (3.2) is written in the matrix form as $x_\nu = (\alpha_1, \alpha_2, \ldots, \alpha_n).C_\nu$. Where C_ν is the matrix whose rows are the basis vectors$\{c_{1\nu}, c_{2\nu}, \ldots, c_{n\nu}\}$ then $x_\nu = q.C_\nu$ has a solution ($\alpha_1, \alpha_2, \ldots, \alpha_n$). Hence $\Omega(C_\nu, x_\nu) \neq \Phi$ and by lemma(2.2), it follows that this equation has a unique minimal solution($q_1, q_2, \ldots, q_n$)(say). Thus $x = \langle x_\mu, x_\nu \rangle$,where

$x_\mu = \sum_{j=1}^{n} p_j c_j$ with $p_j \in F^M$ and $x_\nu = \sum_{j=1}^{n} q_j c_j$ with $q_j \in F^N$ is the unique representation of the intuitionistic fuzzy vector x. Hence the proof.

Theorem 3.3. Let $\{c_1, c_2, \ldots, c_n\}$ be the standard basis of the subspace W in V_n. In the standard fuzzy linear combination of the basis vector c_i , the i^{th} coefficient of c_i is <1,0>.

Proof. Let us compute the standard fuzzy linear combination of the basis vector $c_i = \langle c_{i\mu}, c_{i\nu} \rangle$ in terms of the standard basis $\{c_1, c_2, \ldots, c_n\}$. Since $c_i \in V_n$, let $c_i = \langle c_{i\mu}, c_{i\nu} \rangle = (\langle c_{i1\mu}, c_{i1\nu} \rangle, \langle c_{i2\mu}, c_{i2\nu} \rangle, \ldots, \langle c_{in\mu}, c_{in\nu} \rangle)$,for each i=1,2, ….., n.

$c_i = \langle c_{i\mu}, c_{i\nu} \rangle = \hat{x}_1 \langle c_{1\mu}, c_{1\nu} \rangle + \hat{x}_2 \langle c_{2\mu}, c_{2\nu} \rangle + \ldots + \hat{x}_n \langle c_{n\mu}, c_{n\nu} \rangle$. This implies

$c_{i\mu} = \hat{x}_1 c_{1\mu} + \hat{x}_2 c_{2\mu} + \ldots + \hat{x}_n c_{n\mu}$ and $c_{i\nu} = \hat{x}_1 c_{1\nu} + \hat{x}_2 c_{2\nu} + \ldots + \hat{x}_n c_{n\nu}$ are the standard fuzzy linear combination. This can be expressed as intutitionistic fuzzy relational equation xA = b, where $b = \langle b_\mu, b_\nu \rangle = (\langle c_{i1\mu}, c_{i1\nu} \rangle, \langle c_{i2\mu}, c_{i2\nu} \rangle, \ldots, \langle c_{in\mu}, c_{in\nu} \rangle)$,

$x = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ and

$$A = \langle A_\mu, A_\nu \rangle = \begin{pmatrix} \langle c_{11\mu}, c_{11\nu} \rangle & \langle c_{12\mu}, c_{12\nu} \rangle & \ldots \langle c_{1n\mu}, c_{1n\nu} \rangle \\ \langle c_{21\mu}, c_{21\nu} \rangle & \langle c_{22\mu}, c_{22\nu} \rangle & \ldots \langle c_{2n\mu}, c_{2n\nu} \rangle \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ \langle c_{n1\mu}, c_{n1\nu} \rangle & \langle c_{n2\mu}, c_{n2\nu} \rangle & \ldots \langle c_{nn}, c_{nn\nu} \rangle \end{pmatrix}$$

Let us find the maximum solution of the intuitionistic fuzzy relational equation xA = b by using lemma (2.3).The maximum solution $x = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ is determined by

$$\hat{x} = <\min_{k\in K} \sigma(a_{jk\mu}, b_{k\mu}),\ \max_{k\in K} \sigma(a_{jk\nu}, b_{k\nu})>$$

where $\sigma(a_{jk\mu}, b_{k\mu}) = \begin{cases} b_{k\mu} & \text{if } a_{jk\mu} > b_{k\mu} \\ 1 & \text{otherwise} \end{cases}$ and $\sigma(a_{jk\nu}, b_{k\nu}) = \begin{cases} b_{k\nu} & \text{if } a_{jk\nu} < b_{k\nu} \\ 0 & \text{otherwise} \end{cases}$

$$\begin{aligned}\hat{x}_i &= <\min\{\sigma(a_{i1\mu}, b_{1\mu}), \sigma(a_{i2\mu}, b_{2\mu}), \ldots, \sigma(a_{in\mu}, b_{n\mu})\},\\ &\quad \max\{\sigma(a_{i1\nu}, b_{1\nu}), \sigma(a_{i2\nu}, b_{2\nu}), \ldots, \sigma(a_{in\nu}, b_{n\nu})\}>\\ &= <\min\{\sigma(c_{i1\mu}, c_{i1\mu}), \sigma(c_{i2\mu}, c_{i2\mu}), \ldots, \sigma(c_{in\mu}, c_{in\mu})\},\\ &\quad \max\{\sigma(c_{i1\nu}, c_{i1\nu}), \sigma(c_{i2\nu}, c_{i2\nu}), \ldots, \sigma(c_{in\nu}, c_{in\nu})\}>\\ &= <\min\{1,1, \ldots ,1\}, \max\{0,0, \ldots ,0\}>\end{aligned}$$

$\hat{x}_i = <1,0>$. Hence the proof.

Illustration 3.2. Let us compute the standard fuzzy linear combination of the basis vector C_1 = (<0.5,0.0> ,<0.5,0.5> ,<0.5,0.0>) in terms of the standard basis vector B = {(<0.5,0.0> ,<0.5,0.5> ,<0.5,0.0>) ,(<0.0,1.0> ,<1.0,0.0> ,<0.5,0.5>), (<0.0,1.0> ,<0.5,0.5>, <1.0,0.0>) } of the subspace W of V_3 generated by B.

Let (<0.5,0.0> ,<0.5,0.5> ,<0.5,0.0>) = $\hat{x}_1$(<0.5,0.0> ,<0.5,0.5> ,<0.5,0.0>) + $\hat{x}_2$(<0.0,1.0> ,<1.0,0.0> ,<0.5,0.5>) + $\hat{x}_3$(<0.0,1.0> ,<0.5,0.5>, <1.0,0.0>) .

$$= (\hat{x}_1, \hat{x}_2, \hat{x}_3)\begin{pmatrix} <0.5,0.0> & <0.5,0.5> & <0.5,0.0> \\ <0.0,1.0> & <1.0,0.0> & <0.5,0.5> \\ <0.0,1.0> & <0.5,0.5> & <1.0,0.0> \end{pmatrix}$$

The above expression is of the form xA = b. Where b= (<0.5,0.0> ,<0.5,0.5> ,<0.5,0.0>),

x= $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ and A= $\begin{pmatrix} <0.5,0.0> & <0.5,0.5> & <0.5,0.0> \\ <0.0,1.0> & <1.0,0.0> & <0.5,0.5> \\ <0.0,1.0> & <0.5,0.5> & <1.0,0.0> \end{pmatrix}$

Here $b_{1\mu} = b_{2\mu} = b_{3\mu} = 0.5$, $b_{1\nu} = b_{3\nu} = 0.0$, $b_{2\nu} = 0.5$ and $a_{11\mu} = a_{12\mu} = a_{13\mu} = a_{23\mu} = a_{32\mu} = 0.5$, $a_{21\mu} = a_{31\mu} = 0.0$, $a_{22\mu} = a_{33\mu} = 1.0$, $a_{11\nu} = a_{13\nu} = a_{22\nu} = a_{33\nu} = 0.0$, $a_{12\nu} = a_{23\nu} = a_{32\nu} = 0.5$, $a_{21\nu} = a_{31\nu} = 1.0$.

Let us find the maximum solution x = $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ of xA =b,by using lemma(2.3).

$$\hat{x} = <\min_{k\in K} \sigma(a_{jk\mu}, b_{k\mu}),\ \max_{k\in K} \sigma(a_{jk\nu}, b_{k\nu})>,$$

for j=1,2,3 and $k\in K$ ={1,2,3}.

For j=1, $\hat{x}_1 = <\min_{k\in K} \sigma(a_{1k\mu}, b_{k\mu}),\ \max_{k\in K} \sigma(a_{1k\nu}, b_{k\nu})>$

= < min{1.0, 1.0, 1.0}, max{0.0, 0.0, 0.0}>
= <1.0, 0.0 >

For j=2, $\hat{x}_1$ = < min σ($a_{2k\mu}, b_{k\mu}$) , max σ($a_{2k\nu}, b_{k\nu}$) >
k∈K k∈K
= < min{1.0, 0.5, 1.0}, max{0.0, 0.5, 0.0}>
= <0.5, 0.5 >

For j=3, $\hat{x}_1$ = < min σ($a_{3k\mu}, b_{k\mu}$) , max σ($a_{3k\nu}, b_{k\nu}$) >
k∈K k∈K
= < min{1.0, 1.0, 0.5}, max{0.0, 0.0, 0.0}>
= <0.5, 0.0 > .

Thus x = ($\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$) = (<1.0, 0.0 > , <0.5, 0.5 > , <0.5, 0.0 >).
Hence (<0.5,0.0> ,<0.5,0.5> ,<0.5,0.0>) = (<1.0, 0.0 > (<0.5,0.0> ,<0.5,0.5> ,<0.5,0.0>) + <0.5, 0.5 >(<0.0,1.0>,<1.0,0.0> ,<0.5,0.5>)+<0.5, 0.0 >(<0.0,1.0>,<0.5,0.5>,<1.0,0.0>)
Thus is the standard fuzzy linear combination of the basis vector
C_1 = (<0.5,0.0> ,<0.5,0.5> ,<0.5,0.0>). Similarly it can be verified for other basis vectors.

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