

## **Inspection Cost and Imperfect Quality Items with Multiple Imprecise Goals in Supply Chains in an Uncertain Environment**

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*Received 19 December 2012; accepted 14 January 2013*

**Abstract.** This work presents a Possibilistic Linear Programming (PLP) method of solving integrated Manufacturing and Distribution Planning Decisions (MDPD) problems with multiple imprecise goals in supply chains under uncertain environment. The model aims to minimize total net costs, total delivery time and total imperfect quality items with reference to available supply, machine capacities, labour levels, quota flexibility, cost budget, forecast demand, warehouse space at each destination, inspection cost at each source and imperfect quality items transported from origins to destinations. Triangular distribution numbers are used to represent imprecise numbers. An industrial case is used to demonstrate the application of PLP method for MDPD problem. LINGO software is used to solve the problem.

**Keywords:** Supply Chain, Fuzzy, Multi Objective Linear Programming, Manufacturing Planning Decision, Inspection Cost, Imperfect Quality Item

**AMS Mathematics Subject Classification (2010):** 03E72, 90B05

### **1. Introduction**

Supply Chain Management aims to synchronize customer requirements with the flow of materials from suppliers to retailers and the decision makers must simultaneously handle conflicting goals that govern the use of constrained resources within organizations. These goals are required to be optimized frequently. Chen et al.,[5] and Petrovic et al.,[14] discussed the commonly seen conflicting goals are minimizing the cost, delivery time and number of rejected items and optimizing customer service and flexibility. Integrating manufacturing / distribution planning

decisions in supply chains is a major issue of effective SCM. In real world problem input data and related parameters are often imprecise. They are not deterministic or crisp. So the conventional deterministic mathematical models cannot be applied.

In practical MDPD problems, the decision maker must handle multiple conflicting goals in a framework of fuzzy aspiration level. Li and Lai [12] and Sabri and Beamon[15] presented a possibilistic linear programming (PLP) method for solving MDPD problems with multiple imprecise goals in supply chains under fuzzy environments. The real success in supply chain management is to satisfy the customer completely by supplying all perfect quality items. This can be achieved by inspecting all goods that are transported from various sources. In this work, an inspection cost at each source is introduced to ensure good quality items. But while transporting, items can be broken like glass items. Hence we set the limit for the imperfect quality items that may be transported from sources. They can be reworked or considered as scrap

The proposed method aims to minimize total net costs, total delivery time, total number of imperfect quality items with reference to available supply, capacities, labour levels, quota flexibility, cost budget constraints at each source, inspection cost at each source as well as forecast demand and warehouse space constraints at each destination, percentage of imperfect quality items that reach each destination.

### 1.1. Review of Literature

Zimmerman [17] introduced fuzzy set theory to solve ordinary LP and multiobjective linear programming problem with fuzzy goals and constraint. Hintz and Zimmerman[7] presented an approach based on FLP and approximate reasoning to solve production planning problems. Fung et al[6] presented fuzzy multiproduct aggregate production planning model by Parametric Programming. Li and Lai [12] presented a fuzzy compromise programming method to obtain non-dominated compromise solution for MOLP. Liang T.F[13] presented integrated MDPD problems with multiple imprecise goals in an uncertain environment. Abd El-Wahed and Lee[1] designed an interactive fuzzy programming method to determine compromise solution for distribution planning problems with multiple fuzzy goals. Lee and Kim[11] proposed a hybrid approach to combine analytical and simulation models to solve MDPD problems involving multi product and multi time periods in supply chains, which introduced flexibility capacity constraint.. Ana Maria and Rakesh[2], Jang et al[9], Lee et al [11] and Bilgen and Ozkarahan[3] studied deterministic MDPD problem.

Kumar et al [10] investigated fuzzy MDPD problems since deterministic models are inadequate to solve real world problems. In 1978 Zadeh [17] first created the theory of possibility which is related to fuzzy set theory. He showed that much of the information on which we make decisions are possibilistic rather than probabilistic in nature. Buckley [4] designed a mathematical programming problem in which all parameters may be fuzzy variables based on their possibility distribution and developed PLP. Hsu and Wang [8] developed possibilistic programming model integrating PLP method of Lai. Much of the works concentrated on single goal. But

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the present paper includes multiple goals which are conflicting in nature with imprecise objective functions.

### 2. Problem Formulation

Assume that the logistics center in a supply chain attempts to determine the integrated manufacturing / distribution plan for a homogeneous commodity from  $m$  sources to  $n$  destinations. Each source has a supply of commodity available to distribute to various destinations and each destination has its expected demand to be received from the sources. Generally the capacities demand and cost coefficients are imprecise. Hence this work focuses on developing a PLP method for optimizing the integrated MDPD plan in an uncertain environment. The method simultaneously minimizes the total net costs, total delivery time, total deteriorated items with respect to available supply capacities, labour levels, quota flexibility and cost budget constraints at each source and destination.

#### 2.1 Assumptions

1. All the objective functions are imprecise.
2. The pattern of triangular possibility distribution is adopted to represent the imprecise objective functions and related imprecise numbers.
3. All the objective functions and constraints are linear equations.
4. The linear membership functions are specified for all fuzzy sets involved.
5. The minimum operator is used to aggregate all fuzzy sets.

#### 2.2 Notations

1. Index Sets :
  - i. index for source, for all  $i = 1, 2, \dots, m$ .
  - ii. index for destination, for all  $j = 1, 2, \dots, n$ .
  - iii. index for objectives, for all  $g = 1, 2, \dots, k$ .
2. Decision Variables  
 $Q_{ij}$  : number of units distributed from source  $i$  to destination  $j$ .
3. Objective Functions  
 $\tilde{Z}_1$  - total net costs (\$)  
 $\tilde{Z}_2$  - total delivery time (hours)  
 $\tilde{Z}_3$  - total number of imperfect quality items
4. Parameters  
 $p_{ij}$  - manufacturing cost per unit delivered from source  $i$  to destination  $j$  (\$/unit).  
 $c_{ij}$  - distribution cost per unit delivered from source  $i$  to destination  $j$  (\$/unit).  
 $t_{ij}$  - distribution time per unit delivered from source  $i$  to destination  $j$  (hours/unit).  
 $e_{ij}$  - capacity per truck delivered from source  $i$  to destination  $j$  (units).  
 $\tilde{S}_i$  - total supply available for each source  $i$  (units).  
 $\tilde{D}_j$  - total demand of each destination  $j$  (units).  
 $l_{ij}$  - hours of labour per unit produced by each source  $i$  (units).

- $\tilde{g}_i$  - Inspection cost at each source  $i$  (\$/unit).
- $\tilde{d}_i$  - total number of deteriorated items at each source  $i$ .
- $\tilde{d}_{ij}$  - rate of imperfect quality of items transported from each source  $i$  to destination  $j$  (%)
- $\tilde{W}_{imax}$  - maximum labour levels available for each source  $i$  (man-hour)
- $\tilde{a}_{ij}$  - hours of machine usage per unit produced by each source  $i$  (machine hour unit)
- $M_{imax}$  - maximum machine capacities available for each source  $i$  (machine-hour)
- $\tilde{f}_{ij}$  - quota flexibility for source  $i$  to destination  $j$  (%)
- $\tilde{f}_{imin}$  - low bound of quota flexibility for source  $i$  (units)
- $\tilde{B}_i$  - budget allocated to each source  $i$  (\$)
- $b_{ij}$  - warehouse space per unit delivered from source  $i$  to destination  $j$  (ft<sup>2</sup>/units)
- $V_{jmax}$  - maximum warehouse space available for each destination  $j$  (ft<sup>2</sup>)

### 2.3. Imprecise multi objective possibilistic linear programming model

#### 2.3.1. Objective Functions

The imprecise objective functions are as follows

- 1) Minimize total net costs

$$\text{Min } \tilde{Z}_1 = \sum_{i=1}^m \sum_{j=1}^n [\tilde{p}_{ij} + \tilde{c}_{ij} + \tilde{g}_i] Q_{ij} \quad \dots \quad (1)$$

- 2) Minimize total delivery time

$$\text{Min } \tilde{Z}_2 = \sum_{i=1}^m \sum_{j=1}^n \left[ \frac{\tilde{t}_{ij}}{\tilde{e}_{ij}} \right] Q_{ij} \quad \dots \quad (2)$$

- 3) Minimize total number of imperfect quality items

$$\text{Min } \tilde{Z}_3 = \sum \sum \tilde{d}_{ij} Q_{ij} \quad \dots \quad (3)$$

where  $\tilde{p}_{ij}$ ,  $\tilde{c}_{ij}$ ,  $\tilde{t}_{ij}$  and  $\tilde{d}_{ij}$  are imprecise coefficients with triangular possibility distributions.

The total net costs are the sum of the manufacturing, distribution and inspection costs over the planning horizon. For each of the objective functions in the original PLP model. The Decision Maker (DM) has imprecise goals such as “The objective function should be essentially equal to some value”. Accordingly equations (1), (2) and (3) are imprecise and incorporate the variations in DM judgments regarding the solutions of multi objective optimization problems in a framework of fuzzy aspiration levels.

#### 2.3.2. Constraints

Constraints on total supply available for each source  $i$ .

$$\sum_j^n Q_{ij} \leq \tilde{s}_i \quad \forall i \quad \dots \quad (4)$$

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Constraints on total demand for each destination j.

$$\sum_j^n Q_{ij} \leq \tilde{D}_i \quad \forall j \quad \dots (5)$$

Constraints on labour levels and machine capacities for each source i.

$$\sum_{j=1}^n l_{ij} Q_{ij} \leq \tilde{W}_{imax} \quad \forall i \quad \dots (6)$$

$$\sum_{j=1}^n \tilde{a}_{ij} Q_{ij} \leq M_{imax} \quad \forall i \quad \dots (7)$$

Constraints on budget for each source i.

$$\sum_{j=1}^n [\tilde{p}_{ij} + \tilde{c}_{ij} + \tilde{g}_i] Q_{ij} \leq \tilde{B}_i \quad \forall i \quad \dots (8)$$

constraints on quota flexibility for each source i.

$$\sum f_{ij} Q_{ij} \geq F_{imin} \quad \forall i \quad \dots (9)$$

Constraints on warehouse space for each destination i.

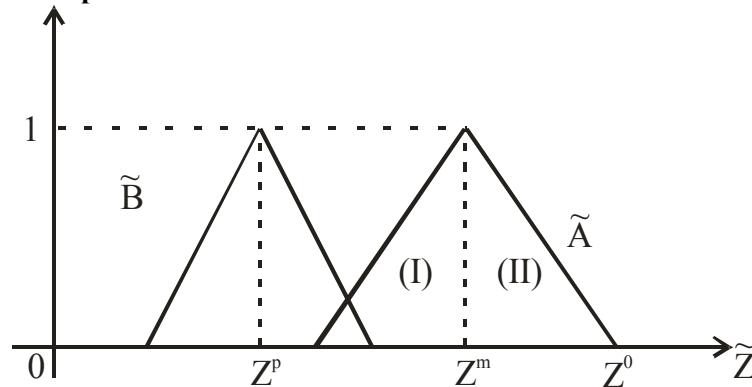
$$\sum_{i=1}^m b_{ij} Q_{ij} \leq V_{jmax} \quad \forall j \quad \dots (10)$$

Constraints on the total number of deteriorated items

$$\sum_{j=1}^n d_{ij} Q_{ij} \leq \tilde{d}_i \quad \forall i \quad \dots (11)$$

Non negativity constraints on decision variables  $Q_{ij} \geq 0 \quad \forall i, \forall j \quad \dots (12)$

### 3. Model the imprecise data



The possibility distribution can be stated as the degree of occurrence of an event with imprecise data. Here DM adopted triangular possibility distribution for all imprecise numbers as they are flexible for fuzzy arithmetic operations. For example  $C_{ij}$  is based on three prominent data as follows.

- (i) The most pessimistic value ( $C_{ij}^p$ ) that has a very low likelihood of belonging to the set of available values (possibility degree = 0 if normalized).

- (ii) The most possible value ( $C_{ij}^m$ ) that definitely belongs to the set of available values (possibility degree = 1 if normalized).
- (iii) The most optimistic value ( $C_{ij}^0$ ) that has a very low likelihood of belonging to the set of available values (possibility degree = 0 if normalized).

### 3.1. Developing an auxiliary MOLP Model

#### 3.1.1. Strategy for solving the imprecise objective function

The strategy involves simultaneously minimizes the most possible goal of the imprecise objective function  $Z_1^m$ , maximizes the possibility of obtaining lower goal value (region I in Fig.1),  $(Z_1^m - Z_1^p)$  and minimizing the risk of obtaining higher goal value (region II in Fig.1),  $(Z_1^0 - Z_1^m)$ . The last two goals are relative measures from  $Z_1^m$ , the most possible value of the imprecise total net costs. Equations (13) to (15) list the results for three new objective functions of the total net cost in equation 1.

$$\text{Min } Z_{11} = Z_1^m = \sum_{i=1}^m \sum_{j=1}^n [p_{ij}^m + c_{ij}^m + g_i^m] Q_{ij} \quad \dots (13)$$

$$\text{Max } Z_{12} = (Z_1^m - Z_1^p) = \sum_{i=1}^m \sum_{j=1}^n [(p_{ij}^m - p_{ij}^p) + (c_{ij}^m - c_{ij}^p) + (g_i^m - g_i^p)] Q_{ij} \quad \dots (14)$$

$$\text{Min } Z_{13} = (Z_1^0 - Z_1^m) = \sum_{i=1}^m \sum_{j=1}^n [(p_{ij}^0 - p_{ij}^m) + (c_{ij}^0 - c_{ij}^m) + (g_i^0 - g_i^m)] Q_{ij} \quad \dots (15)$$

Similarly equations (16 – 18) list this result for the new objective function of total delivery time in equation (2)

$$\text{Min } Z_{21} = Z_2^m = \sum_{i=1}^m \sum_{j=1}^n \left[ \frac{t_{ij}}{e_{ij}} \right] Q_{ij} \quad \dots (16)$$

$$\text{Max } Z_{22} = (Z_2^m - Z_2^p) = \sum_{i=1}^m \sum_{j=1}^n \left[ \frac{t_{ij}^m - t_{ij}^p}{e_{ij}} \right] Q_{ij} \quad \dots (17)$$

$$\text{Min } Z_{23} = (Z_2^0 - Z_2^m) = \sum_{i=1}^m \sum_{j=1}^n \left[ \frac{t_{ij}^0 - t_{ij}^m}{e_{ij}} \right] Q_{ij} \quad \dots (18)$$

Equations (19) – (21) list the results for the new objective functions of total number of deteriorated items in equation (3)

$$\text{Min } Z_{31} = Z_3^m = \sum_{i=1}^m \sum_{j=1}^n d_{ij}^m Q_{ij} \quad \dots (19)$$

$$\text{Max } Z_{32} = (Z_3^m - Z_3^p) = \sum_{i=1}^m \sum_{j=1}^n [d_{ij}^m - d_{ij}^p] Q_{ij} \quad \dots (20)$$

$$\text{Min } Z_{33} = (Z_3^0 - Z_3^m) = \sum_{i=1}^m \sum_{j=1}^n [d_{ij}^0 - d_{ij}^m] Q_{ij} \quad \dots (21)$$

#### 3.1.2. Strategy for solving the imprecise constraints

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The available supply for each source,  $\tilde{S}_i$ , is imprecise and has a triangular possibility distribution with the most and least possible values. The main problem is to obtain a crisp representative number for the imprecise supply. We apply weighted aggregate method to convert  $\tilde{S}_i$  into a crisp number. If the minimum acceptable possibility  $\beta$  is specified, the auxiliary crisp inequality constraint then can be represented as follows

$$\sum_{j=1}^n Q_{ij} \leq w_1 S_{i,\beta}^p + w_2 S_{i,\beta}^m + w_3 S_{i,\beta}^0 \quad .(22)$$

where  $0 \leq w_e \leq 1$ ,  $\sum w_e = 1$  ( $e = 1, 2, 3$ ),  $w_1, w_2, w_3$  represent the corresponding weights of the most pessimistic, most possible and most optimistic values of the imprecise number respectively.

Similarly if the minimum acceptable possibility  $\beta$  is given, the auxiliary crisp inequality constraints of equations (5) and (6) can be represented as follows

$$\sum_{i=1}^m Q_{ij} \geq w_1 D_{i,\beta}^p + w_2 D_{i,\beta}^m + w_3 D_{i,\beta}^0, \quad \forall j \quad \dots (23)$$

$$\sum_{j=1}^n l_{ij} Q_{ij} \leq w_1 W_{i,\beta}^p + w_2 W_{i,\beta}^m + w_3 W_{i,\beta}^0, \quad \forall i \quad \dots (24)$$

Fuzzy ranking concept (Tanaka et al 1984, Ramik and Rimanec 1985, Lai and Hwang 1992) is used to convert imprecise inequality constraint (7), (8) and (11).

$$\sum_{j=1}^n a_{ij,\beta}^p Q_{ij} \leq M_{i \max, \beta}^p \quad \forall i \quad \dots (25)$$

$$\sum_{j=1}^n a_{ij,\beta}^m Q_{ij} \leq M_{i \max, \beta}^m \quad \forall i \quad \dots (26)$$

$$\sum_{j=1}^n a_{ij,\beta}^0 Q_{ij} \leq M_{i \max, \beta}^0 \quad \forall i \quad \dots (27)$$

Inequality constraint (8) can be represented as follows

$$\sum_{j=1}^n (P_{ij,\beta}^p + C_{ij,\beta}^p + g_i^p) Q_{ij} \leq B_{i,\beta}^p \quad \forall i \quad \dots (28)$$

$$\sum_{j=1}^n (P_{ij,\beta}^m + C_{ij,\beta}^m + g_i^m) Q_{ij} \leq B_{i,\beta}^m \quad \forall i \quad \dots (29)$$

$$\sum_{j=1}^n (P_{ij,\beta}^0 + C_{ij,\beta}^0 + g_i^0) Q_{ij} \leq B_{i,\beta}^0 \quad \forall i \quad \dots (30)$$

Inequality constraint (11) can be represented as follows

$$\sum_{j=1}^n d_{ij,\beta}^p Q_{ij} \leq d_{i,\beta}^p \quad \forall i \quad \dots (31)$$

$$\sum_{j=1}^n d_{ij,\beta}^m Q_{ij} \leq d_{i,\beta}^m \quad \forall i \quad \dots (32)$$

$$\sum_{j=1}^n d_{ij,\beta}^0 Q_{ij} \leq d_{i,\beta}^0 \quad \forall i \quad \dots (33)$$

### 3.1.3. Solving the auxiliary MOLP problem

The auxiliary MOLP problem developed above can be converted into equivalent single goal LP problem using Linear Membership Function of Zimmerman (1976, 78) to represent imprecise goal of DM together with fuzzy decision making concept of Bellman and Zadeh (1970), First specifies the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) of the objective functions (13) to (21) of the auxiliary MOLP problem as follows

$$Z_{g_1}^{PIS} = \text{Min } Z_g^m \quad Z_{g_1}^{NIS} = \text{Max } Z_g^m \quad g = 1, 2, 3 \quad \dots (34a)$$

$$Z_{g_2}^{PIS} = \text{Max } (Z_g^m - Z_g^p) \quad Z_{g_2}^{NIS} = \text{Min } (Z_g^m - Z_g^p) \quad g = 1, 2, 3 \quad \dots (34b)$$

$$Z_{g_3}^{PIS} = \text{Min } (Z_g^0 - Z_g^m) \quad Z_{g_3}^{NIS} = \text{Max } (Z_g^0 - Z_g^m) \quad g = 1, 2, 3 \quad \dots (34c)$$

Furthermore the corresponding linear membership functions for each objective function is defined by

$$fg_1(Zg_1) = \begin{cases} 1 & \text{if } Z_{g_1} < Z_{g_1}^{PIS} \\ \frac{Z_{g_1}^{NIS} - Z_{g_1}}{Z_{g_1}^{NIS} - Z_{g_1}^{PIS}} & \text{if } Z_{g_1}^{PIS} \leq Z_{g_1} \leq Z_{g_1}^{NIS} \\ 0 & \text{if } Z_{g_1} > Z_{g_1}^{NIS} \end{cases} \quad g = 1, 2, 3 \quad \dots (35)$$

$$fg_2(Zg_2) = \begin{cases} 1 & \text{if } Z_{g_2} > Z_{g_2}^{PIS} \\ \frac{Z_{g_2} - Z_{g_2}^{NIS}}{Z_{g_2}^{PIS} - Z_{g_2}^{NIS}} & \text{if } Z_{g_2}^{NIS} \leq Z_{g_2} \leq Z_{g_2}^{PIS} \\ 0 & \text{if } Z_{g_2} < Z_{g_2}^{NIS} \end{cases} \quad g = 1, 2, 3 \quad \dots (36)$$

$$fg_3(Zg_3) = \begin{cases} 1 & \text{if } Z_{g_3} < Z_{g_3}^{PIS} \\ \frac{Z_{g_3}^{NIS} - Z_{g_3}}{Z_{g_3}^{NIS} - Z_{g_3}^{PIS}} & \text{if } Z_{g_3}^{PIS} \leq Z_{g_3} \leq Z_{g_3}^{NIS} \\ 0 & \text{if } Z_{g_3} > Z_{g_3}^{NIS} \end{cases} \quad g = 1, 2, 3 \quad \dots (37)$$

The maximum operator of the fuzzy decision making concept of Bellman and Zadeh (1970) is used to aggregate all fuzzy sets. Introducing the auxiliary variable L enables the auxiliary MOLP single goal LP, from that can be solved efficiently using standard simplex method. Consequently the complete ordinary LP model for solving the MDPD problems with multiple imprecise goals can be formulated as follows.

Max L

$$\text{Subject to } L \leq \frac{Z_{g_1}^{NIS} - Z_{g_1}}{Z_{g_1}^{NIS} - Z_{g_1}^{PIS}} \quad g = 1, 2, 3$$



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$$L \leq \frac{Z_{g_2} - Z_{g_2}^{NIS}}{Z_{g_2}^{PIS} - Z_{g_2}^{NIS}} \quad g = 1, 2, 3$$

$$L \leq \frac{Z_{g_3}^{NIS} - Z_{g_3}}{Z_{g_3}^{NIS} - Z_{g_3}^{PIS}} \quad g = 1, 2, 3$$

Equations (9), (10), (22) to (33) and  $Q_{ij} \geq 0, \forall i, \forall j$   
where L value ( $0 \leq L \leq 1$ ) represents the overall DM satisfaction with the determined goal values.

### 3.1.4. Solution procedure algorithm

**Step 1:** Formulate the original imprecise multi objective PLP model according to equations 1 – 12.

**Step 2:** Model the imprecise coefficients and right hand side values using the triangular possibility distributions.

**Step 3:** Develop the new objective functions of auxiliary MOLP problem for each of the imprecise objective functions using equations (13) to (21).

**Step 4:** Given the minimum acceptable possibility  $\beta$  and convert the imprecise constraints into crisp ones using either the weighted average or the fuzzy ranking methods, respectively as equations (21) to (33).

**Step 5:** Specify the corresponding linear membership functions for each of the new objective functions in the auxiliary MOLP problem using equations (35) to (37) and then aggregates the auxiliary MOLP problem into an equivalent ordinary single goal LP model by the minimum operator.

**Step 6:** Solve and modify the model interactively.

A company produces soft drinks from 3 plants and distributes them to 4 destinations. The following table gives the data.

**Table 1: Summarized distribution data**

Source	Destination				Supply (in 000 dozen bottle)
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
S <sub>1</sub>	(0.6,0.8,0.9)*/ (4.8,6.7)**	(2.4,3,3.5)/ (34,40,44)	(1.8,2.1,2.3)/ (10.6,12,13)	(1.5,1.8,2)/ (14,16,17)	(17.2,18,18.8)
S <sub>2</sub>	(1.1,1.3,1.4)/ (8.6,10,11.2)	(3.1,3.6,4.1)/ (27,32,36)	(1.4,1.6,1.7)/ (13.2,15,16.2)	(2.2,2.5,2.7)/ (20,22,23.6)	(22.6,24,25.4)
S <sub>3</sub>	(1.5,1.8,2.0)/ (10.6,12,13)	(3.1,3.5,3.9)/ (26,30,33.2)	(2.1,2.4,2.7)/ (16.2,18,19.4)	(0.8,1,1.1)/ (8.6,10,11)	(12.6,13,13.4)
<b>Demand (in 000 dozen bottle)</b>	(11.6,12,12.4)	(5.7,6,6.3)	(15.5,16,16.5)	(19.2,20,20.8)	

Note: “\*” denotes distribution cost per unit (in \$), “\*\*” denotes delivery time per truck to carry 100 dozen bottles (hours)

**Table 2:**

The inspection cost for one dozen bottles is as follows

Source	Inspection Cost (in \$)
$S_1$	(0.02,0.03,0.04)
$S_2$	(0.06,0.07,0.08)
$S_3$	(0.08,0.09,0.10)

**Table 3: Summarized imprecise manufacturing data of all sources**

Source	$\tilde{P}_{ij}$ (\$/unit)	$\tilde{a}_{ij}$ (machine hr/unit)	$\tilde{B}_i$ (in 000 dollars)	$\tilde{W}_{i\max}$ (in 000 man hours)	$\tilde{M}_{i\max}$ (in 000 machine hours)
$S_1$	(2.4,3,3.5)	(0.19,0.21,0.23)	(70,80,90)	(5.2,5.6,6.0)	(3.6,3.8,4.15)
$S_2$	(2.3,2.7,3.0)	(0.14,0.16,0.18)	(115,125,135)	(4.8,5.0,5.2)	(3.7,3.90,4.35)
$S_3$	(3.0,3.6,4.0)	(0.1,0.12,0.14)	(60,68,76)	(1.9,2.1,2.3)	(1.5,1.60,1.95)

**Table 4: Percentage of imperfect quality items**

Source	Destination			
	$D_1$	$D_2$	$D_3$	$D_4$
$S_1$	(0.0046, 0.005,0.006)	(0.0035, 0.004,0.0046)	(0.0054, 0.006,0.0065)	(0.0008, 0.001,0.0012)
$S_2$	(0.0028, 0.003,0.0032)	(0.006, 0.007,0.0072)	(0.0008, 0.001,0.0012)	(0.0016, 0.002,0.0022)
$S_3$	(0.0016, 0.002,0.0022)	(0.0028, 0.003,0.0032)	(0.0016, 0.002,0.0022)	(0.0035, 0.004,0.0046)

**Table 5: Summarized crisp manufacturing data of all sources**

Source	$l_{ij}$ (man hours/unit)	$b_{ij}$ (ft <sup>2</sup> /unit)	$f_{ij}$ (%)	$F_{i\min}$ (units)
$S_1$	0.30	0.32	0.04	720
$S_2$	0.20	0.28	0.03	650
$S_3$	0.15	0.30	0.05	620

Maximum warehouse space for four distribution centres  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$  are 4000 ft<sup>2</sup>, 1700 ft<sup>2</sup>, 5000 ft<sup>2</sup>, 5800 ft<sup>2</sup> respectively, capacity per truck from each source to various destinations is fixed to carry 100 dozen bottles.

### 3.2 Implementation

The integrated MDPD problem for the above case focuses on developing an interactive PLP method for optimizing the manufacturing / distribution plan in uncertain environment. Solution of MDPD problem is expected to minimize total net worth, total delivery time and total imperfect quality items subject to constraints on

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available capacities, labour levels, quota flexibility, budget constraints at each source, number of imperfect quality items, forecast demand and warehouse space at each distribution center.

The interactive solution procedure using the proposed PLP method is as follows. First formulate the original multi objective PLP model for the MDPD problem according to equation 1 – 12. Second develop the new objective function of the auxiliary MOLP for each of the imprecise objective function using equations 13 – 21. Third, formulate the auxiliary crisp constraints using equations 22 – 33 at  $\beta = 0.5$ . This work sets  $w_2 = 4/6$ ,  $w_1 = w_3 = 1/6$ . The most likely values  $w_2 = 4/6$  are used herein because these values are generally most important ones.

Specify the PIS and NIS for all the new objective functions in the auxiliary MOLP problem and define the corresponding linear membership function for each objective function using equations 35, 36, 37. Both intervals of PIS and NIS must cover the LP solution. The new objective functions of auxiliary MOLP problem are solved using the ordinary single goal LP model and Table 6 presents the corresponding sets of PIS and NIS of the initial solution.

Additionally this auxiliary MOLP problem can be converted into an equivalent ordinary single goal LP form using the minimum operator to aggregate all fuzzy sets. Finally LINGO computer software is used to run this ordinary LP model and obtains the following results. Table 7 lists the optimal manufacturing / distribution plan.

$$\tilde{Z}_1 = (\$188409.3, \$231717.38, \$263901.48)$$

$$\tilde{Z}_2 = (\$5161.17, \$8470.46, \$11779.75) \text{ hours}$$

$$\tilde{Z}_3 = (\$159, \$173, \$197) \text{ units}$$

and over all DM satisfaction is 0.8438 with the determined goal values. The objective values using the proposed PLP method should be precise and have triangular possibility distribution because the related cost/time coefficients are always imprecise in nature.

**Table 6: The PIS and NIS for all the objective function**

Objective Function	Min $Z_{g_1}$ ( $g = 1,2$ )	Max $Z_{g_2}$ ( $g = 1,2$ )	Min $Z_{g_3}$ ( $g = 1,2$ )	(PIS, NIS)
$Z_{11}(\$)$	261488	-	-	(250000,800000)
$Z_{12}(\$)$	-	43380	-	(50000,10000)
$Z_{13}(\$)$	-	-	31160	(30000,80000)
$Z_{21}(\text{hours})$	11306	-	-	(11500,22500)
$Z_{22}(\text{hours})$	-	1075	-	(1200,400)
$Z_{23}(\text{hours})$	-	-	829	(800,2100)
$Z_{31}(\text{units})$	169	-	-	(160,500)
$Z_{32}(\text{units})$	-	23	-	(30,5)
$Z_{33}(\text{units})$	-	-	27	(20,80)

**Table 7: The optimal MDPD plan with the proposed PLP method solution**

Item	Solutions
$Q_{ij}$ (in dozen bottles)	$Q_{11} = 10536, Q_{12} = 0, Q_{13} = 6233, Q_{14} = 1231, Q_{21} = 0,$ $Q_{22} = 5762, Q_{23} = 6007, Q_{24} = 11231, Q_{31} = 1464, Q_{32} = 238,$ $Q_{33} = 3760, Q_{34} = 7538$
L value	$L = 0.8345$
Objective Value (\$)	$Z_{11} = 231717.38 ; Z_{12} = 43308.08 ; Z_{13} = 32184.10 ;$ $\tilde{Z}_1 = (188409.3, 231717.38, 263901.48)^*$ $Z_{21} = 8470.46 ; Z_{22} = 3309.29 ; Z_{23} = 817.46 ;$ $\tilde{Z}_2 = (5161.17, 8470.46, 11779.75)^*$ $Z_{31} = 173 ; Z_{32} = 22 ; Z_{33} = 24 ;$ $\tilde{Z}_3 = (151, 173, 197)^*$

Note : The objective values have triangular distributors of  
 $\tilde{Z}_g = (Z_{g1} - Z_{g2}, Z_{g1}, Z_{g1} + Z_{g3}) \quad g = 1, 2.$

Table 8: Comparison of solutions

Item Objective Function	LP_1 Min $Z_1$	LP_2 Min $Z_2$	LP_3 Min $Z_3$	The proposed PLP method Max L
<b>L</b>	100%	100%	100%	0.8438
<b><math>Z_1</math> (Total net costs, \$)</b>	261488	-	-	(18840903, 231717.38, 263901.48)
<b><math>Z_2</math> (Total delivery time, hours)</b>	-	11306	-	(5161.17, 8470.46, 11779.75)
<b><math>Z_3</math> (Total Imperfect Quality Items)</b>	-	-	169	(151,173,197)

### 3.3. Analysis

The proposed method yields efficient solutions to the integrated MDPD problem which can be solved by ordinary LP model .The Decision Maker specifies the most possible value of each imprecise data as the precise number. Table 7 compares the results obtained using the LP model with those obtained using PLP method. According to Table 7 , minimum value of  $z_1$  by LP-1 is \$261488 , minimum value of  $z_2$  by LP-2 is 11306 hours and minimum value of  $z_3$  by LP-3 is 169 units . These figures reveal that the PLP yields compromise solutions, compared to optimal goal values by crisp single goal LP model and integrated manufacturing/distribution plan is made based on PLP method which has acceptable overall DM satisfaction in uncertain environments.

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### 4. Conclusion

This work presents a possibilistic Programming method for solving the integrated MDPD problems with multiple imprecise goals in uncertain environment. The proposed method aims to minimize the total net costs, total delivery time and the number of imperfect quality items with reference to available supply, inspection costs, labour levels, quota flexibility, budget, percentage of imperfect quality items transported from each source, demand and warehouse space at each destination. An industrial example is given to illustrate the method. LINDO software is used to solve the problem. The interactive method yields an efficient compromise solution and overall Decision Maker satisfaction with the given goal values.

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