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On Fuzzy δg*- Closed Sets in Fuzzy Topological Spaces

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Abstract. In this paper a new class of fuzzy sets called fuzzy δg*-closed sets are introduced and its properties are studied.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh in his classical paper [8]. Thereafter many investigations have been carried out, in the general theoretical field and also in different applied areas, based on this concept. The idea of fuzzy topological space was introduced by Chang[3]. The idea is more or less a generalization of ordinary topological spaces. Different aspects of such spaces have been developed, by several investigators. This paper is also devoted to the development of the theory of fuzzy topological spaces. In [2] the extensions of functions in fuzzy settings were carried out using the concepts of quasi-coincidences and q-neighborhoods by Pu and Liu[5]. Dontchev introduced δg-closed sets in [4] and Veera kumar studied g*-closed sets in [6] and [7]. In this paper,these concepts are generalized to fuzzy topological spaces.

2. Preliminary Notes

Throughout this paper, (X,τ) and (Y, σ) (or simply X and Y) always mean fuzzy topological spaces. For a fuzzy set A of (X,τ), fcl(A) and fint(A) denote the fuzzy closure and fuzzy interior of A respectively. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by AqB iff there exists x∈X such that A(x) + B(x) > 1. If A and B are not quasi-coincident then we write $A\bar{q}B$, then A ≤ B means $A\bar{q}1-B$. A fuzzy point x_p is quasi-coincident with a fuzzy set B denoted by x_pqB iff there exists x ∈ X such that p + B(x) > 1. A fuzzy set in X is said to be fuzzy regular open [1] if fint(cl A)=A. A fuzzy point x_a is said to be a fuzzy δ-cluster point of a fuzzy set A in an fts X if every fuzzy regular open q-nhd U of x_a is q-coincident with A. The union of all fuzzy δ-cluster points of A is called the fuzzy δ-closure of A is denoted by δcl(A). A subset A of a topological space (X, τ) is called a fuzzy g-closed set if fcl(A)≤U whenever A≤U and U is open in (X, τ). The complement of a fg-closed set is called a fg-open set. A subset A of a space X is called fδg-closed if $fcl_δ(A) \le U$ whenever $A \le U$ and U is a fuzzy open set.

3. Fuzzy δg*- closed sets

Definition 3.1. A fuzzy topology is a family τof fuzzy sets in X which satisfies the following conditions:

1. **0,1**∈τ
2. If A, B ∈τ, then $A \wedge B \in \tau$,
3. If $A_i \in \tau$ for each i∈I, then $\vee_i A_i \in \tau$.

Definition 3.2. A fuzzy set A in fts (X, τ) is called fuzzy δg*-closed iff $fcl_δ(A) \le B$, whenever A≤B and B is fuzzy g-open in X.

Theorem 3.3. Every fuzzy δ-closed set is a fuzzy δg*-closed set in a fts X.

Proof: Let A be a fuzzy δ-closed set in a fts X. Let B be a fuzzy g-open set in X such that A≤B. Since A is a fuzzy δ-closed, $fcl_δ(A) = A$. Therefore $fcl_δ(A) = A \le B$. Hence A is a fuzzy δg*-closed set.

Theorem 3.4. If A is fuzzy δ-open and fuzzy δg*-closed in (X, τ), then A is fuzzy δ-closed in (X, τ)

Proof: Let A be fuzzy δ-open and fuzzy δg*-closed in X. For A ≤ A, then $fcl_δ(A) \le A$. But $A \le fcl_δ(A)$, which implies $fcl_δ(A) = A$. Hence A is fuzzy δ-closed.

Theorem 3.5. Let (X, τ) be a fts and A be a fuzzy set of X. Then A is fuzzy δg*-closed if and only if $A\bar{q}B$ implies $fcl_δ(A)\bar{q}B$ for every fuzzy g-closed set B of X.

Proof: Suppose A is a fuzzy δg*-closed set of X. Let B be a fuzzy g-closed set in X such that $A\bar{q}B$. Then A ≤ 1 – B and 1 – B is a fuzzy g-open set of X. Therefore $fcl_\delta(A) \le 1 - B$, as A is fuzzy δg*-closed. Hence $fcl_\delta(A)\bar{q}B$.

Conversely let D be a fuzzy g-open set in X such that A ≤ D. Then $A\bar{q}(1-D)$ and 1 – D is a fuzzy g-closed set in X. By hypothesis, $fcl_\delta(A)\bar{q}(1-D)$ which implies, $cl_\delta(A) \le D$. Hence A is fuzzy δg*-closed.

Theorem 3.6. Let A be a fuzzy δg*-closed set in (X,τ) and x_p be a fuzzy point of (X,τ) such that $x_p q fcl_\delta(A)$, then $fcl_\delta(x_p)qA$.

Proof: Let A be a fuzzy δg*-closed set in (X,τ) and x_p be a fuzzy point of (X,τ) such that $x_p q fcl_\delta(A)$. Suppose $fcl_\delta(x_p)\bar{q}A$, then $fcl_\delta(x_p) \le 1 - A$ and hence $A \le 1 - fcl_\delta(x_p)$. Now $1 - fcl_\delta(x_p)$ is fuzzy δ-open and hence fuzzy open. Moreover, since A is fuzzy δg*-closed, $fcl_\delta(A) \le 1 - fcl_\delta(x_p) \le 1 - x_p$. Hence $x_p \bar{q} fcl_\delta(A)$, which is a contradiction.

Theorem 3.7. If A is a fuzzy δg*-closed set in (X,τ) and $A \le B \le fcl_\delta(A)$, then B is a fuzzy δg*-closed set in (X,τ).

Proof: Let A be a fuzzy δg*-closed set in (X,τ). Given $A \le B \le fcl_\delta(A)$. Let $B \le U$ where U is fuzzy g-open set. Since $A \le B \le U$ and A is a fuzzy δg*-closed set, we get $fcl_\delta(A) \le U$. As $B \le fcl_\delta(A), fcl_\delta(B) \le fcl_\delta(fcl_\delta(A)) = fcl_\delta(A)$ we get $fcl_\delta(B) \le U$. Hence B is a fuzzy δg*-closed set in (X,τ).

Theorem 3.8. If A is a fuzzy δg*-open set in (X,τ) and $f\,int_\delta(A) \le B \le A$, then B is a fuzzy δg*-open set in (X,τ).

Proof: Let A be fuzzy δg*-open set and B be any fuzzy set in X such that $f\,int_\delta(A) \le B \le A$. Then 1 – A is a fuzzy δg*-closed set and $1 - A \le 1 - B \le fcl_\delta(1-A)$, as $1 - f\,int_\delta(A) = fcl_\delta(1-A)$. By Theorem 3.7, 1 – B is a fuzzy δg*-closed. Hence B is fuzzy δg*-open.

Theorem 3.9. Let (Y, τ_y) be a subspace of a fts (X,τ) and A be a fuzzy set of Y. If A is fuzzy δg*-closed in X, then A is a fuzzy δg*-closed in Y.

Proof: Let Y be a subspace of X. Given A≤U, where U is a fuzzy g-open set in Y. We need to prove $fcl_y(A) \leq U$. Since U is fuzzy g-open in Y, we have U = G∩Y where G is fuzzy g-open in X. Hence $A \leq U = G \cap Y$. Therefore A≤G and A is fuzzy δg*-closed in X, we get $fcl(A) \leq G$. Therefore $fcl(A) \cap Y \leq G \cap Y = U$. Hence $fcl_y(A) \leq U$. Therefore A is fuzzy δg*-closed in Y.

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