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On Fuzzy Dot SU-subalgebras

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Abstract. This paper introduces the notion of fuzzy Dot SU-subalgebra and deals with some of their basic but interesting properties by applying the idea of fuzzy subset.

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1. Introduction

The concept of a fuzzy set was introduced by Zadeh [6]. Since then many research work **have been** carried out in the fuzzy settings. Y.Imai and K.Iseki[3] introduced two classes of abstract algebras; BCK-algebras and BCI-algebra. It is known that the **class of** BCK-algebras is **a** proper sub class of the class of BCI-algebras. Recently, Supawadee Keawrahun and Utsanee Leerawat[5] introduced a new algebraic structure named SU-Algebra. In this paper we investigate the fuzzy Dot SU-subalgebra and establish some of their basic properties.

2. Preliminaries

In this section basic definitions of a SU-algebra, **fuzzy** subset and fuzzy subalgebra are recalled. We start with,

Definition 2.1.[5] A SU-algebra is a non-empty set X with a constant 0 and a single binary operation * satisfying the following axioms for any $x, y, z \in X$

i. $((x*y)*(x*z))*(y*z) = 0$;
ii. $x*0 = x$;
iii. If $x*y = 0 \Rightarrow x = y$.

Example 2.2. The following table with $(X = \{0,1,2,3\}, *, 0)$ is a SU-algebra.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Definition 2.3. A binary relation "$\leq$" on X can be defined as $x \leq y$ if and only if $x*y = 0$.

Definition 2.4. A non-empty subset S of a SU-algebra X is said to be a subalgebra if $x*y \in S \quad \forall x, y \in S$.

Definition 2.5. A function $f : X \to Y$ of SU-algebras X and Y is called homomorphism if $f(x*y) = f(x)*f(y) \quad \forall\, x, y \in X$. And $f : X \to Y$ is called anti-homomorphism if $f(x*y) = f(y)*f(x) \quad \forall\, x, y \in X$.

Remark 2.6. If $f : X \to Y$ is a homomorphism on SU-algebras then $f(0_X) = 0_Y$.

Definition 2.7. A fuzzy subset μ in a non-empty set X is a function $\mu : X \to [0,1]$.

Definition 2.8. A fuzzy Subset μ in a SU-algebra X is said to have ***Sup-property*** if for any subset $T \subseteq X$ there exists $x_0 \in T$ such that $\mu(x_0) = \sup_{t \in T} \mu(t)$

Definition 2.9. Let $f : X \to Y$ be a function and μ be a fuzzy subset of X. Then the image of μ under f is a fuzzy subset ν on Y defined by

$$\nu(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} \mu(z) & if \quad f^{-1}(y) = \{x : f(x) = y\} \neq \phi \quad \forall\, y \in Y \\ 0 & otherwise \end{cases}$$

Definition 2.10. Let $f : X \to Y$ be a function and ν be the fuzzy subset of Y. Then the inverse image of ν under f is a fuzzy subset μ of X defined by $\nu(f(x)) = \mu(x) \ \forall\, x \in X$.

Definition 2.11.[4] A fuzzy Subset μ in X is said to be a fuzzy SU-subalgebra of X if $\mu(x*y) \geq \min\{\mu(x), \mu(y)\}$ for any $x, y \in X$.

Example 2.12. Consider the SU-algebra $X = \{0, 1, 2, 3\}$ in Example 2.2. and μ is the fuzzy Subset of X defined by

$$\mu = \begin{cases} 0.3 & ; \quad x = 1, 2 \\ 0.6 & ; \quad x = 0, 3 \end{cases}$$ is fuzzy SU-subalgebra of X.

3. Fuzzy Dot SU-Subalgebra

Here we introduce the **notion** of fuzzy Dot SU-subalgebra in a SU-algebra X. Here after unless otherwise specified X denotes a SU-algebra.

Definition 3.1. A fuzzy Subset μ in X is said to be a fuzzy Dot SU-subalgebra of X if $\mu(x*y) \geq \mu(x).\mu(y)$ for any $x, y \in X$

Example 3.2. Consider the SU-algebra $X = \{0, 1, 2, 3\}$ in Example 2.2. and μ is the fuzzy Subset of X defined by $\mu = \begin{cases} 0.3; & x = 0 \\ 0.4; & x = 1 \\ 0.5; & x = 2 \\ 0.6; & x = 3 \end{cases}$ is fuzzy Dot SU-subalgebra of X.

Remark 3.3. Every fuzzy SU-subalgebra is fuzzy Dot SU-subalgebra but the converse is not true. In the Example.3.2, $\mu(3*3) \geq \min\{\mu(3), \mu(3)\}$ not valid, since we have

$$\mu(3*3) = \mu(0) = 0.3 < \min\{\mu(3), \mu(3)\} = \min\{0.6, 0.6\} = 0.6$$

Theorem 3.4. If μ is a fuzzy Dot SU-subalgebra of X, then $\mu(0) \geq (\mu(x))^2$ and $\mu(0^n * x) \geq (\mu(x))^{2n+1} \quad \forall\, x \in X$ and $n \in \mathbb{N}$ where $0^n * x = 0*(0*(\ldots(0*x)\ldots))$ in which 0 occurs n times.

Proof. Since $x*x = 0 \quad \forall\, x \in X$, we have $\mu(0) = \mu(x*x) \geq \mu(x).\mu(x) = (\mu(x))^2$ and by induction $\mu(0^n * x) \geq (\mu(x))^{2n+1} \quad \forall\, x \in X$ and $n \in \mathbb{N}$

Theorem 3.5. If μ is a fuzzy Dot SU-subalgebra of X, then $K(\mu;1)=\{x\in X\ ;\ \mu(x)=1\}$ is a subalgebra of X.

Proof. Let $x,y\in K(\mu;1)$.

Then $\mu(x)=\mu(y)=1.$

$\Rightarrow \mu(x*y)\geq \mu(x).\mu(y)=1.$

Then $\mu(x*y)=1 \Rightarrow x*y\in K(\mu;1)$.

Theorem 3.6. Intersection of two fuzzy Dot SU-subalgebras of X is also a fuzzy Dot SU-subalgebra of X.

Proof. Let μ and σ be any two fuzzy Dot SU-subalgebras of X and $\lambda=\mu\cap\sigma$ where $\lambda(x)\geq \min\{\mu(x),\sigma(x)\}$. We have to prove $\lambda(x*y)\geq\lambda(x).\lambda(y)\ \ \forall\, x\,,\,y\in X$.

For $\lambda(x*y)=\min\{\mu(x*y)\,,\sigma(x*y)\}$

$$\geq \min\{\{\mu(x).\mu(y)\},\{\sigma(x).\sigma(y)\}\}$$

$$\geq \min\{\mu(x).\sigma(x)\}.\min\{\mu(y).\sigma(y)\}$$

$$=\lambda(x).\lambda(y)\qquad \forall\, x\,,\,y\in X\ .$$

The above theorem can be generalized as follows.

Theorem 3.7. The intersection of a family of fuzzy Dot SU-subalgebras of X is also an fuzzy Dot SU-subalgebra of X.

Theorem 3.8. Let f be a homomorphism from SU-algebras X onto Y and μ be an fuzzy Dot SU-subalgebra of X with Sup-property. Then the image of μ is also fuzzy Dot SU-subalgebra of Y.

Proof. Let $a,b\in Y$ with $x_0\in f^{-1}(a)$ and $y_0\in f^{-1}(b)$ such that

$$\mu(x_0)=\sup_{t\in f^{-1}(a)}\mu(t)\quad\text{and}\quad \mu(y_0)=\sup_{t\in f^{-1}(b)}\mu(t)\ .$$

Let ν be the image of μ .

Now by definitions 2.8 and 2.9,

$$\nu(a*b)=\sup_{t\in f^{-1}(a*b)}\mu(t)\ \geq \mu(x_0*y_0)\geq \mu(x_0).\mu(y_0)$$

$$=\sup_{t\in f^{-1}(a)}\mu(t)\ .\ \sup_{t\in f^{-1}(b)}\mu(t)=\nu(a)\,.\,\nu(b)\,.$$

Hence the image ν is a fuzzy Dot SU-subalgebra of Y.

Theorem 3.9. Let f be a homomorphism from SU-algebras X to Y and ν be a fuzzy Dot SU-subalgebra of Y. Then the inverse image of ν , is also fuzzy Dot SU-subalgebra of X.

Proof. Let $x,y\in X$ and μ be the inverse image of ν . Then

$$\mu(x*y)=\nu\{f(x*y)\}=\nu\{f(x)*f(y)\}\geq \nu\{f(x)\}.\nu\{f(y)\}=\mu(x)\,.\,\mu(y)\,.$$

Hence the inverse image of ν , is fuzzy Dot SU-subalgebra of X.

Remark 3.10. One can verify the theorem **3.8** and **3.9** for an anti-homomorphism on SU-algebras.

4. Conclusion

In this article we have extended the notions of fuzzy Dot SU-subalgebras of SU-algebras and verified some of their basic properties and the characteristic of their homomorpic(antihomomorphic) image(pre image). Also in [5] Supawadee Keawrahun and Utsanee Leerawat says that the structure SU-algebra becomes a TM-algebra, QS-algebra and a BF-algebra and Andrzej Walendziak[1] proved that a BF-algebra is BG-algebra. Hence we conclude that, the results we have proved for SU-algebras are obviously true in TM-algebras, QS-algebras, BF-algebras and BG-algebras.

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