General Multiplicative Zagreb Indices of $TUC_4C_8[m, n]$ and $TUC_4[m, n]$ Nanotubes

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Abstract. The topological indices correlate certain physicochemical properties such as boiling point, stability of chemical compounds. In this paper, we determine the multiplicative Zagreb index, multiplicative hyper Zagreb indices and general multiplicative Zagreb indices of $TUC_4C_8[m, n]$ and $TUC_4[m, n]$ nanotubes.

Keywords: multiplicative Zagreb index, multiplicative hyper Zagreb indices, general multiplicative Zagreb indices

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1. Introduction

In this paper, we consider finite simple undirected graphs. Let $G$ be a graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex $v$ is the number of vertices adjacent to $v$, we refer to [1] for undefined term and notation.

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences. In chemical science, the physico-chemical properties of chemical compounds are often modeled by means of a molecular graph based structure descriptors, which are referred to as topological indices.

In [11], Todeshine et al. introduced the first and second multiplicative Zagreb indices. These indices are defined as

\[ II_1(G) = \prod_{v \in V(G)} d_G(v)^2, \quad II_2(G) = \prod_{u \in E(G)} d_G(u)d_G(v). \]

In [3], Eliasi et al. introduced a new multiplicative version of the first multiplicative Zagreb index as

\[ II^*_1(G) = \prod_{u \in E(G)} (d_G(u) + d_G(v)). \]

In [6], Kulli introduced the first and second multiplicative hyper-Zagreb indices of a graph. These indices are defined as
In [10], Kulli, Stone, Wang and Wei introduced the general first and second multiplicative Zagreb indices. These indices are defined as

\[ MZ^1_u(G) = \prod_{u \in V(G)} (d_G(u) + d_G(v))^u, \quad MZ^2_u(G) = \prod_{u \in V(G)} (d_G(u)d_G(v))^u. \]

Recent development of molecular descriptors may be found in [2, 4, 5, 7, 8, 9, 12, 13, 14].

In this paper, we compute the multiplicative Zagreb index, the multiplicative hyper Zagreb indices and general multiplicative Zagreb indices of \( TUC_4C_8[m, n] \) nanotubes and \( TUC_4[m, n] \) nanotubes.

2. Results for \( TUSC_4C_8(S) \) nanotubes

We consider \( TUSC_4C_8(S) \) nanotubes which is a family of nanostructures. These structures are made up of cycles \( C_4 \) and \( C_8 \). These nanotubes usually symbolized as \( TUC_4C_8[m, n] \) for any \( m, n \in \mathbb{N} \), in which \( m \) is the number of octagons \( C_8 \) in the first row and \( n \) is the number of octagons \( C_8 \) in the first column as depicted in Figure 1 [15].

![Figure 1: TUSC_4C_8 nanotubes](image)

We compute the first multiplicative Zagreb index of \( TUSC_4C_8[m, n] \) nanotubes.

**Theorem 2.1.** Let \( G = TUC_4C_8[m, n] \). Then

\[ II_1(G) = 2^{8m^2} \times 3^{16mn}. \]

**Proof:** Let \( G = TUC_4C_8[m, n] \) as depicted in Figure 1. By Algebraic method, we get \( |V(G)| = 8mn + 4m \). From Figure 1, it is easy to see that there are two partitions of the vertex set:

\[ V_2 = \{v \in V(G)| d_G(v) = 2\}, \quad |V_2| = 2m + 2m \]

\[ V_3 = \{v \in V(G)| d_G(u) = 3\}, \quad |V_3| = 8mn. \]

We determine \( II_1(G) \), we see that

\[ II_1(G) = \prod_{u \in V(G)} d_G(u)^2 = \prod_{u \in V_2} d_G(u)^2 \times \prod_{u \in V_3} d_G(u)^2 \]

and hence

\[ II_1(G) = 2^{8m^2} \times 3^{16mn}. \]

We now compute the general first and second multiplicative Zagreb indices of \( TUC_4C_8[m, n] \) nanotubes.
Theorem 2.2. Let $G = TUC_4C_8 [m, n]$. Then

(1) $MZ_1^*(G) = 4^{2am} \times 5^{4am} \times 6^{4(12mn-2m)}$.

(2) $MZ_2^*(G) = 4^{2am} \times 6^{4am} \times 9^{4(12mn-2m)}$.

Proof: Let $G = TUC_4C_8 [m, n]$. By algebraic method, we obtain three partitions of the edge set of $TUC_4C_8 [m, n]$ nanotubes as follows:

$E_4 = E^*_4 \{ e = uv \in E(G) \mid d_G(u) = d_G(v) = 2 \}, \quad |E_4| = |E^*_4| = 2m$

$E_6 = E^*_6 \{ e = uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3 \}, \quad |E_6| = |E^*_6| = 4m$

$E_9 = E^*_9 \{ e = uv \in E(G) \mid d_G(u) = d_G(v) = 3 \}, \quad |E_9| = |E^*_9| = 12mn - 2m$.

Now to determine $MZ_1^*(G)$, we see that

$MZ_1^*(G) = \prod_{u \in V(G)} \left[ d_G(u) + d_G(v) \right]^a$

$= \prod_{u \in E_6} \left[ d_G(u) + d_G(v) \right]^a \times \prod_{u \in E_9} \left[ d_G(u) + d_G(v) \right]^a$

$= \left[ (2+2)^a \right]^{2m} \times \left[ (2+3)^a \right]^{4m} \times \left[ (3+3)^a \right]^{(12mn-2m)} = 4^{2am} \times 5^{4am} \times 6^{4(12mn-2m)}$

To see the second result, we have

$MZ_2^*(G) = \prod_{u \in V(G)} \left[ d_G(u) d_G(v) \right]^a$

$= \prod_{u \in E_6} \left[ d_G(u) d_G(v) \right]^a \times \prod_{u \in E_9} \left[ d_G(u) d_G(v) \right]^a$

$= \left[ (2 \times 2)^a \right]^{2m} \times \left[ (2 \times 3)^a \right]^{4m} \times \left[ (3 \times 3)^a \right]^{(12mn-2m)} = 4^{2am} \times 6^{4am} \times 9^{4(12mn-2m)}$

An immediate corollary is the first and second multiplicative Zagreb indices of $TUC_4C_8 [m, n]$ nanotubes.

Corollary 2.3. Let $G = TUC_4C_8 [m, n]$. Then

(1) $H_1^*(G) = 4^{2am} \times 5^{4am} \times 6^{(12mn-2m)}$.

(2) $H_2^*(G) = 4^{2am} \times 6^{4am} \times 9^{(12mn-2m)}$.

An immediate another corollary is the first and second multiplicative hyper Zagreb indices of $TUC_4C_8 [m, n]$ nanotubes.

Corollary 2.4. Let $G = TUC_4C_8 [m, n]$. Then

(1) $H H_1^*(G) = 4^{4m} \times 5^{8m} \times 6^{24mn-4m}$.

(2) $H H_2^*(G) = 4^{4m} \times 6^{8m} \times 9^{24mn-4m}$.

3. Results for $TUHRC_4(S)$ nanotubes

In this section, we focus on the structures of a family of nanostructure which are called $TUHRC_4(S)$ nanotubes. These nanotubes usually symbolized as $TUC_4[m, n]$ for any $m, n \in N$, in which $m$ is the number of cycle $C_4$ in the first row and $n$ in the number of cycles $C_4$ in the first column as depicted in Figure 2 [15].
We compute the first multiplicative Zagreb index of $TUC_4[m, n]$ nanotubes.

**Theorem 3.1.** Let $G$ be the $TUC_4[m, n]$ nanotubes. Then

$$I_1(G) = 2^{4m} \times 6^{4mn}.$$

**Proof:** Let $G$ be the $TUC_4[m, n]$ nanotubes as depicted in Figure 2. By algebraic method, we get $|V(G)| = 2m(n+1)$. From Figure 2, it is easy to see that there are two partitions of the vertex set of $G$ as follows:

- $V_1 = \{ v \in V(G) \mid d_G(v) = 2 \}$, $|V_1| = 2m$.
- $V_2 = \{ v \in V(G) \mid d_G(v) = 4 \}$, $|V_2| = 2mn$.

We determine $I_1(G)$, we see that

$$I_1(G) = \prod_{u \in V_1} d_G(u)^2 = \prod_{u \in V_2} d_G(u)^2 \times \prod_{u \in V_1} d_G(u)^2$$

and hence

$$I_1(G) = (2^2)^{2m} \times (4^4)^{2mn} = 2^{4m} \times 4^{4mn}.$$

We now determine the general first and second multiplicative Zagreb indices of $TUC_4[m, n]$ nanotubes.

**Theorem 3.2.** Let $G$ be the $TUC_4[m, n]$ nanotubes. Then

1. $MZ_1^+(G) = 6^{4am} \times 8^{(8mn-2m)a}$.
2. $MZ_1^-(G) = 2^{(16mn+4m)a}$.

**Proof:** Let $G = TUC_4[m, n]$. By algebraic method, we obtain two partitions of the edge set of $G$ as follows:

- $E_1 = E_1^+ = \{ uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4 \}$, $|E_1^+| = 4m$.
- $E_2 = E_1^- = \{ uv \in E(G) \mid d_G(u) = d_G(v) = 4 \}$, $|E_1^-| = 4mn - 2m$.

Now to determine $MZ_1^+(G)$, we see that

$$MZ_1^+(G) = \prod_{uv \in E_1^+} [d_G(u) + d_G(v)]^a$$

$$= \prod_{u \in E_1^+} [d_G(u) + d_G(v)]^a \times \prod_{v \in E_1^+} [d_G(u) + d_G(v)]^a$$

$$= \left[ (2 + 4)^a \right]^{4m} \times \left[ (4 + 4)^a \right]^{(4mn-2m)a} = 6^{4am} \times 8^{(4mn-2m)a}$$

To see the second result, we get

$$MZ_1^-(G) = \prod_{uv \in E_1^-} [d_G(u)d_G(v)]^a.$$
\[
\prod_{u \in V(G)} \left[ d_G(u) d_G(v) \right]^a \times \prod_{v \in E(G)} \left[ d_G(u) d_G(v) \right]^a = 2^{(16m+4n)a}
\]

An immediate corollary is the first and second multiplicative Zagreb indices of \( TUC_4[m,n] \) nanotubes.

**Corollary 3.3.** Let \( G = TUC_4[m,n] \). Then

1. \( II_1^+(G) = 6^{4m} \times 8^{4mn-2m} \).
2. \( II_2^+(G) = 2^{16mn+4m} \).

An immediate another corollary is the first and second multiplicative hyper Zagreb indices of \( TUC_4[m,n] \) nanotubes.

**Corollary 3.4.** Let \( G = TUC_4[m,n] \). Then

(a) \( HII_1^+(G) = 6^{8m} \times 8^{36mn-4m} \).
(b) \( HII_2^+(G) = 2^{2^{16mn+4m}} \).

**REFERENCES**