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General Multiplicative Zagreb Indices of $TUC_4C_8[m, n]$ and $TUC_4[m, n]$ Nanotubes

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Abstract. The topological indices correlate certain physicochemical properties such as boiling point, stability of chemical compounds. In this paper, we determine the multiplicative Zagreb index, multiplicative hyper Zagreb indices and general multiplicative Zagreb indices of $TUC_4C_8[m, n]$ and $TUC_4[m, n]$ nanotubes.

Keywords: multiplicative Zagreb index, multiplicative hyper Zagreb indices, general multiplicative Zagreb indices

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*90

1. Introduction

In this paper, we consider finite simple undirected graphs. Let G be a graph with a vertex set $V(G)$ and an edge set $e(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. we refer to [1] for undefined term and notation.

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences. In chemical science, the physico-chemical properties of chemical compounds are often modeled by means of a molecular graph based structure descriptors, which are referred to as topological indices.

In [11], Todeshine et al. introduced the first and second multiplicative Zagreb indices. These indices are defined as

$$II_1(G) = \prod_{v \in V(G)} d_G(u)^2, \qquad II_2(G) = \prod_{uv \in E(G)} d_G(u) d_G(v).$$

In [3], Eliasi et.al. introduced a new multiplicative version of the first multiplicative Zagreb index as

$$II_1^*(G) = \prod_{uv \in E(G)} \left(d_G(u) + d_G(v)\right).$$

In [6], Kulli introduced the first and second multiplicative hyper-Zagreb indices of a graph. These indices are defined as

$$HII_1(G) = \prod_{uv \in E(G)} \left(d_G(u) + d_G(v)\right)^2, \qquad HII_2(G) = \prod_{uv \in E(G)} \left(d_G(u) d_G(v)\right)^2.$$

In [10], Kulli, Stone, Wang and Wei introduced the general first and second multiplicative Zagreb indices. These indices are defined as

$$MZ_1^a(G) = \prod_{uv \in E(G)} \left(d_G(u) + d_G(v)\right)^a, \qquad MZ_2^a(G) = \prod_{uv \in E(G)} \left(d_G(u) d_G(v)\right)^a.$$

Recent development of molecular descriptors may be found in [2, 4, 5, 7, 8, 9, 12, 13, 14].

In this paper, we compute the multiplicative Zagreb index, the multiplicative hyper Zagreb indices and general multiplicative Zagreb indices of TUC_4C_8 [m, n] nanotubes and TUC_4[m, n] nanotubes.

2. Results for $TUSC_4C_8$ (S) nanotubes

We consider $TUSC_4C_8(S)$ nanotubes which is a family of nanostructures. These structures are made up of cycles C_4 and C_8. These nanotubes usually symbolized as TUC_4C_8 [m, n] for any m, $n \in N$, in which m is the number of octagons C_8 in the first row and n is the number of octagons C_8 in the first column as depicted in Figure 1 [15].

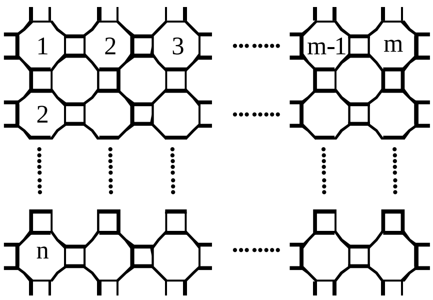

Figure 1:

We compute the first multiplicative Zagreb index of $TUSC_4C_8[m, n]$ nanotubes.

Theorem 2.1. Let $G = TUC_4C_8$ [m, n]. Then

$$II_1(G) = 2^{8m} \times 3^{16mn}.$$

Proof: Let $G = TUC_4C_8$ [m, n] as depicted in Figure 1. By Algebraic method, we get $|V(G)| = 8mn + 4m$. From Figure 1, it is easy to see that there are two partitions of the vertex set:

$$V_2 = \{v \in V(G) \mid d_G(v) = 2\}, \qquad |V_2| = 2m + 2m$$

$$V_3 = \{v \in V(G) \mid d_G(u) = 3\}, \qquad |V_3| = 8mn.$$

We determine $II_1(G)$, we see that

$$II_1(G) = \prod_{u \in V(G)} d_G(u)^2 = \prod_{u \in V_1} d_G(u)^2 \times \prod_{u \in V_3} d_G(u)^2$$

and hence

$$II_1(G) = \left(2^2\right)^{4m} \times \left(3^2\right)^{8mn} = 2^{8m} \times 3^{16mn}.$$

We now compute the general first and second multiplicative Zagreb indices of TUC_4C_8 [m, n] nanotubes.

Theorem 2.2. Let $G = TUC_4C_8\ [m, n]$. Then

(1) $MZ_1^a(G) = 4^{2am} \times 5^{4am} \times 6^{a(12mn-2m)}$.

(2) $MZ_2^a(G) = 4^{2am} \times 6^{4am} \times 9^{a(12mn-2m)}$.

Proof: Let $G = TUC_4C_8\ [m, n]$. By algebraic method, we obtain three partitions of the edge set of $TUC_4C_8[m, n]$ nanotubes as follows:

$$E_4 = E_4^* = \{e = uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, \quad |E_4| = |E_4^*| = 2m$$

$$E_5 = E_6^* = \{e = uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, \quad |E_5| = |E_6^*| = 4m$$

$$E_6 = E_9^* = \{e = uv \in E(G) \mid d_G(u) = d_G(v) = 3\}, \quad |E_6| = |E_9^*| = 12mn - 2m.$$

Now to determine $MZ_1^a(G)$, we see that

$$MZ_1^a(G) = \prod_{uv\in E(G)} \left[d_G(u) + d_G(v)\right]^a$$
$$= \prod_{uv\in E_4} \left[d_G(u) + d_G(v)\right]^a \times \prod_{uv\in E_5} \left[d_G(u) + d_G(v)\right]^a \times \prod_{uv\in E_6} \left[d_G(u) + d_G(v)\right]^a$$
$$= \left[(2+2)^a\right]^{2m} \times \left[(2+3)^a\right]^{4m} \times \left[(3+3)^a\right]^{12mn-2m} = 4^{2am} \times 5^{4am} \times 6^{a[12mn-2m]}$$

To see the second result, we have

$$MZ_2^a(G) = \prod_{uv\in E(G)} \left[d_G(u) d_G(v)\right]^a$$
$$= \prod_{uv\in E_4^*} \left[d_G(u) d_G(v)\right]^a \times \prod_{uv\in E_6^*} \left[d_G(u) d_G(v)\right]^a \times \prod_{uv\in E_9^*} \left[d_G(u) d_G(v)\right]^a$$
$$= \left[(2\times2)^a\right]^{2m} \times \left[(2\times3)^a\right]^{4m} \times \left[(3\times3)^a\right]^{12mn-2m} = 4^{2am} \times 6^{4am} \times 9^{a[12mn-2m]}$$

An immediate corollary is the first and second multiplicative Zagreb indices of $TUC_4C_8\ [m, n]$ nanotubes.

Corollary 2.3. Let $G = TUC_4C_8\ [m, n]$. Then

(1) $II_1^*(G) = 4^{2m} \times 5^{4m} \times 6^{12mn-2m}$.

(2) $II_2(G) = 4^{2m} \times 6^{4m} \times 9^{12m-2m}$.

An immediate another corollary is the first and second multiplicative hyper Zagreb indices of $TUC_4C_8[m, n]$ nanotubes.

Corollary 2.4. Let $G = TUC_4C_8\ [m, n]$. Then

(1) $\mathrm{H}II_1(G) = 4^{4m} \times 5^{8m} \times 6^{24mn-4m}$.

(2) $\mathrm{H}II_2(G) = 4^{4m} \times 6^{8m} \times 9^{24mn-4m}$.

3. Results for $TUHRC_4(S)$ nanotubes

In this section, we focus on the structures of a family of nanostructure which are called $TUHRC_4(S)$ nanotubes. These nanotubes usually symbolized as $TUC_4[m, n]$ for any $m, n \in N$, in which m is the number of cycle C_4 in the first row and n in the number of cycles C_4 in the first column as depicted in Figure 2 [15].

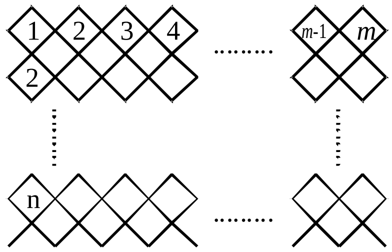

Figure 2:

We compute the first multiplicative Zagreb index of $TUC_4[m, n]$ nanotubes.

Theorem 3.1. Let G be the TUC_4 $[m, n]$ nanotubes. Then

$$II_1(G) = 2^{4m} \times 6^{4mn}.$$

Proof: Let G be the $TUC_4[m, n]$ nanotubes as depicted in Figure 2. By algebraic method, we get $|V(G)| = 2m(n+1)$. From Figure 2, it is easy to see that there are two partitions of the vertex set of G as follows:

$$V_2 = \{v \in V(G) \mid d_G(v) = 2\}, \qquad |V_2| = 2m.$$

$$V_4 = \{v \in V(G) \mid d_G(v) = 4\}, \qquad |V_4| = 2mn.$$

We determine $II_1(G)$, we see that

$$II_1(G) = \prod_{u \in V(G)} d_G(u)^2 = \prod_{u \in V_2} d_G(u)^2 \times \prod_{u \in V_4} d_G(u)^2$$

and hence

$$II_1(G) = \left(2^2\right)^{2m} \times \left(4^2\right)^{2mn} = 2^{4m} \times 4^{4mn}.$$

We now determine the general first and second multiplicative Zagreb indices of TUC_4 $[m, n]$ nanotubes.

Theorem 3.2. Let G be the TUC_4 $[m, n]$ nanotubes. Then

(1) $\quad MZ_1^a(G) = 6^{4am} \times 8^{(8mn-2m)a}.$

(2) $\quad MZ_2^a(G) = 2^{(16mn+4m)a}.$

Proof: Let $G = TUC_4[m, n]$. By algebraic method, we obtain two partitions of the edge set of G as follows:

$$E_6 = E_8^* = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}, \qquad |E_6| = |E_8^*| = 4m.$$

$$E_8 = E_{16}^* = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}, \qquad |E_8| = |E_{16}^*| = 4mn - 2m.$$

Now to determine $MZ_1^a(G)$, we see that

$$\begin{aligned} MZ_1^a(G) &= \prod_{uv \in E(G)} \left[d_G(u) + d_G(v)\right]^a \\ &= \prod_{uv \in E_6} \left[d_G(u) + d_G(v)\right]^a \times \prod_{uv \in E_8} \left[d_G(u) + d_G(v)\right]^a \\ &= \left[(2+4)^a\right]^{4m} \times \left[(4+4)^a\right]^{4mn-2m} = 6^{4am} \times 8^{(4mn-2m)a} \end{aligned}$$

To see the second result, we get

$$MZ_2^a(G) = \prod_{uv \in E(G)} \left[d_G(u) d_G(v)\right]^a$$

$$= \prod_{uv \in E_8^*} \left[d_G(u) d_G(v) \right]^a \times \prod_{uv \in E_{16}^*} \left[d_G(u) d_G(v) \right]^a = 2^{(16mn+4m)a}$$

An Immediate corollary is the fist and second multiplicative Zagreb indices of $TUC_4[m, n]$ nanotubes.

Corollary 3.3. Let $G = TUC_4\,[m, n]$. Then

(1) $II_1^*(G) = 6^{4m} \times 8^{4mn-2m}$.

(2) $II_2(G) = 2^{16mn+4m}$.

An Immediate another corollary is the first and second multiplicative hyper Zagreb indices of TUC_4 [m, n] nanotubes.

Corollary 3.4. Let $G = TUC_4\,[m, n]$. Then

(a) $HII_1(G) = 6^{8m} \times 8^{8mn-4m}$.

(b) $HII_2(G) = 2^{2(16mn+4m)}$.

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