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Intuitionistic Fuzzy Soft Graph

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Abstract. We introduce the notions of intuitionistic fuzzy soft graph, strong intuitionistic fuzzy soft graph, complete intutionistic fuzzy soft graph in this paper. Also studied about union of two Intuitionistic fuzzy soft graph and proved that the collection of intuitionistic fuzzy soft graph is closed under finite union.

Keywords: Intuitionistic fuzzy soft graph, strong intuitionistic fuzzy soft graph, complete intuitionistic fuzzy soft graph, union of intuitionistic fuzzy soft graph.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

Molodtsov [13] introduced the concept of soft set that can be seen as a new mathematical theory for dealing with uncertainties. Molodtsov applied this theory to several directions [13, 14, 15] and then formulated the notions of Soft number, Soft derivative, Soft integral, etc. in [16]. The soft set theory has been applied to many different fields with greatness. Maji [11] worked on theoretical study of soft sets in detail. The algebraic structure of soft set theory dealing with uncertainties has also been studied in more detail. Aktas and Cagman [2] introduced definition of soft groups, and derived their basic properties. The most appreciate theory to deal with uncertainties is the theory of fuzzy sets, developed by Zadeh [22] in 1965. But it has an inherent difficulty to set the membership function in each particular cases. The generalization of Zadeh's fuzzy set called intuitionistic fuzzy set was introduced by Atanassov [4] which is characterized by a membership function and a non-membership function. In Zadeh's fuzzy set, the sum of membership degree and non-membership degree is equal to one. In Atanassov's intuitionistic fuzzy set the sum of membership degree and non- membership degree does not exeed one.

Maji et al. [9] presented the concept of fuzzy soft sets by embedding ideas of fuzzy set in [22]. In fact the notion of fuzzy soft set is more generalized than that of fuzzy set and soft set. There after many papers devoted to fuzzify the concept of soft set theory which leads to a series of mathematical models such as fuzzy soft set [1,9,17], generalized fuzzy soft set [13, 21], possibility fuzzy soft set [3] and so on. There after Maji and his coauthor [10] introduced the notion of intuitionistic fuzzy soft set which is based on a combination of intuitionistic fuzzy sets and soft set models and they studied the properties of intuitionistic fuzzy soft set.

In 1736, Euler first introduced the concept of graph theory. The theory of graph is extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operation research, optimization and computer science, etc.. The first definition of fuzzy graphs was proposed by Kaffmann [8] in 1973, from Zadeh's fuzzy relations [22]. But Rosenfeld [19] introduced another elaborated definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts. The first definition of intuitionistic fuzzy graph was introduced by Atanassov [5] in1999. Karunambigai and Parvathy [7] introduced intuitionistic fuzzy graph as a special case of Atanassov's intuitionistic fuzzy graph. Soft graph was introduced by Thumbakara and George [18]. In 2015, Mohinta and Samanta [20] introduced the concept of fuzzy soft graph.

In this paper, our aim is to introduce the notions of intuitionistic fuzzy soft graph, strong intuitionistic fuzzy soft graph, complete intutionistic fuzzy soft graph. Also studied about union of two intuitionistic fuzzy soft graph and proved that the collection of intuitionistic fuzzy soft graph is closed under finite union.

2. Preliminaries

Definition 2.1. *A fuzzy set of a base set* $V = \{v_1, v_2, \ldots, v_n\}$ *(non-empty set) is specified by its membership function* σ *; where* $\sigma : V \to [0,1]$ *assigning to each* $v_i \in V$ *, the degree or grade to which* $V \in \sigma$*.*

Definition 2.2. *A fuzzy graph* $G = (\sigma, \mu)$ *is a pair of function* $\sigma : V \to [0,1]$ *and* $\mu : V \times V \to [0,1]$, *where for all* $v_i, v_j \in V$ *we have* $\mu(v_i, v_j) \le \sigma(v_i) \wedge \sigma(v_j)$ *for each* $(v_i, v_j) \in V \times V$ *. Here* σ *and* μ *are respectively called fuzzy vertex and fuzzy edge of the fuzzy graph* $G = (\sigma, \mu)$*.*

Definition 2.3. *Let* $G_1 = (\sigma_1, \mu_1)$ *and* $G_2 = (\sigma_2, \mu_2)$ *be two fuzzy graphs over the set* V *. Then the Union of* G_1 *and* G_2 *is another fuzzy graph* $G_3 = (\sigma_3, \mu_3)$ *over the set* V *,where* $\sigma_3 = \sigma_1 \vee \sigma_2 = max\{\sigma_1(v_i), \sigma_2(v_i)\}$ *for every* $v_i \in V$ and $i = 1,2\ldots n$ *and* $\mu_3(v_i, v_j) = max\{\mu_1(v_i, v_j), \mu_2(v_i, v_j)\}$ *for every* $v_i, v_j \in V$ and $i, j = 1,2\ldots n$

Definition 2.4. *Let* V *be a non-empty set. An intuitionistic fuzzy set* A *in* V *defined as* $A = \{(V, \mu_A(v), \gamma_A(v)) \mid v \in V\}$ *which is characterised by a membership function* $\mu_A : V \to [0,1]$ *and the non-membership function* $\gamma_A : V \to [0,1]$ *and satisfying* 1. $0 \le \mu_A(v) + \gamma_A(v) \le 1$ for every $v \in V$.

2. $0 \le \mu_A(v), \gamma_A(v), \pi_A(v) \le 1$ for every $v \in V$.
3. $\pi_A(v) = 1 - \mu_A(v) - \gamma_A(v)$.

where π_A is called the intuitionistic fuzzy index of the element v *in* A; the value denotes the measure of non-determinancy.

Obviously if $\pi_A(v) = 0$ for every v in V, then the intuitionistic fuzzy set A is just Zadeh's fuzzy set.

Definition 2.5. *An intuitionistic fuzzy graph is defined as* $G = (V, E, \mu, \gamma)$ *where*
1. $V = \{v_1, v_2, \ldots, v_n\}$ (non-empty set) such that $\mu_1 : V \to [0,1]$ and $\gamma_1 : V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V$, $i = 1,2\ldots n$
2. $E \subset V \times V$ *where* $\mu_2 : V \times V \to [0,1]$ *and* $\gamma_2 : V \times V \to [0,1]$ are such that

(i) $\mu_2(v_i, v_j) \le min\{\mu_1(v_i), \mu_1(v_j)\}$

(ii) $\gamma_2(v_i, v_j) \le max\{\gamma_1(v_i), \gamma_1(v_j)\}$ and

(iii) $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$, $0 \le \mu_2(v_i, v_j), \gamma_2(v_i, v_j), \pi(v_i, v_j) \le 1$

where $\pi(v_i, v_j) = 1 - \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)$ for every $(v_i, v_j) \in E$, $i, j = 1,2,\ldots,n$

Let U be an initial universal set, P be a set of parameters, $\mathscr{P}(U)$ be the power set of U and $A \subseteq P$.

Definition 2.6. *A pair* (F, A) *is called a soft set over* U *if and only if* F *is a mapping of* A *into the set of all subsets of the set* U.

Definition 2.7. *A pair* $(\overline{F}, A)$ *is called fuzzy soft set over* U*, where* $\overline{F}$ *is a mapping given by* $\overline{F} : A \to I^U$ *;* I^U *denotes the collection of all fuzzy subsets of* U; $A \subseteq P$.

Definition 2.8. *A pair* $(\breve{F}, A)$ *is called an* intuitionistic fuzzy soft set *over* U*, where* $\breve{F}$ *is a mapping given by* $\breve{F} : A \to IF^U$ *;* IF^U *denotes the collection of all intuitionistic fuzzy subsets of* U; $A \subseteq P$.

Definition 2.9. *Let* $V = \{v_1, v_2, \ldots, v_n\}$ *(non-empty set),P(parameterset) and* $A \subseteq P$*. Also let*

1. $\rho : A \to I^U(V)$ ($I^U(V)$ denotes collection of all fuzzy subsets in V)

   $a \mapsto \rho(a) = \rho_a$ (say), $a \in A$ and $\rho_a : V \to [0,1]$,

   (A, ρ) Fuzzy soft vertex.

2. $\mu : A \to I^U(V \times V)$, ($I^U(V \times V)$ denotes collection of all fuzzy subsets in $V \times V$)

   $a \mapsto \mu(a) = \mu_a$ (say), $a \in A$ and $\mu_a : V \times V \to [0,1]$ $(v_i, v_j) \mapsto \mu_a(v_i, v_j)$

   (A, μ) Fuzzy soft edge.

Then $((A, \rho), (A, \mu))$ is called fuzzy soft graph if and only if

$\mu_a(v_i, v_j) \le \rho_a(v_i) \wedge \rho(v_j)$ for each $(v_i, v_j) \in V \times V$,
for every $a \in A$ *and* $i, j = 1,2,\ldots,n$.

3. Intuitionistic fuzzy soft graph

Definition 3.1. *Let* $G = (V, E)$ *be a simple graph,* $V = \{v_1, v_2, \ldots, v_n\}$ *(non-empty set),* $E \subseteq V \times V$, $P(\textit{parameter set})$ *and* $A \subseteq P$ *.Also let*

1. μ_1 is a membership function defined on V by
$\mu_1 : A \to IF^U(V)$ ($IF^U(V)$ denotes collection of all intuitionistic fuzzy subsets in V)
$a \mapsto \mu_1(a) = \mu_{1a}$ (say), $a \in A$ and $\mu_{1a} : V \to [0,1], v_i \mapsto \mu_{1a}(v_i)$
(A, μ_1) Intuitionistic fuzzy soft vertex of membership function and
γ_1 is a non-membership function defined on V by
$\gamma_1 : A \to IF^U(V)$ ($IF^U(V)$ denotes collection of all intuitionistic fuzzy subsets in V)
$a \mapsto \gamma_1(a) = \gamma_{1a}$ (say), $a \in A$ and $\gamma_{1a} : V \to [0,1]$, $v_i \mapsto \gamma_{1a}(v_i)$
(A, γ_1) Intuitionistic fuzzy soft vertex of non-membership function such that $0 \le \mu_{1a}(v_i) + \gamma_{1a}(v_i) \le 1$ for every $v_i \in V, = 1,2\ldots n$ and $a \in A$.
2. μ_2 is a membership function defined on E by
$\mu_2 : A \to IF^U(V \times V)$ ($IF^U(V \times V)$ denotes collection of all intuitionistic fuzzy subsets in E)
$a \mapsto \mu_2(a) = \mu_{2a}$ (say), $a \in A$ and $\mu_{2a} : V \times V \to [0,1], (v_i, v_j) \mapsto \mu_{2a}(v_i, v_j)$
γ_2 is the non membership function defined on E by
$\gamma_2 : A \to IF^U(V \times V)$ ($IF^U(V \times V)$ denotes collection of all intuitionistic fuzzy subsets in $V \times V$)
$a \mapsto \gamma_2(a) = \gamma_{2a}$ (say), $a \in A$ and $\gamma_{2a} : V \times V \to [0,1], (v_i, v_j) \mapsto \gamma_{2a}(v_i, v_j)$

where (A, μ_2), (A, γ_2) are Intuitionistic fuzzy soft edge of membership function and non- membership function satisfying

(i) $\mu_{2a}(v_i, v_j) \le min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$
(ii) $\gamma_{2a}(v_i, v_j) \le max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ and
(iii) $0 \le \mu_{2a}(v_i, v_j) + \gamma_{2a}(v_i, v_j) \le 1$,

$$0 \le \mu_{2a}(v_i, v_j), \gamma_{2a}(v_i, v_j) \le 1, \textit{ forevery}(v_i, v_j) \in E, i, j = 1,2,\ldots,n \textit{ and } a \in A$$

Then $G^* = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ is said to be the intuitionistic fuzzy soft graph (IFSG) and this IFSG is denoted by $G^*_{A,V,E}$.

Example 3.1. Consider a simple graph $G = (V, E)$ where $V = \{v_1, v_2, v_3\}$ and $E = \{(v_1, v_2), (v_2, v_3), (v_1, v_3)\}$. Let $A = \{a_1, a_2, a_3\}$ be the parameter set. Then the intuitionistic fuzzy soft graph (IFSG), $G^*_{A,V,E} = (V, E, (A,\mu_1)$, (A,γ_1) , (A,μ_2) , $(A,\gamma_2))$ is described in Table 1 and Figure 1.

1(a)

μ_{1a}	v_1	v_2	v_3
a_1	0.8	0.6	0
a_2	0.9	0.65	0.5
a_3	0.2	0.75	0.8

1(b)

γ_{1a}	v_1	v_2	v_3
a_1	0.1	0.2	0
a_2	0.05	0.25	0.4
a_3	0.6	0.1	0.15

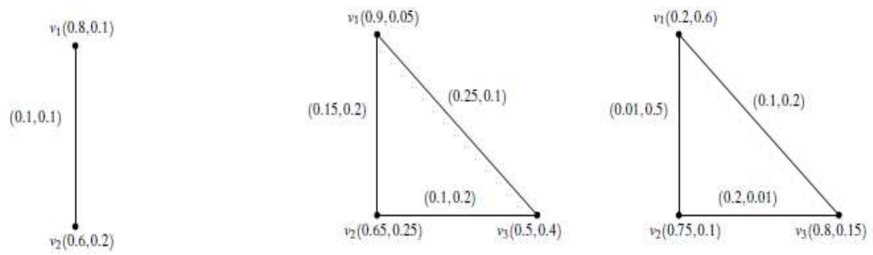
1(c)

μ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0	0
a_2	0.15	0.25	0.1
a_3	0.01	0.1	0.2

1(d)

γ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0	0
a_2	0.2	0.1	0.2
a_3	0.5	0. 2	0.01

Table 1:



1(a) IFSG corresponding to the parameter a_1

1(b) IFSG corresponding to the parameter a_2

1(c) IFSG corresponding to the parameter a_3

Figure 1:

Definition 3.2. *An intuitionistic fuzzy soft graph* $G^* = (V, E, (A,\mu_1), (A,\gamma_1)$, (A,μ_2), $(A,\gamma_2))$ *is said to be* strong intuitionistic fuzzy soft graph *if* $\mu_{2a}(v_i, v_j) = min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ *and* $\gamma_{2a}(v_i, v_j) = max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ *for every* $(v_i, v_j) \in E$ *and* $a \in A$ *and is said to be* complete intuitionistic fuzzy soft graph *if* $\mu_{2a}(v_i, v_j) = min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ *and* $\gamma_{2a}(v_i, v_j) = max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ *for every* $v_i, v_j \in V$ *and* $a \in A$.

Example 3.2. Consider a simple graph $G = (V, E)$ where $V = \{v_1, v_2, v_3\}$ and

$E=\{(v_1,v_2),(v_2,v_3),(v_1,v_3)\}$. Let $A=\{a_1,a_2,a_3\}$ be the parameter set. Then the strong intuitionistic fuzzy softgraph, $G^*_{A,V,E}=(V,E,(A,\mu_1),(A,\gamma_1),(A,\mu_2),(A,\gamma_2))$ is described in Table 2 and Figure 2.

2(a)

μ_{1a}	v_1	v_2	v_3
a_1	0.2	0.3	0.4
a_2	0.6	0.05	0.35
a_3	0.5	0.2	0.1

2(b)

γ_{1a}	v_1	v_2	v_3
a_1	0.4	0.3	0.55
a_2	0.1	0.2	0.25
a_3	0.2	0.1	0.2

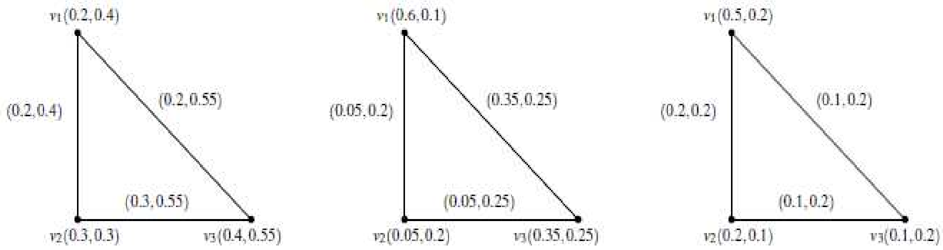
2(c)

μ_{2a}	(v_1,v_2)	(v_2,v_3)	(v_1,v_3)
a_1	0.2	0.3	0.2
a_2	0.05	0.05	0.35
a_3	0.2	0.1	0.1

2(d)

γ_{2a}	(v_1,v_2)	(v_2,v_3)	(v_1,v_3)
a_1	0.4	0.55	0.55
a_2	0.2	0.25	0.25
a_3	0.2	0.2	0.2

Table 2:



2(a) Strong IFSG corresponding to the parameter a_1 2(b) Strong IFSG corresponding to the parameter a_2 2(c) Strong IFSG corresponding to the parameter a_3

Figure 2:

Example 3.3. Consider a simple graph $G=(V,E)$ where $V=\{v_1,v_2,v_3\}$ and $E=\{(v_1,v_2),(v_2,v_3),(v_1,v_3),(v_1,v_1),(v_2,v_2),(v_3,v_3)\}$. Let $A=\{a_1,a_2,a_3\}$ be the parameter set. Then the complete intuitionistic fuzzy soft graph, $G^*_{A,V,E}=(V,E,(A,\mu_1)$, (A,γ_1), (A,μ_2), $(A,\gamma_2))$ is described in Table 3 and Figure 3.

3(a)

μ_{1a}	v_1	v_2	v_3
a_1	0.1	0.2	0
a_2	0.2	0.1	0
a_3	0	0.1	0

3(b)

γ_{1a}	v_1	v_2	v_3
a_1	0.4	0.3	0
a_2	0.3	0.2	0
a_3	0	0.3	0

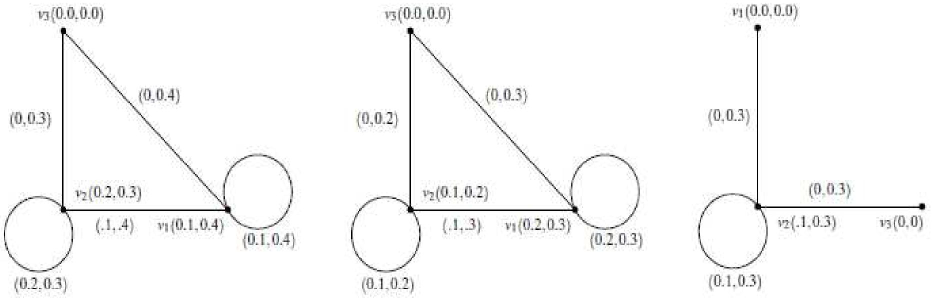
3(c)

μ_{2a}	(v_1,v_2)	(v_2,v_3)	(v_1,v_3)	(v_1,v_1)	(v_2,v_2)	(v_3,v_3)
a_1	0.1	0	0	0.1	0.2	0
a_2	0.1	0	0	0.2	0.1	0
a_3	0	0	0	0	0.1	0

3(d)

γ_{2a}	(v_1,v_2)	(v_2,v_3)	(v_1,v_3)	(v_1,v_1)	(v_2,v_2)	(v_3,v_3)
a_1	0.4	0.3	0.4	0.4	0.3	0
a_2	0.3	0.2	0.3	0.3	0.2	0
a_3	0.3	0.3	0	0	0.3	0

Table 3:



3(a) Complete IFSG corresponding to the parameter a_1 3(b) Complete IFSG corresponding to the parameter a_2 3(c) Complete IFSG corresponding to the parameter a_3

Figure 3:

Definition 3.3. *Let* $G = (V, E)$ *be a simple graph,* $P(parameter\ set)$. *Also let* $V_1, V_2 \subseteq V$, $E_1, E_2 \subset E$, $A, B \subseteq P$ *and* $G^*_{A,V_1,E_1} = (V_1, E_1, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *and* $G^*_{B,V_2,E_2} = (V_2, E_2, (B, \mu_1^{'}), (B, \gamma_1^{'}), (B, \mu_2^{'}), (B, \gamma_2^{'}))$ *be two intuitionistic fuzzy soft graphs with* $V_1 \cap V_2 \neq \phi$. *Then the* union of intuitionistic fuzzy soft graphs $G^*_{C,V_3,E_3} = G^*_{A,V_1,E_1} \cup G^*_{B,V_2,E_2}$ *with the condition* $\mu_{2a}(v_i, v_j) \geq max(\mu_{1a}^{'}(v_k))$ *and*

$\gamma_{2a}(v_i, v_j) \geq max(\gamma'_{1a}(v_k))$ *for every* $v_i, v_j, v_k \in V, a \in P$ *and for any* $i, j, k = 1,2,\ldots,n$ is defined to be $G^*_{C,V_3,E_3} = (V_3, E_3, (C, \mu''_1), (C, \gamma''_1), (C, \mu''_2), (C, \gamma''_2))$ where $C = A \cup B$, $V_3 = V_1 \cup V_2$ and

$$
\begin{aligned}
\mu''_{1a}(v_i) &= \mu_{1a}(v_i) && \text{forevery } v_i \in V_1 \,|\, V_2 \text{ and } a \in A \,|\, B \\
&= 0 && \text{forevery } v_i \in V_2 \,|\, V_1 \text{ and } a \in A \,|\, B \\
&= \mu_{1a}(v_i) && \text{forevery } v_i \in V_1 \cap V_2 \text{ and } a \in A \,|\, B \\
&= \mu'_{1a}(v_i) && \text{forevery } v_i \in V_2 \,|\, V_1 \text{ and } a \in B \,|\, A \\
&= 0 && \text{forevery } v_i \in V_2 \,|\, V_1 \text{ and } a \in B \,|\, A \\
&= \mu'_{1a}(v_i) && \text{forevery } v_i \in V_1 \cap V_2 \text{ and } a \in B \,|\, A \\
&= \mu_{1a}(v_i) \vee \mu'_{1a}(v_i) && \text{forevery } v_i \in V_1 \cap V_2 \text{ and } a \in A \cap B \\
&= \mu_{1a}(v_i) && \text{forevery } v_i \in V_1 \,|\, V_2 \text{ and } a \in A \cap B \\
&= \mu'_{1a}(v_i) && \text{forevery } v_i \in V_2 \,|\, V_1 \text{ and } a \in A \cap B
\end{aligned}
$$

$$
\begin{aligned}
\gamma''_{1a}(v_i) &= \gamma_{1a}(v_i) && \text{forevery } v_i \in V_1 \,|\, V_2 \text{ and } a \in A \,|\, B \\
&= 0 && \text{forevery } v_i \in V_2 \,|\, V_1 \text{ and } a \in A \,|\, B \\
&= \gamma_{1a}(v_i) && \text{forevery } v_i \in V_1 \cap V_2 \text{ and } a \in A \,|\, B \\
&= \gamma'_{1a}(v_i) && \text{forevery } v_i \in V_2 \,|\, V_1 \text{ and } a \in B \,|\, A \\
&= 0 && \text{forevery } v_i \in V_2 \,|\, V_1 \text{ and } a \in B \,|\, A \\
&= \gamma'_{1a}(v_i) && \text{forevery } v_i \in V_1 \cap V_2 \text{ and } a \in B \,|\, A \\
&= \gamma_{1a}(v_i) \wedge \gamma'_{1a}(v_i) && \text{forevery } v_i \in V_1 \cap V_2 \text{ and } a \in A \cap B \\
&= \gamma_{1a}(v_i) && \text{forevery } v_i \in V_1 \,|\, V_2 \text{ and } a \in A \cap B \\
&= \gamma'_{1a}(v_i) && \text{forevery } v_i \in V_2 \,|\, V_1 \text{ and } a \in A \cap B
\end{aligned}
$$

$$
\begin{aligned}
&\mu''_{2a}(v_i, v_j) \\
&= \mu_{2a}(v_i, v_j) && \text{forevery } (v_i, v_j) \in (V_1 \times V_1) \,|\, (V_2 \times V_2) \text{ and } a \in A \,|\, B \\
&= 0 && \text{forevery } (v_i, v_j) \in (V_2 \times V_2) \,|\, (V_1 \times V_1) \text{ and } a \in A \,|\, B \\
&= \mu_{2a}(v_i, v_j) && \text{forevery } (v_i, v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2) \text{ and } a \in A \,|\, B \\
&= \mu'_{2a}(v_i, v_j) && \text{forevery } (v_i, v_j) \in (V_2 \times V_2) \,|\, (V_1 \times V_1) \text{ and } a \in B \,|\, A \\
&= 0 && \text{forevery } (v_i, v_j) \in (V_1 \times V_1) \,|\, (V_2 \times V_2) \text{ and } a \in B \,|\, A \\
&= \mu'_{2a}(v_i, v_j) && \text{forevery } (v_i, v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2) \text{ and } a \in B \,|\, A \\
&= \mu_{2a}(v_i, v_j) \vee \mu'_{2a}(v_i, v_j) && \text{forevery } (v_i, v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2) \text{ and } a \in A \cap B \\
&= \mu_{2a}(v_i, v_j) && \text{forevery } (v_i, v_j) \in (V_1 \times V_1) \,|\, (V_2 \times V_2) \text{ and } a \in A \cap B \\
&= \mu'_{2a}(v_i, v_j) && \text{forevery } (v_i, v_j) \in (V_2 \times V_2) \,|\, (V_1 \times V_1) \text{ and } a \in A \cap B
\end{aligned}
$$

$\gamma''_{2a}(v_i, v_j)$

$= \gamma_{2a}(v_i, v_j)$ forevery $(v_i, v_j) \in (V_1 \times V_1) \mid (V_2 \times V_2)$ and $a \in A \mid B$

$= 0$ forevery $(v_i, v_j) \in (V_2 \times V_2) \mid (V_1 \times V_1)$ and $a \in A \mid B$

$= \gamma_{2a}(v_i, v_j)$ forevery $(v_i, v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$ and $a \in A \mid B$

$= \gamma'_{2a}(v_i, v_j)$ forevery $(v_i, v_j) \in (V_2 \times V_2) \mid (V_1 \times V_1)$ and $a \in B \mid A$

$= 0$ forevery $(v_i, v_j) \in (V_1 \times V_1) \mid (V_2 \times V_2)$ and $a \in B \mid A$

$= \gamma'_{2a}(v_i, v_j)$ forevery $(v_i, v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$ and $a \in B \mid A$

$= \gamma_{2a}(v_i, v_j) \wedge \mu'_{2a}(v_i, v_j)$ forevery $(v_i, v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$ and $a \in A \cap B$

$= \gamma_{2a}(v_i, v_j)$ forevery $(v_i, v_j) \in (V_1 \times V_1) \mid (V_2 \times V_2)$ and $a \in A \cap B$

$= \gamma'_{2a}(v_i, v_j)$ forevery $(v_i, v_j) \in (V_2 \times V_2) \mid (V_1 \times V_1)$ and $a \in A \cap B$

Example 3.4. Consider a simple graph $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{(v_1, v_2), (v_1, v_3), (v_3, v_4), (v_3, v_5)\}$. Let $V_1 = \{v_1, v_2, v_3\}$, $E_1 = \{(v_1, v_2), (v_1, v_3)\}$ and $V_2 = \{v_3, v_4, v_5\}$, $E_2 = \{(v_3, v_4), (v_3, v_5)\}$. Let $P = \{a_1, a_2, a_3, a_4\}$ be the parameter set. Also let $A, B \subseteq P$ such that $A = \{a_1, a_2, a_3\}$ and $B = \{a_2, a_3, a_4\}$. Let $G^*_{A,V_1,E_1} = (V_1, E_1, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ be the IFSG described in Table 4 and Figure 4 and $G^*_{B,V_2,E_2} = (V_2, E_2, (A, \mu'_1), (A, \gamma'_1), (A, \mu'_2), (A, \gamma'_2))$ be the IFSG described in Table 5 and Figure 5. Then the union of G^*_{A,V_1,E_1} and G^*_{B,V_2,E_2} is $G^*_{C,V_3,E_3} = (V_3, E_3, (C, \mu''_1), (C, \gamma''_1), (C, \mu''_2), (C, \gamma''_2))$ given by Table 6 and Figure 6 where $C = A \cup B$, $V_3 = V_1 \cup V_2$ *and* $E_3 = E_1 \cup E_2$.

4(a)

μ_{1a}	v_1	v_2	v_3
a_1	0.3	0.2	0.3
a_2	0.3	0.4	0.5
a_3	0.2	0.4	0.3

4(b)

γ_{1a}	v_1	v_2	v_3
a_1	0.3	0.3	0.4
a_2	0.4	0.3	0.3
a_3	0.4	0.3	0.3

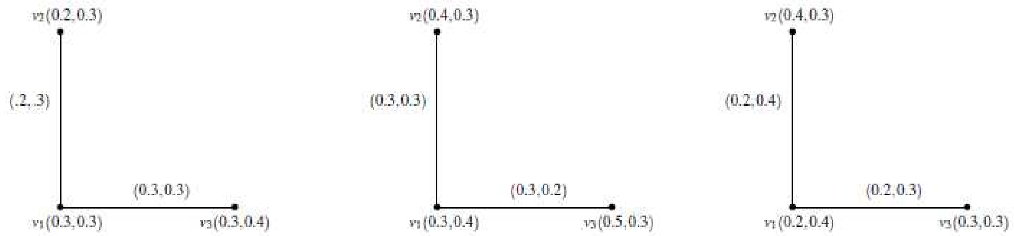
4(c)

μ_{2a}	(v_1, v_2)	(v_1, v_3)
a_1	0.2	0.3
a_2	0.3	0.3
a_3	0.2	0.2

4(d)

γ_{2a}	(v_1, v_2)	(v_1, v_3)
a_1	0.3	0.3
a_2	0.3	0.2
a_3	0.4	0.3

Table 4:



4(a) IFSG Corresponding to the parameter a_1

4(b) IFSG Corresponding to the parameter a_2

4(c) IFSG Corresponding to the parameter a_3

Figure 4:

5(a)

μ'_{1a}	v_3	v_4	v_5
a_2	0.1	0.2	0.1
a_3	0.1	0.2	0.2
a_4	0.1	0.1	0.2

5(b)

γ'_{1a}	v_3	v_4	v_5
a_2	0.1	0.2	0.1
a_3	0.2	0.1	0.1
a_4	0.1	0.1	0.1

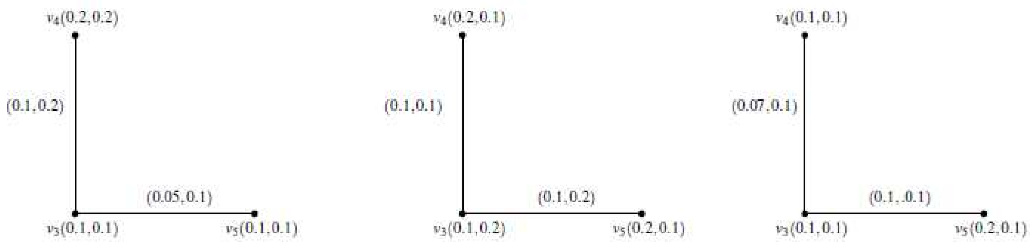
5(c)

μ'_{2a}	(v_3,v_4)	(v_3,v_5)
a_2	0.1	0.05
a_3	0.1	0.1
a_4	0.07	0.1

5(d)

γ'_{2a}	(v_3,v_4)	(v_3,v_5)
a_2	0.2	0.1
a_3	0.1	0.2
a_4	0.1	0.1

Table 5:



5(a) IFSG corresponding to the parameter a_2

5(b) IFSG corresponding to the parameter a_3

5(c) IFSG corresponding to the parameter a_4

Figure 5:

6(a)

μ''_{1a}	v_1	v_2	v_3	v_4	v_5
a_1	0.3	0.2	0.3	0	0
a_2	0.3	0.4	0.5	0.3	0.1
a_3	0.2	0.4	0.3	0.2	0.2
a_4	0	0	0.1	0.1	0.2

6(b)

γ''_{1a}	v_1	v_2	v_3	v_4	v_5
a_1	0.3	0.3	0.4	0	0
a_2	0.4	0.3	0.1	0.2	0.1
a_3	0.4	0.3	0.2	0.1	0.1
a_4	0	0	0.1	0.1	0.1

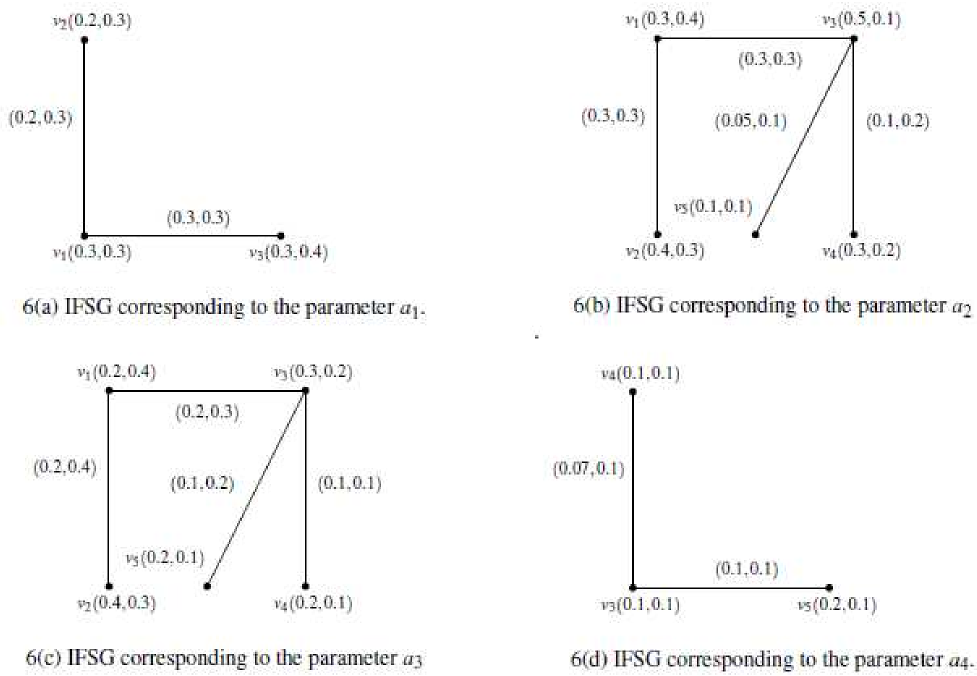
6(c)

μ''_{2a}	(v_1,v_2)	(v_1,v_3)	(v_3,v_4)	(v_3,v_5)
a_1	0.2	0.3	0	0
a_2	0.3	0.3	0.1	0.05
a_3	0.2	0.2	0.1	0.1
a_4	0	0	0.07	0.1

6(d)

γ''_{2a}	(v_1,v_2)	(v_1,v_3)	(v_3,v_4)	(v_3,v_5)
a_1	0.3	0.3	0	0
a_2	0.3	0.2	0.2	0.1
a_3	0.4	0.3	0.1	0.2
a_4	0	0	0.1	0.1

Table 6:



6(a) IFSG corresponding to the parameter a_1.

6(b) IFSG corresponding to the parameter a_2

6(c) IFSG corresponding to the parameter a_3

6(d) IFSG corresponding to the parameter a_4.

Figure 6:

Proposition 3.4. *Let* $G^*_{C,V_3,E_3} = (V_3, E_3, (C, \mu''_1), (C, \gamma''_1), (C, \mu''_2), (C, \gamma''_2))$ *be the union* *of two intuitionistic fuzzy soft graphs* $G^*_{A,V_1,E_1} = (V_1, E_1, (A,\mu_1), (A,\gamma_1),$ $(A,\mu_2), (A,\gamma_2))$ *and* $G^*_{B,V_2,E_2} = (V_2, E_2, (B,\mu_1^{'}), (B,\gamma_1^{'}), (B,\mu_2^{'}), (B,\gamma_2^{'}))$. *Then* $G^*_{C,V_3,E_3} = G^*_{A,V_1,E_1} \cup G^*_{B,V_2,E_2}$ *is an Intuitionistic fuzzy soft graph.*

Proof: Consider the case $a \in A \mid B.$

If $v_i \in V_1 \mid V_2$; by definition3.3 $\mu_{1a}^{''}(v_i) = \mu_{1a}(v_i)$ and $\gamma_{1a}^{''}(v_i) = \gamma_{1a}(v_i)$

$\Rightarrow 0 \le \mu_{1a}^{''}(v_i) + \gamma_{1a}^{''}(v_i) = \mu_{1a}(v_i) + \gamma_{1a}(v_i) \le 1$, since G^*_{A,V_1,E_1} is an IFSG.

If $v_i \in V_2 \mid V_1$; by definition3.3 $\mu_{1a}^{''}(v_i) = 0$ and $\gamma_{1a}^{''}(v_i) = 0$

Then clearly $0 \le \mu_{1a}^{''}(v_i) + \gamma_{1a}^{''}(v_i) \le 1$.

Also, if $v_i \in V_1 \cap V_2$; by definition3.3 $\mu_{1a}^{''}(v_i) = \mu_{1a}(v_i)$ and $\gamma_{1a}^{''}(v_i) = \gamma_{1a}(v_i)$

$\Rightarrow 0 \le \mu_{1a}^{''}(v_i) + \gamma_{1a}^{''}(v_i) = \mu_{1a}(v_i) + \gamma_{1a}(v_i) \le 1$, since G^*_{A,V_1,E_1} is an IFSG.

Similarly, when $a \in B \mid A$ in all cases $0 \le \mu_{1a}^{''}(v_i) + \gamma_{1a}^{''}(v_i) \le 1$

Consider the case when $a \in A \cap B$

If $v_i \in V_1 \mid V_2$ or $v_i \in V_2 \mid V_1$ clearly by definition3.3 $0 \le \mu_{1a}^{''}(v_i) + \gamma_{1a}^{''}(v_i) \le 1$

Now if $a \in A \cap B$ and $v_i \in V_1 \cap V_2$,

$\mu_{1a}^{''}(v_i) = \mu_{1a}(v_i) \vee \mu_{1a}^{'}(v_i)$ and $\gamma_{1a}^{''}(v_i) = \gamma_{1a}(v_i) \wedge \gamma_{1a}^{'}(v_i)$

Consider the cases $\mu_{1a}^{''}(v_i) = \mu_{1a}(v_i)$ *and* $\gamma_{1a}^{''}(v_i) = \gamma_{1a}(v_i)$,

$\mu_{1a}^{''}(v_i) = \mu_{1a}^{'}(v_i)$ *and* $\gamma_{1a}^{''}(v_i) = \gamma_{1a}^{'}(v_i)$

Since G^*_{A,V_1,E_1} and G^*_{B,V_2,E_2} are IFSG, clearly in these cases $0 \le \mu_{1a}^{''}(v_i) + \gamma_{1a}^{''}(v_i) \le 1$.

When $\mu_{1a}^{''}(v_i) = \mu_{1a}(v_i)$ and $\gamma_{1a}^{''}(v_i) = \gamma_{1a}^{'}(v_i)$

By the condition in the definition 3.3 $\gamma_{2a}(v_i, v_j) \ge \gamma_{1a}^{'}(v_i)$ *for every* $v_i, v_j \in V$

$\Rightarrow \gamma_{1a}^{'}(v_i) \le \gamma_{2a}(v_i, v_j)$ *for every* $v_i, v_j \in V$.

Also we have $\gamma_{2a}(v_i, v_j) \le max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ by definition 3.1.

$\Rightarrow \gamma_{2a}(v_i, v_j) \le \gamma_{1a}(v_i)$ *for every* $v_i \in V \Rightarrow \gamma_{1a}^{'}(v_i) \le \gamma_{1a}(v_i)$

$\therefore 0 \le \mu_{1a}^{''}(v_i) + \gamma_{1a}^{''}(v_i) = \mu_{1a}(v_i) + \gamma_{1a}^{'}(v_i) \le \mu_{1a}(v_i) + \gamma_{1a}(v_i) \le 1$

Similarly, when $\mu_{1a}^{''}(v_i) = \mu_{1a}^{'}(v_i)$ and $\gamma_{1a}^{''}(v_i) = \gamma_{1a}(v_i)$

By the condition in the definition 3.3 $\mu_{2a}(v_i, v_j) \ge \mu_{1a}^{'}(v_i)$ *for every* $v_i, v_j \in V$

$\Rightarrow \mu_{1a}^{'}(v_i) \le \mu_{2a}(v_i, v_j)$ *for every* $v_i, v_j \in V$.

Also we have $\mu_{2a}(v_i, v_j) \le min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ by definition 3.1.

$\Rightarrow \mu_{2a}(v_i, v_j) \le \mu_{1a}(v_i)$ *for every* $v_i, v_j \in V$

$\Rightarrow \mu_{1a}^{'}(v_i) \le \mu_{1a}(v_i)$

$\therefore 0 \le \mu''_{1a}(v_i)+\gamma''_{1a}(v_i) = \mu'_{1a}(v_i)+\gamma_{1a}(v_i) \le \mu_{1a}(v_i)+\gamma_{1a}(v_i) \le 1$

Now to check the condition 2 of definition 3.1.

By using the definition 3.3,in all cases clearly got the inequalities $2(i)$ *and* $2(ii)$ of definition 3.1.That is $\mu''_{2a}(v_i,v_j) \le min\{\mu''_{1a}(v_i), \mu''_{1a}(v_j)\}$ and $\gamma''_{2a}(v_i,v_j) \le max\{\gamma''_{1a}(v_i), \gamma''_{1a}(v_j)\}$ *for every* $v_i, v_j \in V$ *and* $a \in P$

Now to check the condition $2(iii)$ of definition 3.1

If $a \in A|B$ *and* $(v_i,v_j) \in (V_1 \times V_1)|(V_2 \times V_2)$ *or* $(v_i,v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$.

Then by definition 3.3

$\mu''_{2a}(v_i,v_j) = \mu_{2a}(v_i,v_j)$ *and* $\gamma''_{2a}(v_i,v_j) = \gamma_{2a}(v_i,v_j)$ *for every* $(v_i,v_j) \in E$.

$\Rightarrow 0 \le \mu''_{2a}(v_i,v_j)+\gamma''_{2a}(v_i,v_j) = \mu_{2a}(v_i,v_j)+\gamma_{2a}(v_i,v_j) \le 1)$ *for every* $(v_i,v_j) \in E$.

Also if $a \in A|B$ *and* $(v_i,v_j) \in (V_2 \times V_2)|(V_1 \times V_1)$.

Then by definition 3.3 $\mu''_{2a}(v_i,v_j) = 0$ *and* $\gamma''_{2a}(v_i,v_j) = 0$.

Obviously, $0 \le \mu''_{2a}(v_i,v_j)+\gamma''_{2a}(v_i,v_j) \le 1$, Since G^*_{A,V_1,E_1} is an IFSG. Similarly if $a \in B|A$ *and* $(v_i,v_j) \in (V_2 \times V_2)|(V_1 \times V_1)$ *or* $(v_i,v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$.

By definition 3.3

$\mu''_{2a}(v_i,v_j) = \mu'_{2a}(v_i,v_j)$ *and* $\gamma''_{2a}(v_i,v_j) = \gamma'_{2a}(v_i,v_j)$ *for every* $(v_i,v_j) \in E$.

$\Rightarrow 0 \le \mu''_{2a}(v_i,v_j)+\gamma''_{2a}(v_i,v_j) = \mu'_{2a}(v_i,v_j)+\gamma'_{2a}(v_i,v_j) \le 1$.

Also if $a \in B|A$ *and* $(v_i,v_j) \in (V_1 \times V_1)|(V_2 \times V_2)$. Then by definition 3.3 $\mu''_{2a}(v_i,v_j) = 0$ *and* $\gamma''_{2a}(v_i,v_j) = 0$. Obviously $0 \le \mu''_{2a}(v_i,v_j)+\gamma''_{2a}(v_i,v_j) \le 1$, Since G^*_{B,V_2,E_2} is an IFSG. Now consider the case when $a \in A \cap B$.

If $a \in A \cap B$ and $(v_i,v_j) \in (V_1 \times V_1)|(V_2 \times V_2)$, by definition3.3,

$0 \le \mu''_{2a}(v_i,v_j)+\gamma''_{2a}(v_i,v_j) = \mu_{2a}(v_i,v_j)+\gamma_{2a}(v_i,v_j) \le 1$.

Similarly if $a \in A \cap B$ and $(v_i,v_j) \in (V_2 \times V_2)|(V_1 \times V_1)$ by definition3.3,

$0 \le \mu''_{2a}(v_i,v_j)+\gamma''_{2a}(v_i,v_j) = \mu'_{2a}(v_i,v_j)+\gamma'_{2a}(v_i,v_j) \le 1$

Now consider the case when $a \in A \cap B$ and $(v_i,v_j) \in (V_1 \times V_1) \cap (V_2 \times V_2)$

By definition 3.3 $\mu''_{2a}(v_i,v_j) = \mu_{2a}(v_i,v_j)$ *or* $\mu'_{2a}(v_i,v_j)$ and $\gamma''_{2a}(v_i,v_j) = \gamma_{2a}(v_i,v_j)$ *or* $\gamma'_{2a}(v_i,v_j)$. The condition $2(iii)$ is clear when $\mu''_{2a}(v_i,v_j) = \mu_{2a}(v_i,v_j)$ and $\gamma''_{2a}(v_i,v_j) = \gamma_{2a}(v_i,v_j)$ and also when $\mu''_{2a}(v_i,v_j) = \mu'_{2a}(v_i,v_j)$ and $\gamma''_{2a}(v_i,v_j) = \gamma'_{2a}(v_i,v_j)$

Now consider the case $\mu''_{2a}(v_i,v_j) = \mu_{2a}(v_i,v_j)$ and $\gamma''_{2a}(v_i,v_j) = \gamma'_{2a}(v_i,v_j)$

By the condition in the definition 3.3 $\mu_{1a}(v_i,v_j) \ge max\{\mu'_{1a}(v_k)\}$ *for any*

$i, j, k = 1,2,\ldots,n$ and $\gamma_{1a}(v_i, v_j) \geq max\{\gamma'_{1a}(v_k)\}$ *for any* $i, j, k = 1,2\ldots,n$. Also $\mu_{2a}(v_i, v_j) \leq max\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ *for every* $v_i, v_j \in V$ by definition 3.1.

$$\begin{aligned}
\therefore 0 \leq \mu''_{2a}(v_i, v_j) + \gamma''_{2a}(v_i, v_j) &= \mu_{2a}(v_i, v_j) + \gamma'_{2a}(v_i, v_j) \text{ for everyy } v_i, v_j \in V \\
&\leq \mu_{2a}(v_i, v_j) + max\{\gamma'_{1a}(v_i), \gamma'_{1a}(v_j)\} \text{ for everyy } v_i, v_j \in V \\
&\leq \mu_{2a}(v_i, v_j) + \gamma'_{1a}(v_i) \text{ for everyy } v_i, v_j \in V \\
&\leq \mu_{2a}(v_i, v_j) + \gamma_{2a}(v_i, v_j) \text{ for everyy } v_i, v_j \in V \\
&\leq 1
\end{aligned}$$

Similarly when $\mu''_{2a}(v_i, v_j) = \mu'_{2a}(v_i, v_j)$ and $\gamma''_{2a}(v_i, v_j) = \gamma_{2a}(v_i, v_j)$

We know that $\mu'_{2a}(v_i, v_j) \leq max\{\mu'_{1a}(v_i), \mu'_{1a}(v_j)\}$ *for every* $v_i, v_j \in V$

$$\begin{aligned}
\therefore 0 \leq \mu''_{2a}(v_i, v_j) + \gamma''_{2a}(v_i, v_j) &= \mu'_{2a}(v_i, v_j) + \gamma_{2a}(v_i, v_j) \text{ for everyy } v_i, v_j \in V \\
&\leq max\{\mu'_{1a}(v_i), \mu'_{1a}(v_j)\} + \gamma_{2a}(v_i, v_j) \text{ for everyy } v_i, v_j \in V \\
&\leq \mu'_{1a}(v_i) + \gamma_{2a}(v_i, v_j) \text{ for everyy } v_i, v_j \in V \\
&\leq \mu_{2a}(v_i, v_j) + \gamma_{2a}(v_i, v_j) \text{ for everyy } v_i, v_j \in V \\
&\leq 1
\end{aligned}$$

Thus in all cases $G^*_{C,V_3,E_3} = G^*_{A,V_1,E_1} \cup G^*_{B,V_2,E_2}$ is an Intuitionistic fuzzy soft graph.

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