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Common Fixed Point of Semi-Weakly Compatible Maps on $\in$-Chainable Intutionistic Fuzzy Metric Space

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Abstract. In this paper, we introduce the concept of semi weakly compatibility of maps on a $\in$-chainable intuitionistic fuzzy metric space introduced by Manro et al. and established some results regarding common fixed point for such mappings in this newly defined space. Our results generalize the result of Kumar et al. Manro et al. and others.

Keywords: $\in$- chainable intuitionistic fuzzy metric space, weakly compatible maps, semi-weakly compatible maps.

AMS Mathematics Subject Classification (2010): 47H10, 54H25

1. Introduction

Zadeh [25] initiated the concept of fuzzy sets in 1965. Since then, due to the wide applicability of this notion in various fields, many authors have expansively developed the theory of fuzzy sets and its applications. In this context Deng [8], Erecg [9], Fang [10], Kaleva and Seikkala [15], Kramosil and Michalek [16] have introduced the concept of fuzzy metric spaces in different ways. In 1994 George and Veeramani [11] modified the definition of fuzzy metric space of [16].

Atanassov [5] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets and later many authors developed the theory of intuitionistic fuzzy sets. Park [19] introduced the notion of intuitionistic fuzzy metric spaces which is based on the idea of intuitionistic fuzzy set due to Atanassov [5] and the concept of a fuzzy metric space given by George and Veeramani [11] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space given by Kramosil and Michalek [16]. Alaca et al [3] proved the well known Banach fixed point theorem of Banach [6] in the setting of intuitionistic fuzzy metric space. Letter on Turkoglu et al [23], Saadati and Park [20] and many others studied the concept of intuitionistic fuzzy metric space and its applications.

In metric fixed point theory, after the classical result of Jungck [12] of common fixed point of two commuting maps. Sessa [21] initiated the weaker condition than that of commutativity namely weak commutativity of maps. A weaker condition of these notions namely, compatibility of maps has been introduced by Jungck [13]. Further Jungck and Rhoades [14] have introduced a weaker class among all commutative conditions namely weakly compatibility of maps and gave results regarding common fixed points in their respective papers.

In 2010 and 2011 Manro et al. [18] and Kumar et al. [17] introduced the notion of $\in$ - chainable intuitionistic fuzzy metric space akin to the notion of $\in$ - chainable fuzzy metric space introduced by Cho, Jung [7] and proved common fixed point theorems for four weakly compatible mappings in this newly defined structure.

In this paper, we have introduced a new concept namely semi-weakly compatibility of maps and extend the results of Kumar et al [17] for six such maps as oppose two four maps.

2. Preliminaries

Throughout this paper for the symbols and basic definitions, we refer [1, 3, 15, 16]. Here we describe some relevant definitions and results for further use.

Definition 2.1. [22] A binary operation $*: [0, 1]\times[0, 1] \to [0, 1]$ is called a continuous t-norm if $*$ satisfies the following conditions:
(1) $*$ is commutative and associative (2) $*$ is continuous (3) $a * 1 = a$ for all $a \in [0, 1]$
(4) $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$.
Examples: (i) $a * b = \min\{a, b\}$ and (ii) $a * b = ab$.

Definition 2.2. [22] A binary operation $\lozenge: [0, 1]\times[0, 1] \to [0, 1]$ is called a continuous t-conorm if $\lozenge$ satisfies the following conditions:
(1) $\lozenge$ is commutative and associative
(2) $\lozenge$ is continuous
(3) $a\lozenge 0 = a$, for all $a \in [0, 1]$
(4) $a \lozenge b \leq c \lozenge d$, whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$.

Examples: (i) $a \lozenge b = \max\{a, b\}$ and (ii) $a \lozenge b = \min\{1, a + b\}$.

Definition 2.3. [1,3] A 5-tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2\times [0,\infty)$ satisfying the following conditions:
(IMF 1) $M(x, y, t) + N(x, y, t) \leq 1$, for all $x, y \in X$ and $t > 0$,
(IMF 2) $M(x, y, 0) = 0$, for all $x, y \in X$,
(IMF 3) $M(x, y, t) = 1$, for all $x, y \in X$ and $t > 0$ if and only if $x = y$,
(IMF 4) $M(x, y, t) = M(y, x, t)$, for all $x, y \in X$ and $t > 0$,
(IMF 5) $M(x, y, t)*M(y, z, s) \leq M(x, z, t + s)$, for all $x, y, z \in X$ and $s, t > 0$,
(IMF 6) For all $x, y \in X$, $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left continuous,
(IMF 7) $\lim_{t\to\infty}M(x, y, t) = 1$, for all $x, y \in X$ and $t > 0$,
(IMF 8) $N(x, y, 0) = 1$, for all $x, y \in X$,
(IMF 9) $N(x, y, t) = 0$, for all $x, y \in X$ and $t > 0$ if and only if $x = y$,
(IMF 10) $N(x, y; t) = N(y, x, t)$, for all $x, y \in X$ and $t > 0$,
(IMF 11) $N(x, y, t)\lozenge N(y; z; s) \geq N(x, z, t + s)$, for all $x, y, z \in X$ and $s, t > 0$,

(IMF 12) For all $x, y \in X$, $N(x, y, \cdot) : [0, \infty) \to [0, 1]$ is right continuous,

(IMF 13) $\lim_{t\to\infty} N(x, y, t) = 0$, for all $x, y \in X$ and $t > 0$.

The pair (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 2.4. [2] In intuitionistic fuzzy metric space M(x, y, *) is non-decreasing and N(x, y, ◊) is non-increasing for all $x, y \in X$, whenever the t-norm and t-conorm are defined by $a * a \geq a$ and $(1\text{-}a) \lozenge (1\text{-}a) \leq (1\text{-}a)$, for all $a \in [0, 1]$.

Remark 2.5. [19] Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1\text{-}M, *, \lozenge)$ such that t-norm $*$ and t-conorm $\lozenge$ are associated,
i.e., $x \lozenge y = 1- ((1 - x) * (1 - y))$ for all $x, y \in X$.

Example 2.6. [19] Let (X, d) be a metric space. Define t-norm $a * b = \min\{a, b\}$ and t-conorm

$a \lozenge b = \max\{a, b\}$ and for all $x, y \in X$ and $t > 0$, M and N are defined by

$$M(x, y, t) = t/[\, t + d(x, y]) \text{ and } N(x, y, t) = d(x, y)/[\, t + d(x, y)].$$

Then $(X, M, N, *, \lozenge)$ is an intuitionistic fuzzy metric space induced by the metric d. It is obvious that $N(x, y, t) = 1 - M(x, y, t)$.

Definition 2.7. [1] A sequence $\{x_n\}$ in an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ is said to be.

(i) **convergent** to a point $x \in X$ if for all $t > 0$,

$$\lim_{n\to\infty} M(x_n,x,t) = 1 \text{and} \lim_{n\to\infty} N(x_n,x,t) = 0.$$

Since $*$ and $\lozenge$ are continuous, the limit is uniquely determined from (IMF5) and (IMF11) in Definition 2.3 respectively.

(ii) **Cauchy sequence** if for all $t > 0$, $p > 0$, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

(iii) The intuitionistic fuzzy metric space X is said to be **complete** if and only if every Cauchy sequence in X is convergent.

Lemma 2.8. [1] Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0, 1)$ such that:

$$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t) \quad \text{and} \quad N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$$

for all $t > 0$ and $n = 1, 2, 3, \dots$, then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2.9. Let $(X, M, N, *, \lozenge)$ be intuitionistic fuzzy metric space and for all x, y in X, $t > 0$ and if for a number k in (0, 1), $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ Then $x = y$.

Definition 2.10. Two self mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ are said to be

(i) **weakly commuting** if
$M(ABx, BAx, t) \geq M(Ax, Bx, t)$ and $N(ABx, BAx, t) \leq N(Ax, Bx, t)$, $\forall\, x \in X$, $t > 0$.
(ii) **compatible** if for all $t > 0$,
$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ and $\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$,
whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.**(c.f. 24)**
(iii) **weakly compatible** if for $x \in X$ and $t > 0$, $ABx = BAx$ implies that
$M(ABx, BAx, t) = 1$ and $N(ABx, BAx, t) = 0$,
(iv) **semi weakly compatible** if $M(ABz, BAz, t) = 1$ and $N(ABz, BAz, t) = 0$, where z is the fixed point of either A or B.

Proposition 2.11. For two self maps A and B on an intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$, the notion of commutativiy $\Rightarrow$ weakly commutativity $\Rightarrow$ compatibility $\Rightarrow$ weakly compatibility $\Rightarrow$ commutativity at common fixed points, but the converse is not true always.
Proof: If A and B are commuting maps, then $ABx = BAx$ for all x in X, then
$1 = M(ABx, BAx, t) \geq M(Ax, Bx, t)$ and $0 = N(ABx, BAx, t) \leq N(Ax, Bx, t)$ for all x in X and $t > 0$ implies that A and B are weakly commuting maps.

If A and B are weakly commuting maps and there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = y \in X$, then for all $t > 0$, we have
$M(ABx_n, BAx_n, t) \geq M(Ax_n, Bx_n, t) \to 1$ and $N(ABAx_n, BAx_n, t) \leq M(Ax_n, Bx_n, t) \to 0$ as $n \to \infty$ implies that A and B are compatible maps.

If A and B are compatible maps and take $x_n = x$ for all n, then $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = Ax(= Bx) \in X$. Therefore for all $t > 0$, we have
$M(ABx, BAx, t) = M(ABx_n, BAx_n, t) \to 1$and $N(ABx, BAx, t) = N(ABx_n, BAx_n, t) \to 0$ as $n \to \infty$ yields that A and B are weakly compatible maps.

If suppose that A and B are weakly compatible maps and x is the common fixed point of A and B then $x = Ax = Sx$ implies that $M(ABx, BAx, t) = 1$ and $N(ABx, BAx, t) = 0$ implies that $ABx = BAx$.

Proposition 2.12. Let A and B be compatible and continuous self-maps on an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$. If there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = y \in X,$$

where y is fixed point of either A or B. Then A and B are semi weakly compatible maps.
Proof: Suppose that y is a fixed point of A then $Ay = y$. By the continuity of A, $ABx_n \to Ay$ as $n \to \infty$. Now for $s, t > 0$

$$M(BAx_n, Ay, s + t) \geq M(BAx_n, ABx_n, s) * M(ABx_n, Ay, t) \to 1 \text{ and}$$
$$N(BAx_n, Ay, s + t) \leq N(BAx_n, ABx_n, s) * N(ABx_n, Ay, t) \to 1.$$

Letting $n \to \infty$ and using the compatibility of A and B, we have $BAx_n \to Ay$. By the continuity of B, $BSAx_n \to By$. Now by the uniqueness of the limit $Ay = By = y$ implies that $ABy = Ay = y = By = BAy$. Hence A and B are semi weakly compatible maps.

Remark 2.13. From the propositions (2.11) and (2.12), it is clear that for two self maps A and B on an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$. (i) Commutativiy $\Rightarrow$ semi weakly compatibility of maps. (ii) Compatibility $\Rightarrow$ semi weakly compatibility of maps, if both the maps are continuous and

$$y = \lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n \in X$$

is a fixed point of either A or B. But the converse of (i) an (ii) are not true always as we can see in the following examples.

Example 2.14. Let (X, M, N, $*$, $\Diamond$) be an intuitionistic fuzzy metric space, where X = R, $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ $\forall$ x, y $\in$ X, t > 0. Define self maps A and B on X by $Ax = (x + 1)/3$, $Bx = 3x^2$. Then we have

$M(ABx, BAx, t) = \frac{t}{t+|\frac{3x^2+1}{3} - \frac{(x+1)^2}{3}|} \neq 1$and $N(ABx, BAx, t) = \frac{|\frac{3x^2+1}{3} - \frac{(x+1)^2}{3}|}{t+|\frac{3x^2+1}{3} - \frac{(x+1)^2}{3}|} \neq 0$ $\forall$ x $\in$ X, t > 0, implies that A and B are non-commuting maps. On the other hand at the fixed point 0 of B, we have

$M(AB0, BA0, t) = \frac{t}{t+|\frac{1}{3} - \frac{1}{3}|} = 1$and $N(AB0, BA0, t) = \frac{|\frac{1}{3} - \frac{1}{3}|}{t+|\frac{1}{3} - \frac{1}{3}|} = 0$, implies that the maps A and B are semi weakly compatible.

Example 2.15. Let (X,M,N,$*$,$\Diamond$) be an intuitionistic fuzzy metric space, where X = [0, 2], $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ $\forall$x, y $\in$ X, t > 0. Define self maps A and B on X by

$$Ax = \begin{cases} 2, \ if\ x = 2 \\ \frac{x+1}{2}, \ otherwise \end{cases}, \quad Bx = \begin{cases} 2, \ if\ x = 1 \\ \frac{2x+2}{3}, \ otherwise \end{cases}.$$

Then it is easy to verify that A and B are non-commuting, non-continuous and non-compatible maps. On the other hand at the fixed point 1 of A, we have

$$M(AB1, BA1, t) = \frac{t}{t+|2-2|} = 1 \quad \text{and} \quad N(AB1, BA1, t) = \frac{|2-2|}{t+|2-2|} = 0$$

implies that the maps A and B are semi weakly compatible.

Definition 2.16. [18] Let (X, M, N,*, $\Diamond$) be intuitionistic fuzzy metric space. A finite sequence $x = x_0, x_1, x_2, \ldots, x_n = y$ is called $\in$ - chain from x to y if there exists a positive number $\in > 0$ such that $M(x_i, x_{i-1}, t) > 1-\in$ and $N(x_i, x_{i-1}, t) < 1-\in$ for all t > 0 and i = 1, 2, . . . , n.

An intuitionistic fuzzy metric space (X, M, N,*, $\Diamond$) is called $\in$ - chainable if for any x, y in X, there exists an $\in$ - chain from x to y.

3. Results

In a paper, anro et al [18] and Kumar et al [17] introduced the concept of $\in$ - chainable intuitionistic fuzzy metric space and proved some results regarding common fixed point theorems for four weakly compatible mappings on $\in$- chainable intuitionistic fuzzy metric space.

Here we extend the results of Kumar et al [17] by introducing the concept semi-weakly compatibility of maps and prove the following common fixed point theorems for six such maps as opposed to four mappings of $\in$- chainable intuitionistic fuzzy metric space.

Theorem 3.1. Let A, B, S, T, P and Q be self maps of a complete $\in$ - chainable intuitionistic fuzzy metric spaces (X, M, N, *, ◊) with continuous t-norm * and continuous t-conorm ◊ defined by $a * a \geq a$ and $(1-a)◊(1-a) \leq (1-a)$ for all $a \in [0, 1]$ satisfying the following conditions:

(3.1.1) $A(X) \subseteq QT(X)$ and $B(X) \subseteq PS(X)$,

(3.1.2) and PS are continuous,

(3.1.3) the pairs (A, PS) and (B, QT) are weakly compatible,

(3.1.4) the pairs (P, S) and (Q, T) are commuting maps,

(3.1.5) the pairs (P, A), (S, A), (Q, B) and (T, B) are semi-weakly compatible mappings.

(3.1.6) there exist $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq M(PSx, QTy, t)*M(Ax, PSx, t)*M(By, QTy, t)*M(Ax, QTy, t)*M(By, PSx, 2t)$$

and

$$N(Ax, By, qt) \leq N(PSx, QTy, t)◊N(Ax, PSx, t)◊N(By, QTy, t)◊N(Ax, QTy, t)◊N(By, PSx, 2t),$$

for all $x, y \in X$ and $t > 0$.Then A, B, S and T have a unique common fixed point in X.

Proof: Since $A(X) \subseteq QT(X)$, therefore for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = QTx_1$ and for the point x_1, we can choose a point $x_2 \in X$ such that $Bx_1 = PSx_2$ as $B(X) \subseteq PS(X)$. Inductively, we can find a sequence $\{y_n\}$ in X as follows $y_{2n-1} = QTx_{2n-1} = Ax_{2n-2}$ and $y_{2n} = PSx_{2n} = Bx_{2n-1}$ for n = 1, 2, ...

Now using (3.1.6), we have

$$\begin{aligned} M(y_{2n}, y_{2n+1}, qt) &= M(Ax_{2n-1}, Bx_{2n}, qt) \\ &\geq M(PS_{2n-1}, QT_{2n}, t)*M(Ax_{2n-1}, PSx_{2n-1}, t)*M(Bx_{2n}, QTx_{2n}, t) \\ &\quad *M(Ax_{2n-1}, QTx_{2n}, t)*M(Bx_{2n}, PSx_{2n-1}, 2t) \end{aligned}$$

and

$$\begin{aligned} N(y_{2n}, y_{2n+1}, qt) &= N(Ax_{2n-1}, Bx_{2n}, qt) \\ &\leq N(PSx_{2n-1}, QTx_{2n}, t)◊N(Ax_{2n-1}, PSx_{2n-1}, t)◊N(Bx_{2n}, QTx_{2n}, t) \\ &\quad ◊N(Ax_{2n-1}, QTx_{2n}, t)◊N(Bx_{2n}, PSx_{2n-1}, 2t) \end{aligned}$$

$$\Rightarrow \quad M(y_{2n}, y_{2n+1}, qt) \geq M(y_{2n-1}, y_{2n}, t)*M(y_{2n}, y_{2n-1}, t)*M(y_{2n+1}, y_{2n}, t)*M(y_{2n}, y_{2n-1}, t) *M(y_{2n+1}, y_{2n-1}, 2t)$$

and

$$N(y_{2n}, y_{2n+1}, qt) \leq N(y_{2n-1}, y_{2n}, t)◊N(y_{2n}, y_{2n-1}, t)◊N(y_{2n+1}, y_{2n}, t)◊N(y_{2n}, y_{2n-1}, t) ◊N(y_{2n+1}, y_{2n-1}, 2t)$$

$$\Rightarrow \quad M(y_{2n}, y_{2n+1}, qt) \geq M(y_{2n-1}, y_{2n}, t) \quad \text{and} \quad N(y_{2n}, y_{2n+1}, qt) \leq N(y_{2n-1}, y_{2n}, t).$$

Similarly, we can obtain

$$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t) \quad \text{and} \quad N(y_{2n+1}, y_{2n+2}, qt) \leq N(y_{2n}, y_{2n+1}, t).$$

In general, for all n even or odd, we have

$$M(y_n, y_{n+1}, qt) \geq M(y_{n-1}, y_n, t) \text{ and } N(y_n, y_{n+1}, qt) \leq N(y_{n-1}, y_n, t).$$

Hence, we can conclude that $\{y_n\}$ is Cauchy sequence in X. Now, by completeness of X the sequence $\{y_n\}$ and its subsequences $\{QTx_{2n-1}\}$, $\{Ax_{2n-2}\}$, $\{PSx_{2n}\}$ and $\{Bx_{2n-1}\}$ also converges to some z in X. Since X is $\in$- chainable, there exists $\in$ - chain from x_n to x_{n+1}, that is, there exists a finite sequence $x_n = y_1, y_2, \ldots, y_l = x_{n+1}$ such that $M(y_i, y_{i-1}, t) > 1 - \in$ and $N(y_i, y_{i-1}, t) < 1 - \in$ for all $t > 0$ and $i = 1, 2, \ldots, l$. Thus we have

$M(x_n, x_{n+1}, t) \geq M(y_1,y_2, t/l)*M(y_2,y_3, t/l)*...*M(y_{l-1}, y_l, t/l) > (1-\in)*(1-\in)*...*(1-\in) \geq (1-\in)$ and

$N(x_n, x_{n+1}, t) \leq N(y_1,y_2, t/l)\Diamond N(y_2,y_3, t/l)\Diamond ... \Diamond N(y_{l-1},y_l, t/l) < (1-\in)\Diamond(1-\in)\Diamond...\Diamond(1-\in) \leq (1-\in)$.

Now for m > n,

$$M(x_n, x_m, t) \geq M(x_n,x_{n+1}, t/m\text{-}n) * M(x_{n+1},x_{n+2}, t/m\text{-}n) * ... * M(x_{m-1},x_m, t/m\text{-}n) > (1 - \in) * (1 - \in) * ... * (1 - \in) \geq (1 - \in)$$ and

$$N(x_n, x_m, t) \leq N(x_n,x_{n+1}, t/m\text{-}n) \Diamond N(x_{n+1},x_{n+2}, t/m\text{-}n) \Diamond ... \Diamond N(x_{m-1}, x_m, t/m\text{-}n) < (1 - \in) \Diamond (1 - \in) \Diamond ... \Diamond (1 - \in) \leq (1 - \in).$$

Therefore $\{x_n\}$ is a Cauchy sequence in X and hence there exists x in X such that $x_n \to x$.

Now from (3.1.2), $Ax_{2n-2} \to Ax$, $PSx_{2n} \to PSx$ as limit $n \to \infty$. By uniqueness of limits, we have Ax = z = PSx. Since pair (A, PS) is weakly compatible, therefore A(PS)x = (PS)Ax and so Az = PSz.

Again from (3.1.2), we have $A(PS)x_{2n} \to A(PS)x$ and therefore $A(PS)x_{2n} \to PSz$. Also, from continuity of PS, we have $(PS)(PS)x_{2n} \to (PS)z$.

Using (3.1.6), we get

$$M(APSx_{2n}, Bx_{2n-1}, qt) \geq M(PSPSx_{2n}, QTx_{2n-1}, t)*M(APSx_{2n}, PSPSx_{2n}, t)*M(Bx_{2n-1}, QTx_{2n-1}, t) *M(APSx_{2n}, QTx_{2n-1}, t)*M(Bx_{2n-1}, PSx_{2n}, 2t)$$

and

$$N(APSx_{2n}, Bx_{2n-1}, qt) \leq N(PSPSx_{2n}, QTx_{2n-1}, t) \Diamond N(APSx_{2n}, PSPSx_{2n}, t) \Diamond N(Bx_{2n-1}, QTx_{2n-1},t) \Diamond N(APSx_{2n}, QTx_{2n-1}, t)\Diamond N(Bx_{2n-1}, PSx_{2n}, 2t).$$

Proceeding limit as $n \to \infty$, we have

$$M(PSz, z, qt) \geq M(PSz, z, t) * M(PSz, PSz, t) * M(z, z, t) * M(PSz, z, t) * M(PSz, z, 2t)$$

and

$$N(PSz, z, qt) \leq N(PSz, z, t) \Diamond N(PSz, PSz, t) \Diamond N(z, z, t) \Diamond N(PSz, z, t) \Diamond N(PSz, z, 2t).$$

From Lemma 2.9, we get PSz = z and hence Az = PSz = z.

Since $A(X) \subseteq QT(X)$, there exists v in X such that QTv = Az = z.

From (3.1.6), we have

$$M(Ax_{2n}, Bv, qt) \geq M(PSx_{2n}, QTv, t) * M(Ax_{2n}, PSx_{2n}, t) * M(Bv, QTv, t) *M(Ax_{2n}, QTv,t)* M(Bv, PSx_{2n}, 2t)$$

and

$$N(Ax_{2n}, Bv, qt) \leq N(PSx_{2n}, QTv, t) \Diamond N(Ax_{2n}, PSx_{2n}, t) \Diamond N(Bv, QTv, t) \Diamond N(Ax_{2n}, QTv, t) \Diamond M(Bv, PSx_{2n}, 2t).$$

Letting $n \to \infty$, we have

$$M(z, Bv, qt) \geq M(z, QTv, t) * M(z, z, t) * M(Bv, QTv, t) * M(z, QTv, t) *M(Bv, z, 2t) = M(z, z, t) * M(z, z, t) * M(Bv, z, t) * M(z, z, t) * M(Bv, z, 2t) \geq M(Bv, z, t)$$ and

$$N(z, Bv, qt) \leq N(z, QTv, t) \Diamond N(z, z, t) \Diamond N(Bv, QTv, t) \Diamond N(z, QTv, t) \Diamond N(Bv, z, 2t) = N(z, z, t) \Diamond N(z, z, t) \Diamond N(Bv, z, t) \Diamond N(z, z, t) \Diamond N(Bv, z, 2t) \leq N(Bv, z, t).$$

By Lemma 2.9, we have Bv = z and therefore, we have QTv = Bv = z. Since (B, QT) is weakly compatible, therefore, (QT)Bv = B(QT)v and hence QTz = Bz.

Again from (3.1.6), we have

$$M(Ax_{2n}, Bz, qt) \geq M(PSx_{2n}, QTz, t) * M(Ax_{2n}, PSx_{2n}, t) * M(Bz, QTz, t)$$

$$*M(Ax_{2n}, QTz, t) * M(Bz, z, 2t)$$

and

$$N(Ax_{2n}, Bz, qt) \le N(PSx_{2n}, QTz, t) \lozenge N(Ax_{2n}, PSx_{2n}, t) \lozenge N(Bz, QTz, t) \lozenge N(Ax_{2n}, QTz, t) \lozenge N(Bz, z, 2t).$$

Letting $n \to \infty$, we have

$$M(z, Bz, qt) \ge M(z, QTz, t) * M(z, z, t) * M(Bz, QTz, t) * M(z, QTz, t) * M(Bz, z, 2t)$$
$$= M(z, Bz, t) * M(z, z, t) * M(Bz, Bz, t) * M(z, Bz, t) * M(Bz, z, 2t) \ge M(z, Bz, t)$$

and

$$N(z, Bv, qt) \le N(z, QTz, t) \lozenge N(z, z, t) \lozenge N(Bz, QTz, t) \lozenge N(z, QTz, t) \lozenge N(Bz, z, 2t)$$
$$= N(z, Bz, t) \lozenge N(z, z, t) \lozenge N(Bz, Bz, t) \lozenge N(z, Bz, t) \lozenge N(Bz, z, 2t) \le N(z, Bz, t),$$

which implies that $Bz = z$. Therefore, $Az = PSz = Bz = QTz = z$. Hence A, B, PS and QT have common fixed point z in X.

For the uniqueness of z, let w be another common fixed point of A, B, PS and QT. Then from (3.1.6), we have

$$M(z, w, qt) = M(Az, Bw, qt) \ge M(PSz, QTw, t) * M(Az, PSz, t) * M(Bw, QTw, t) * M(Az, QTw, t) * M(Bw, PSz, 2t) \ge M(z, w, t)$$

and

$$N(z, w, qt) = N(Az, Bw, qt) \ge N(PSz, QTw, t) \lozenge N(Az, PSz, t) \lozenge N(Bw, QTw, t) \lozenge N(Az, QTw, t) \lozenge N(Bw, PSz, 2t) \le N(z, w, t).$$

By lemma 2.9, $z = w$. Hence A, B, PS and QT have unique common fixed point z in X. From (3.1.4 & 3.1.5)), we have $Pz = P(PSz) = P(SPz) = (PS)Pz$; $Pz = PAz = APz$ and $Sz = S(PSz) = (SP)Sz = (PS)Sz$; $Sz = SAz = ASz$, implies that Pz and Sz are common fixed points of (PS, A) therefore $z = Pz = Sz = Az = PSz$. Similarly, Qz and Tz are common fixed points of (QT, B) therefore $z = Qz = Tz = Bz = QTz$. Hence z is the common fixed point of A, B, S, T, P and Q. Further, since z is the unique common fixed point of A, B, PS and QT consequently it is the unique common fixed point of A, B, S, T, P and Q.

If we take $P = Q = I$ (the identity map) in theorem 3.1, we have the result of Kumar et al [17] as a corollary and the following corollary.

Corollary 3.2. Let $(X, M, N, *, \lozenge)$ be complete $\in$ - chainable intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying

(3.2.1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$,

(3.2.2) A and S are continuous,

(3.2.3) the pairs (A, S) and (B, T) are weakly compatible,

(3.2.4) there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \ge M(Sx, Ty, t) \text{ and } N(Ax, By, qt) \le N(Sx, Ty, t)\ \forall\, x, y \in X \text{ and } t > 0.$$

Then A, B, S and T have a unique common fixed point in X.

If we take $P = Q = S = T = I$ (the identity map) in Corollary 3.2, we have the following corollary:

Corollary 3.3. Let $(X, M, N, *, \lozenge)$ be complete $\in$ - chainable intuitionistic fuzzy metric space and let A and B be self mappings of X satisfying the following condition:

(3.3.1) there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \ge M(x, y, t) \text{ and } N(Ax, By, qt) \le N(x, y, t)\ \forall\, x, y \in X \text{ and } t > 0.$$

Then A and B have a unique common fixed point in X provided if the pair (A, B) is weakly compatible maps.

By taking B = P = Q = S = T = I (the identity map) in Corollary 3.3, we have the following intuitionistic fuzzy version of Banach contraction theorem:

Corollary 3.4. Let (X, M, N, *, ◊) be complete $\in$ - chainable intuitionistic fuzzy metric space and let A be self mappings of X satisfying the following condition:

(3.4.1) there exists $q \in (0, 1)$ such that

$M(Ax, By, qt) \geq M(x, y, t)$ and $N(Ax, By, qt) \leq N(x, y, t)$ $\forall$ $x, y \in$ X and $t > 0$.

Then A has a unique common fixed point in X.

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