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Common Fixed Point Theorems for Weakly Compatible Mappings Using Common Property (*E.A*) in Modified Intuitionistic Fuzzy Metric Space

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Abstract. The aim of present paper is to prove common fixed point theorems for four self mappings in modified intuitionistic fuzzy metric spaces using the common property (E.A.) satisfying an implicit relation.

Keywords: Modified intuitionistic fuzzy metric space, the property (E. A.), the common property (E. A.)

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1.Introduction

Zadeh [22] investigated the concept of a fuzzy set in his seminal paper. In the last two decades there has been a tremendous development and growth in fuzzy mathematics in 1965.Many authors utilize this concept in topology and analysis extensively developed the theory of fuzzy sets along with their applications (e. g., [3,6–9],12,13,16,19). In 1986, Atanassov [2] introduced and studied the concept of intuitionistic fuzzy sets. Using the idea of intuitionistic fuzzy set, a generalization of fuzzy metric space was introduced by park [15] which is now known as modified intuitionistic fuzzy metric space where in notions of continuous t–norm and continuous t–conorm are employed. Since the topology induce by intuitionistic fuzzy metric coincides with the topology induced by fuzzy metric [6], Saadati et al. [18] reframed the idea of intuitionistic fuzzy metric spaces and proposed a new notion under the name of modified intuitionistic fuzzy metric spaces by introducing the idea of continuous t-representable. Amari and Moutawakli [1] and Liu et al. [14] respectively, define the property (E.A.) and common property (E.A.) and utilize the same to prove common fixed point theorems in metric spaces. Imdad et al. [10] proved common fixed point via L^*common property (E.A.), we utilize the notion of the property (E.A.) and common property (E.A.) to prove some common fixed point theorems in modified intuitionistic fuzzy metric spaces.

2. Basic definitions and preliminaries

Lemma 2.1. [4] Consider the set L^* and operation $\leq_{L^*}$ defined by

$$L^* = \{(x_1, x_2): (x_1, x_2) \in [0,1]^2 \ and \ x_1 + x_2 \leq 1\}$$

$(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1 \ and \ x_2 \geq y_2 \ , for \ every \ (x_1, x_2), (y_1, y_2) \in L^*$.
Then $(L^*, \leq_{L^*})$ is a complete lattice.

Definition 2.1. [2] An intuitionistic fuzzy set $\mathcal{A}_{\zeta,\eta}$ in a universe U is an object $\mathcal{A}_{\zeta,\eta} = \{(\zeta_{\mathcal{A}}(u), \eta_{\mathcal{A}}(u): u \in U)\}$, where, for all $u \in U$, $\zeta_{\mathcal{A}}(u) \in [0,1] \ and \ \eta_{\mathcal{A}}(u) \in [0,1]$ are called the membership degree and the non – membership degree respectively, of $u \in \mathcal{A}_{\zeta,\eta}$ and further more they satisfy $\zeta_{\mathcal{A}}(u) + \eta_{\mathcal{A}}(u) \leq 1$.
for every $z_i = (x_i, y_i) \in L^* , if \ c_i \in [0,1]$ such that $\sum_{i=1}^{n} c_i = 1$ then it is easy to see that

$$c_1(x_1, y_1) + \ \dots\dots + c_n(x_n, y_n) = \sum_{i=1}^{n} c_i(x_i, y_i) = \left(\sum_{i=1}^{n} c_i x_i \, , \sum_{i=1}^{n} c_i y_i\right) \in L^*$$

We denote its unit by 0_{L^*}= (0, 1) and 1_{L^*}= (1, 0). Classically, a triangular norm * = T on [0, 1] is defined as an increasing, commutative, associative mapping $T: [0,1]^2 \to [0,1]$ satisfying $T(1, x) = 1 * x = x, for \ all \ x \in [0,1]$. A triangular co - norm $S = \lozenge$ is defined as an increasing, commutative, associative mapping $S: [0,1]^2 \to [0,1]$ satisfying $S(0, x) = 0 * x = x, for \ all \ x \in [0,1]$using the lattice $(L^*, \leq_{L^*})$ these definitions can straightforwardly be extended.

Definition 2.2. [5] A triangular norm (t - norm) on L^* is a mapping $T: (L^*)^2 \to L^*$ satisfying the following conditions:

(I) $(\forall x \in L^*)(T(x, 1_{L^*}) = x)$(boundary condition),
(II) $(\forall (x, y) \in (L^*)^2)(T(x, y) = T(y, x))$(commutativity),
(III) $(\forall (x, y, z) \in (L^*)^3)(T\big(x, T(y, Z)\big) = T(T(x, y), z))$(associativity),
(IV) $\big(\forall \big(x, x', y, y'\big) \in (L^*)^4\big)(x \leq_{L^*} x')$ and $\Big(y \leq_{L^*} y' \to T(x, y) \leq_{L^*} T\big(x', y'\big)\Big)$ (monotonicity).

Definition 2.3. [4, 5] A continuous t – norm on L^* is called continuous t–representable if and only if there exist a continuous t – norm * and a continuous t–conorm ◊ on [0, 1] such that, for all $x = (x_1, x_2), y = (y_1, y_2) \in L^*$,

$$T(x, y) = (x_1 * y_1, x_2 \lozenge y_2).$$

Now, we define a sequence $\{T^n\}$ recursively by $\{T^1 = T\}$ and
$T^n\big(x^{(1)}, \dots\dots, x^{(n+1)}\big) = T(T^{n-1}\big(x^{(1)}, \dots\dots, x^{(n)}\big), x^{(n+1)})$ for $n \geq 2 \ and \ x^i \in L^*$.

Definition 2.4. [4, 5] A negator on L^* is any decreasing mapping $N: L^* \to L^*$ satisfying $N(0_{L^*}) = 1_{L^*}$ and $\mathrm{N}(1_{L^*}) = 0_{L^*}$. If $N\big(N(x)\big) = x, \ for \ all \ x \in L^*$ then N is called an involutive negator. A negator on [0, 1] is a decreasing mapping $N : [0,1] \to [0,1]$

satisfying $N(0) = 1$ *and* $N(1) = 0$. N_s denotes the standard negator on [0, 1] defined as (for all $x \in [0,1]$) $N_S(x) = 1 - x$.

Definition 2.5. [18] Let M, N are fuzzy sets from X^2 x$(0,\infty)$ to [0, 1] such that $M(x,y,t) + N(x,y,t) \leq 1 for\ all\ x,y \in X\ and\ t > 0$. The 3–tuple $(X, M_{M,N}, T)$ is said to be a modified intuitionistic fuzzy metric space if X is an arbitrary (non – empty) set, T is continuous representable and $M_{M,N}$ is a mapping X^2 x$(0,\infty) \to L^*$(an intuitionistic fuzzy set) satisfying the following conditions for every $x, y \in X\ and\ t, s > 0$;

(I) $M_{M,N}(x,y,t) >_{L^*} 0_{L^*}$,

(II) $M_{M,N}(x,y,t) = 1_{L^*}, if\ and\ only\ if\ x = y$,

(III) $M_{M,N}(x,y,t) = M_{M,N}(y,x,t)$

(IV) $M_{M,N}(x,y,t+s) >_{L^*} T\left(M_{M,N}(x,z,t), M_{M,N}(z,y,s)\right)$,

(V) $M_{M,N}(x,y,.) : (0,\infty) \to L^*$is continuous.

In this case $M_{M,N}$is called a modified intuitionistic fuzzy metric. Hence,

$$M_{M,N}(x,y,t) = \big(M(x,y,t), N(x,y,t)\big).$$

Remark 2.1. [21] In a modified intuitionistic fuzzy metric space $(X, M_{M,N}, T), M(x,y,.)$ is non-decreasing and $N(x,y,.)$ is non–increasing function for $all\ x,y \in X$. Hence $(X, M_{M,N}, T)$ is non- decreasing function for all $x, y \in X$.

Example 2.1. [8] Let (X, d) be a metric space. Denote $T(a,b) = (a_1b_1, \min\{a_2b_2, 1\})$ $for\ all\ a = (a_1,a_2)\ and\ b = (b_1,b_2) \in L^*$ and let M and N be fuzzy sets on $X^2 \times (0,\infty)$ defined as follows:

$$M_{M,N}(x,y,t) = \big(M(x,y,t), N(x,y,t)\big) = \left(\frac{ht^n}{ht^n + md(x,y)}, \frac{md(x,y)}{ht^n + md(x,y)}\right)$$

for all $h, m, n, t \in R^+$.Then $(X, M_{M,N}, T)$ is a modified intuitionistic fuzzy metric space.

Definition 2.6. [8] A sequence $\{x_n\}$ in a modified intuitionistic fuzzy space $(X, M_{M,N}, T)$ is called Cauchy sequence if for each $0 < \varepsilon < 1\ and\ t > 0$, there exist $n_0 \in \mathbb{N}$ such that

$$M_{M,N}(x_n, y_m, t) >_{L^*} (N_s(\mathcal{E}), \mathcal{E})$$

and for each $m, n \geq n_0$ here N_s is standard negator. The sequence $\{x_n\}$ is said to be convergent to $x \in X$ in the modified intuitionistic fuzzy metric space $(X, M_{M,N}, T)$ and denoted by $x_n \to M_{M,N}x$ if $M_{M,N}(x_n, x, t) \to 1_{L^*}$ whenever $n \to \infty$ for every $t > 0$. A modified intuitionistic fuzzy metric space is said to be complete if and only if every Cauchy sequence is convergent.

Lemma 2.2. [17] Let $M_{M,N}$ be a modified intuitionistic fuzzy metric. Then for any $t > 0$, $M_{M,N}(x,y,t)$ is non – decreasing with respect to t, in $(L^*, \leq_{L^*})$, for all $x, y \in X$.

Definition 2.7. [18] Let $(X, M_{M,N}, T)$ be a modified intuitionistic fuzzy metric space. M is said to be continuous on $X \times X \times (0,\infty)$ *if* $\lim_{n\to\infty} M_{M,N}(x_n, y_n, t_n) = M_{M,N}(x, y, t)$, whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X \times X \times (0,\infty)$ converges to a point $(x, y, t) \in X \times X \times (0,\infty)$, *i.e.*

$$\lim_{n\to\infty} M_{M,N}(x_n, x, t) = \lim_{n\to\infty} M_{M,N}(y_n, y, t) = 1_{L^*}$$

and

$$\lim_{n\to\infty} M_{M,N}(x, y, t_n) = M_{M,N}(x, y, t).$$

Definition 2.8. [18] Let f and g be mappings from a modified intuitionistic fuzzy metric space $(X, M_{M,N}, T)$ into itself. Then the pair of these mappings is said to be weakly compatible if they commute at their coincidence point, that is

$$fx = gx \text{ implies that } fgx = gfx.$$

Definition 2.9. [18] Let f and g be mappings from a modified intuitionistic fuzzy metric space $(X, M_{M,N}, T)$ into itself. Then the mappings are said to be compatible if

$$\lim_{n\to\infty} M_{M,N}(fgx_n, gfx_n, t) = 1_{L^*}, \qquad \forall\, t > 0$$

whenever $\{x_n\}$ is a sequence such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x \in X.$$

Definition 2.12. [20] Let f and g be mappings of a modified intuitionistic fuzzy metric space $(X, M_{M,N}, T)$. We say that f and g satisfy the property (E. A.) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} M_{M,N}(fx_n,\, u, t) = \lim_{n\to\infty} M_{M,N}(gx_n,\, u, t) = 1_{L^*}$$

for some $u \in X$ *and* $t > 0$.

Example 2.2.[18] Let $(X, M_{M,N}, T)$ be a modified intuitionistic fuzzy metric space, where X =R and $M_{M,N}(x, y, t) = \left(\frac{1}{t+|x-y|}, \frac{|x-y|}{t+|x-y|}\right)$ for every $x, y \in X$ *and* $t > 0$. Define self maps f and g on X as follows:

$$fx = 2x + 1, \quad gx = x + 2.$$

Consider the sequence $\left\{x_n = 1 + \frac{1}{n}, n = 1, 2, \dots\right\}$ thus we have

$$\lim_{n\to\infty} M_{M,N}(fx_n,\, 3, t) = \lim_{n\to\infty} M_{M,N}(gx_n,\, 3, t) = 1_{L^*}$$

for every $t > 0$. Then f and g satisfy the property (E. A.).

Definition 2.12. [20] Two pairs (f, S) and (g, T) of self- mappings of a modified intuitionistic fuzzy metric space $(X, M_{M,N}, T)$ are said to satisfy the common property (E. A.) if there exists two sequences $\{x_n\}$ *and* $\{y_n\}$ in X such that

$$\lim_{n\to\infty} M_{M,N}(fx_n,\ u,t) = \lim_{n\to\infty} M_{M,N}(Sx_n,\ u,t) = \lim_{n\to\infty} M_{M,N}(gy_n,\ u,t)$$
$$= \lim_{n\to\infty} M_{M,N}(Ty_n,\ u,t) = 1_{L^*}$$
for some $u \in X\ and\ t > 0$.

Remark 2.3. Note that the above examples holds even with the t- norm a * b = min{a, b} and t- conorm a ◊ b= max{a, b}and hence (M, N) is a modified intuitionistic fuzzy metric with respect to any continuous t – norm and continuous t – conorm.

Implicit relation

Let M_5 denotes the set of all real valued continuous function $F(t_1, t_2, \dots, t_5): {L^*}^5 \to L^*$, which are non decreasing and satisfying the following conditions: (for all $u, 1 \in L^*$,
$$u = (u_1, u_2)\ and\ 1 = 1_{L^*} = (1, 0))$$
(A) $F(u, 1, u, 1, u) \geq_{L^*} 0_{L^*}\ implies\ u \geq_{L^*} 1$
(B) $F(u, 1, 1, u, u) \geq_{L^*} 0_{L^*}\ implies\ u \geq_{L^*} 1$
(C) $F(u, u, 1, 1, u) \geq_{L^*} 0_{L^*}\ implies\ u \geq_{L^*} 1$.

Example 2.2. Define F: ${L^*}^5 \to L^*$ as
$F(t_1, t_2, t_3, t_4, t_5) = 11t_1 - 12t_2 + 6t_3 - 8t_4 + 3t_5$. F satisfies all condition (A), (B), (C). Therefore, $F \in \mathrm{M}_5$.

3. Main results

We now establish the following results:

Theorem 3.1. Let *A*, *B*, *S and T* be self-mappings of a modified intuitionistic fuzzy metric space $(X, M_{M,N}, T)$ satisfying the following conditions that:
(i) the pair (A, S) (or (B, T)) satisfies the property (E.A));
(ii) for any x, y ∈ X, $F \in \mathrm{M}_5$ and for all t > 0, there exists α ∈(0,1) such that
$$F(M_{M,N}(Ax, By, \alpha t), M_{M,N}(Sx, Tx, t), M_{M,N}(Sx, Ax, t), M_{M,N}(Ty, By, t),$$
$$M_{M,N}(Ax, Ty, t) * M_{M,N}(Sx, By, t)) \geq_{L^*} 0_{L^*}$$
(ii) *A*(*X*) ⊂*T*(*X*) (*or B*(*X*) ⊂ *S*(*X*)).
Then the pairs (A, S) and (B, T) share the common property (E.A).
Proof. Suppose that the pair (*A*, *S*) satisfies property (*E.A*), then there exists a sequence $\{ x_n \}$ in *X* such that $\lim_{n\to\infty} A\,x_n = \lim_{n\to\infty} S\,x_n = z$ for some z ∈ X. Since *A*(*X*) ⊂*T*(*X*), therefore, for each x_n , there exist y_n in *X* such that $\mathrm{A}x_n = Ty_n$. This gives,
$\lim_{n\to\infty} A\,x_n = \lim_{n\to\infty} S\,x_n = \lim_{n\to\infty} Ty_n = z$.
Now, we claim that $\lim_{n\to\infty} By_n = z$.
Applying inequality (ii), we obtain
$$F(M_{M,N}(Ax_n, By_n, \alpha t), M_{M,N}(Sx_n, Ty_n, t), M_{M,N}(Sx_n, Ax_n, t), M_{M,N}(Ty_n, By_n, t),$$
$$M_{M,N}(Ax_n, Ty_n, t) * M_{M,N}(Sx_n, By_n, t)) \geq_{L^*} 0_{L^*}$$

Taking limit as $n \to \infty$

$$F(M_{M,N}\left(z, \lim_{n\to\infty} By_n, \alpha t\right), M_{M,N}(z,z,t), M_{M,N}(z,z,t), M_{M,N}\left(z, \lim_{n\to\infty} By_n, t\right), M_{M,N}(z,z,t) * M_{M,N}\left(z, \lim_{n\to\infty} By_n, t\right)) \geq_{L^*} 0_{L^*}$$

implies that

$$F(M_{M,N}\left(z, \lim_{n\to\infty} By_n, t\right), 1_{L^*}, 1_{L^*}, M_{M,N}\left(z, \lim_{n\to\infty} By_n, t\right), M_{M,N}\left(z, \lim_{n\to\infty} By_n, t\right)) \geq_{L^*} 0_{L^*}$$

Using (B)

$$M_{M,N}\left(z, \lim_{n\to\infty} By_n, t\right) \geq 1_{L^*}$$

Hence $M_{M,N}\left(z, \lim_{n\to\infty} By_n, t\right) = 1_{L^*}$.

Therefore $\lim_{n\to\infty} By_n = z$.

Hence the pairs (A, S) and (B, T) share the common property.

Similarly, if the pair (B, T) satisfies property (*E.A*) and $B(X) \subset S(X)$, then pairs (A, S) and (B, T) share the common property (*E.A*).

Theorem 3.2. Let *A*, *B*, *S and T* be self-mappings of a modified intuitionistic fuzzy metric space $(X, M_{M,N}, T)$satisfying the following conditions that:

(i) for any x, y ∈ X, $F \in M_5$ and for all t > 0, there exists $\alpha \in (0,1)$ such that

$$F(M_{M,N}(Ax, By, \alpha t), M_{M,N}(Sx, Ty, t), M_{M,N}(Sx, Ax, t), M_{M,N}(Ty, By, t), M_{M,N}(Ax, Ty, t) * M_{M,N}(Sx, By, t)) \geq_{L^*} 0_{L^*}$$

(ii) the pairs (A, S) and (B, T) share the property (E.A);

(iii) S(X) and T(X) are closed subsets of X.

Then each of the pairs (A, S) and (B, T) have a point of coincidence. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.

Proof. Since the pairs (A, S) and (B, T) share the property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} A\, x_n = \lim_{n\to\infty} S\, x_n = \lim_{n\to\infty} Ty_n = z$ for some z ∈ X. S(X) is closed subset of X, there exists a point $u \in X$ such that $z = Su$.

We, now claim that $Au = z$. By (i), we have

$$F(M_{M,N}(Au, By_n, \alpha t), M_{M,N}(Su, Ty_n, t), M_{M,N}(Su, Au, t), M_{M,N}(Ty_n, By_n, t), M_{M,N}(Au, Ty_n, t) * M_{M,N}(Su, By_n, t)) \geq_{L^*} 0_{L^*}$$

Taking limit as $n \to \infty$,

$$F(M_{M,N}(Au, z, \alpha t), M_{M,N}(z,z,t), M_{M,N}(z, Au, t), M_{M,N}(z,z,t), M_{M,N}(Au, z, t) * M_{M,N}(z,z,t)) \geq_{L^*} 0_{L^*}$$

$$F(M_{M,N}(Au, z, t), 1_{L^*}, M_{M,N}(z, Au, t), 1_{L^*}, M_{M,N}(Au, z, t)) \geq_{L^*} 0_{L^*}$$

Using implicit relations (A) we have

$M_{M,N}(Au, z, t) \geq 1_{L^*}$.

Hence $M_{M,N}(Au, z, t) = 1_{L^*}$.

Therefore, $Au = z = Su$ which shows that u is a coincidence point of the pair (A, S). Since $T(X)$ is also a closed subset of X, therefore, $\lim_{n\to\infty} Ty_n = z$ in $T(X)$ and hence there exists $v \in X$ such that $Tv = z = Au = Su$. Now, we show that $Bv = z$.
By using inequality (i), we have

$$F(M_{M,N}(Au,Bv,\alpha t), M_{M,N}(Su,Tv,t), M_{M,N}(Su,Au,t), M_{M,N}(Tv,Bv,t), M_{M,N}(Au,Tv,t) * M_{M,N}(Su,Bv,t)) \geq_{L^*} 0_{L^*}$$

it follows

$$F(M_{M,N}(z,Bv,\alpha t), M_{M,N}(z,z,t), M_{M,N}(z,z,t), M_{M,N}(z,Bv,t), M_{M,N}(z,z,t) * M_{M,N}(z,Bv,t)) \geq_{L^*} 0_{L^*}$$

$$F(M_{M,N}(z,Bv,t), 1_{L^*}, 1_{L^*}, M_{M,N}(z,Bv,t), M_{M,N}(z,Bv,t)) \geq_{L^*} 0_{L^*}$$

Using implicit relations (*B*) we get

$M_{M,N}(z,Bv,t) \geq 1_{L^*}$.

Hence, $M_{M,N}(z, Bv, t) = 1_{L^*}$.

Therefore, $Bv = z = Tv$, which shows that v is a coincidence point of the pair (B, T).
Moreover, since the pairs (A, S) and (B, T) are weakly compatible and $Au = Su$, $Bv = Tv$, therefore, $Az = ASu = SAu = Sz$, $Bz = BTv = TBv = Tz$.

Next, we claim that $Az = z$ for showing the existence of a fixed point of A. By using inequality (i), we have

$$F(M_{M,N}(Az,Bv,\alpha t), M_{M,N}(Sz,Tv,t), M_{M,N}(Sz,Az,t), M_{M,N}(Tv,Bv,t), M_{M,N}(Az,Tv,t) * M_{M,N}(Sz,Bv,t)) \geq_{L^*} 0_{L^*}$$

$$F(M_{M,N}(Az,z,\alpha t), M_{M,N}(Az,z,t), M_{M,N}(Az,Az,t), M_{M,N}(z,z,t), M_{M,N}(Az,z,t) * M_{M,N}(Az,z,t)) \geq_{L^*} 0_{L^*}$$

$$F(M_{M,N}(Az,z,t), M_{M,N}(Az,z,t), 1, 1, M_{M,N}(Az,z,t)) \geq_{L^*} 0_{L^*}$$

On using implicit relations (*C*)

$M_{M,N}(Az,z,t) \geq 1_{L^*}$.

Hence, $M_{M,N}(Az, z, t) = 1_{L^*}$. Therefore, $Az = z = Sz$.

Similarly, we can prove that $Bz = Tz = z$. Hence, $Az = Bz = Sz = Tz = z$, which implies that z is a common fixed point of A, B, S and T.

Uniqueness. Let w be another common fixed points of A, B, S and T. Then by using (i),

$$F(M_{M,N}(Az,Bw,\alpha t), M_{M,N}(Sz,Tw,t), M_{M,N}(Sz,Az,t), M_{M,N}(Tw,Bw,t), M_{M,N}(Az,Tw,t) * M_{M,N}(Sz,Bw,t) \geq_{L^*} 0_{L^*}$$

it follows that

$$F(M_{M,N}(z,w,\alpha t), M_{M,N}(z,w,t), M_{M,N}(z,z,t), M_{M,N}(w,w,t), M_{M,N}(z,w,t) * M_{M,N}(z,w,t)) \geq_{L^*} 0_{L^*}$$

$$F(M_{M,N}(z,w,t), M_{M,N}(z,w,t), 1_{L^*}, 1_{L^*}, M_{M,N}(z,w,t)) \geq_{L^*} 0_{L^*}$$

Using implicit relations (*C*)

$M_{M,N}(z, w, t) \geq 1_{L^*}$.

Hence, $M_{M,N}(z, w, t) = 1_{L^*}$.

Therefore, z = w, i.e., mappings A, B, S and T have a unique common fixed point.
Taking B = A and T = S in the Theorem 3.2. yields following corollary:

Corollary 3.1. Let A and S be self-mappings of a modified intuitionistic fuzzy metric space $(X, M_{M,N}, T)$satisfying the following conditions that

(i) the pair (A, S) share the property (E.A);

(ii) for any x, y ∈ X, $F \in$ M_5 and for all t > 0, there exists $\alpha \in(0,1)$ such that

(iii) $$F(M_{M,N}(Ax, Ay, \alpha t), M_{M,N}(Sx, Sy, t), M_{M,N}(Sx, Ax, t), M_{M,N}(Sy, Ay, t), M_{M,N}(Ax, Sy, t) * M_{M,N}(Sx, Ay, t)) \geq_{L^*} 0_{L^*}$$

(iv) S(X) is a closed subset of X.

Then A and S each have a point of coincidence. Moreover, if the pair (A, S) is weakly compatible, then A and S have a unique common fixed point.

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