Intern. J. Fuzzy Mathematical Archive
Vol. 12, No. 1, 2017, 11-21
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 14 February 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v12n1a2

International Journal of
Fuzzy Mathematical
Archive

Intuitionistic Fuzzy BRK-ideal of BRK-algebra with Interval-valued Membership and Non Membership Functions

Osama Rashad El-Gendy
Foundation Academy for Sciences and Technology
Batterjee Medical College for Sciences & Technology, Saudi Arabia
Email: dr.usamaelgendy@gmail.com

Received 8 January 2017; accepted 28 January 2017

Abstract. In this paper we introduce the notion of intuitionistic fuzzy BRK-ideal of BRK-algebra with interval-valued membership and non membership functions and investigate some interesting properties. The image and inverse image of interval valued intuitionistic fuzzy BRK-ideal are also discussed.

Keywords: BRK-algebra; BRK-ideal; fuzzy BRK-ideal; interval-valued fuzzy BRK-ideal.

AMS Mathematics Subject Classification (2010): 06F35, 03G25, 08A27

1. Introduction

It is known that mathematical logic is a discipline used in sciences and humanities with different point of view. Non-classical logic takes the advantage of the classical logic (two-valued logic) to handle information with various facts of uncertainty. The non-classical logic has become a formal and useful tool for computer science to deal with fuzzy information and uncertain information. The notion of logical algebras: BCK-algebras [11] was initiated by Imai and Iséki in 1966 as a generalization of both classical and non-classical positional calculus. In the same year, Iséki introduced BCI-algebras [12] as a super class of the class of BCK-algebras. In 1983, Hu and Li introduced BCH-algebras [10]. They demonstrated that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. In [15], Neggers et al. introduced Q-algebras which is a generalization of BCK / BCI-algebras and obtained several results. In 2002, Neggers and Kim [14] introduced a new notion, called a B-algebra, and obtained several results. In 2007, Walendziak [18] introduced a new notion, called a BF-algebra, which is a generalization of B-algebra. In [13], Kim and Kim introduced BG-algebra as a generalization of B-algebra. In [16], T. Senapati, M. Bhowmik and M. Pal introduced Interval-valued intuitionistic fuzzy BG-subalgebras also see [6, 17] .In 2012, Bandaru [5] introduces a new notion, called BRK-algebra which is a generalization of BCK / BCI / BCH / Q / QS / BM-algebras. Interval-valued fuzzy sets were first introduced by Zadeh [19] as a generalization of fuzzy sets. An interval-valued fuzzy set is a fuzzy set whose membership function is many-valued and forms an interval in the membership scale. This idea gives the simplest method to capture the imprecision of the membership grades for a fuzzy set. Thus, interval-valued fuzzy sets provide a more adequate description of uncertainty than the traditional fuzzy sets. It is therefore important to use interval-valued fuzzy sets in applications. One of the main applications is in fuzzy control and the most computationally intensive part of fuzzy control is defuzzification. Since the transition of interval-valued fuzzy sets usually increases the amount of computations, it is vitally important to design some faster algorithms for the necessarily defuzzification. On the other hand, Atanassov [1,2,3] introduced the notion of intuitionistic fuzzy sets as an extension of fuzzy set in which not only a membership degree is given, but also a non-membership degree is involved. Considering the increasing interest in intuitionistic fuzzy sets, it is useful to determine the position of intuitionistic fuzzy sets in a frame of different theories of imprecision. With the above background, Atanassov and Gargov [4] introduced the notion of interval-valued intuitionistic fuzzy sets which is a common generalization of intuitionistic fuzzy sets and interval-valued fuzzy sets. The fuzzy structure for BRK-ideal was introduced in [7]. Since then, the concepts and results of BRK-algebra have been broadened to the fuzzy BRK-ideals setting frames also see [8, 9]. In this paper we first apply the concept of interval valued intuitionistic fuzzy sets to BRK-algebra. Then we introduce the notion of Intuitionistic Fuzzy BRK-ideal of BRK-algebra with Interval-valued Membership and Non Membership Functions and investigate some of interesting properties. We study the homomorphic image and inverse image of interval valued intuitionistic fuzzy BRK-ideal of BRK-algebra.

2. Preliminaries

Definition 2.1[5]. A BRK-algebra is a non-empty set *X* with a constant 0 and a binary operation "$*$" satisfying the following conditions:

(BRK_1) $x * 0 = x$,

(BRK_2) $(x * y) * x = 0 * y$, for all $x, y \in X$.

In a BRK-algebra *X*, a partially ordered relation $\leq$ can be defined by $x \leq y$ if and only if $x * y = 0$.

Definition 2.2 [5]. If $(X; *, 0)$ is a BRK-algebra, the following conditions hold:

(1) $x * x = 0$,

(2) $x * y = 0 \Rightarrow 0 * x = 0 * y$,

(3) $0 * (x * y) = (0 * x) * (0 * y)$, for all $x, y \in X$

Definition 2.3. A non empty subset *S* of a BRK-algebra *X* is said to be BRK-subalgebra of *X*, if $x, y \in S$, implies $x * y \in S$.

Definition 2.4. (BRK-ideal of BRK-algebra). A non empty subset *I* of a BRK-algebra *X* is said to be a BRK-ideal of *X* if it satisfies:

(I_1) $0 \in I$,

(I_2) $0 * (x * y) \in I$ and $0 * y \in I$ imply $0 * x \in I$ for all $x, y \in X$.

Definition 2.5. Let *X* be a BRK-algebra. A fuzzy set μ in *X* is called a fuzzy BRK-ideal of *X* if it satisfies:

(FI$_1$) $\mu(0) \geq \mu(x)$,

(FI$_2$) $\mu(0 * x) \geq \min\{\mu(0 * (x * y)), \mu(0 * y)\}$, for all $x, y \in X$.

Example 2.6. Let $X = \{0, a, b, c\}$. Define $*$ on X as the following table:

$*$	0	a	b	c
0	0	b	b	0
a	a	0	0	b
B	b	0	0	b
c	c	a	a	0

Then $(X, *; 0)$ is a BRK-algebra, and A = {0, b, c} is a BRK-ideal of BRK-algebra X.

3. Interval-valued intuitionistic fuzzy homomorphism of BRK-algebra interval-valued intuitionistic fuzzy set

A mapping $A = (\tilde{\mu}, \tilde{\nu}) : X \to D[0,1] \times D[0,1]$ is called an interval-valued intuitionistic fuzzy set (i-v IF set, briefly) in X if $0 \leq \mu_A^{\bullet} + \nu_A^{\bullet} \leq 1$ and $0 \leq \mu_A^{\bullet\bullet} + \nu_A^{\bullet\bullet} \leq 1$ for all $x \in X$. Where $\mu_A^{\bullet}(x), \nu_A^{\bullet}(x)$, $\mu_A^{\bullet\bullet}(x)$ and $\nu_A^{\bullet\bullet}(x)$ are fuzzy sets of X such that $\mu_A^{\bullet} \leq \mu_A^{\bullet\bullet}$ and $\nu_A^{\bullet} \leq \nu_A^{\bullet\bullet}$. $A^{\bullet\bullet} = \{(x, \mu_A^{\bullet\bullet}, \nu_A^{\bullet\bullet}) \mid x \in X\}$ and $A^{\bullet} = \{(x, \mu_A^{\bullet}, \nu_A^{\bullet}) \mid x \in X\}$ are intuitionistic fuzzy sets. Let $D[0,1]$ be the family of all closed sub-interval of $[0,1]$. The mapping

$\tilde{\mu}_A(x) = [\mu_A^{\bullet}(x), \mu_A^{\bullet\bullet}(x)] : X \to D[0,1]$ and $\tilde{\nu}_A(x) = [\nu_A^{\bullet}(x), \nu_A^{\bullet\bullet}(x)] : X \to D[0,1]$

denote the degree membership and degree of non-membership for all $x \in X$ respectively. For simplicity we use the symbols forms $A^{\bullet} = (X, \mu_A^{\bullet}, \nu_A^{\bullet})$ and $A^{\bullet\bullet} = (X, \mu_A^{\bullet\bullet}, \nu_A^{\bullet\bullet})$. The refined minimum (briefly r min) and order "≤" on the subintervals $D_1 = [a_1, b_1]$ and $D_2 = [a_2, b_2]$ of $D[0,1]$ is defined by, $r\min(D_1, D_2) = [\min\{a_1, a_2\}, \min\{b_1, b_2\}]$ where $D_1 \leq D_2 \Leftrightarrow a_1 \leq a_2$ and $b_1 \leq b_2$. Similarly we can define $\geq$ and = .

Definition 3.1. An interval-valued IFS $A = (X, \tilde{\mu}, \tilde{\nu})$ is called interval-valued intuitionistic fuzzy BRK-ideal of BRK-algebra X if it satisfies the following

(IVS 1) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$

(IVS 2) $\tilde{\nu}_A(0) \leq \tilde{\nu}_A(x)$

(IVS 3) $\tilde{\mu}_A(0 * x) \geq \min\{\tilde{\mu}_A(0 * (x * y)), \tilde{\mu}_A(0 * y)\}$

(IVS 4) $\tilde{\nu}_A(0 * x) \leq \max\{\tilde{\nu}_A(0 * (x * y)), \tilde{\nu}_A(0 * y)\}$, for all $x, y \in X$.

Example 3.2. Consider a BRK-algebra $X = \{0, a, b, c\}$ with following table:

$*$	0	a	b	c
0	0	a	0	a
a	a	0	a	0
B	b	a	0	a
c	c	b	c	0

Let *A* be an interval-valued intuitionistic fuzzy set in *X* by

$$\tilde{\mu}_{\mathrm{A}} = \begin{pmatrix} 0 & a & b & c \\ [0.8,0.9] & [0.8,0.9] & [0.2,0.4] & [0.5,0.6] \end{pmatrix}$$

$$\lambda_{\mathrm{A}} = \begin{pmatrix} 0 & a & b & c \\ [0.1,0.2] & [0.1,0.2] & [0.6,0.8] & [0.5,0.7] \end{pmatrix}$$

Then routine calculations give that *A* is an interval-valued intuitionistic fuzzy BRK-ideal of *X*.

Definition 3.3. (Homomorphism of BRK-algebra). Let $(X,*,0)$ and $(Y,*',0')$ be BRK-algebras. A mapping $f: X \to Y$ is said to be a homomorphism if $f(x*y) = f(x)*' f(y)$, for all $x, y \in X$. For any interval-valued intuitionistic fuzzy set $B = (Y, \tilde{\mu}_B, \tilde{\nu}_B)$ in Y we define a new interval-valued $fB = (X, f\tilde{\mu}_B, f\tilde{\nu}_B)$ in *X*, by $f\tilde{\mu}_B = \tilde{\mu}_B(f(x))$ and $f\tilde{\nu}_B = \tilde{\nu}_B(f(x))$ for all $x \in X$.

Proposition 3.4. [3] Let $(X,*,0)$ and $(Y,*',0')$ be BRK-algebras, and a mapping $f: X \to Y$ be a homomorphism of BRK-algebras, then the kernel of f denoted by($\ker(f)$) is a BRK-ideal.

Theorem 3.5. Let $(X,*,0)$ and $(Y,*',0')$ be BRK-algebras. An onto homomorphic image of an i-v intuitionistic fuzzy BRK-ideal of X is also an i-v intuitionistic fuzzy BRK-ideal of *Y*.

Proof: Let $f: X \to Y$ be an onto homomorphism of BRK-algebras. Suppose $A = (Y, \tilde{\mu}_A, \tilde{\nu}_A)$ is the image of an i-v intuitionistic fuzzy BRK-ideal $fA = (X, f\tilde{\mu}_A, f\tilde{\nu}_A)$ of *X*. We have to prove that $A = (Y, \tilde{\mu}_A, \tilde{\nu}_A)$ is an i-v intuitionistic fuzzy BRK-ideal of *Y*.

Since $f: X \to Y$ is onto, then for any $x', y' \in Y$ there exist $x, y \in X$ such that $f(x) = x'$ and $f(y) = y'$ also we consider $f(0) = 0'$. Then we have

$$\tilde{\mu}_A(0') = \tilde{\mu}_A(f(0)) = f\tilde{\mu}_A(0) \geq f\tilde{\mu}_A(x) = \tilde{\mu}_A(f(x)) = \tilde{\mu}_A(x')$$

And

$$\tilde{\nu}_A(0') = \tilde{\nu}_A(f(0)) = f\tilde{\nu}_A(0) \leq f\tilde{\nu}_A(x) = \tilde{\nu}_A(f(x)) = \tilde{\nu}_A(x')$$

Also

$$\tilde{\mu}_A(0' *' x') = \tilde{\mu}_A(f(0) *' f(x)) = \tilde{\mu}_A(f(0*x))$$
$$= f\tilde{\mu}_A(0*x) \geq \min\{f\tilde{\mu}_A(0*(x*y)), f\tilde{\mu}_A(0*y)\}$$
$$= \min\{\tilde{\mu}_A(f(0*(x*y))), \tilde{\mu}_A(f(0*y))\}$$
$$= \min\{\tilde{\mu}_A(f(0) *' f(x*y)), \tilde{\mu}_A(f(0) *' f(y))\}$$
$$= \min\{\tilde{\mu}_A(f(0) *' (f(x) *' f(y))), \tilde{\mu}_A(f(0) *' f(y))\}$$
$$= \min\{\tilde{\mu}_A(0' *' (x' *' y')), \tilde{\mu}_A(0' *' y')\}$$

and

$\tilde{\nu}_A(0'*'x') = \tilde{\nu}_A(f(0)*'f(x)) = \tilde{\nu}_A(f(0*x))$
$= f\tilde{\nu}_A(0*x) \le \max\{f\tilde{\nu}_A(0*(x*y)), f\tilde{\nu}_A(0*y)\}$
$= \max\{\tilde{\nu}_A(f(0*(x*y))), \tilde{\nu}_A(f(0*y))\}$
$= \max\{\tilde{\nu}_A(f(0)*'f(x*y)), \tilde{\nu}_A(f(0)*'f(y))\}$
$= \max\{\tilde{\nu}_A(f(0)*'(f(x)*'f(y))), \tilde{\nu}_A(f(0*y))\}$
$= \max\{\tilde{\nu}_A(0'*'(x'*'y')), \tilde{\nu}_A(0'*'y')\}$
Hence, $A = (Y, \tilde{\mu}_A, \tilde{\nu}_A)$ is an i-v intuitionistic fuzzy BRK-ideal of *Y*.

Theorem 3.6. Let $(X,*,0)$ and $(Y,*',0')$ be BRK-algebras. An onto homomorphic inverse image of an i-v intuitionistic fuzzy BRK-ideal of Y is also an i-v intuitionistic fuzzy BRK-ideal of *X*.
Proof: Let $f: X \to Y$ be an onto homomorphism of BRK-algebras. Suppose $fA = (X, f\tilde{\mu}_A, f\tilde{\lambda}_A)$ is the inverse image of an i-v intuitionistic fuzzy BRK-ideal $A = (Y, \tilde{\mu}_A, \lambda_A)$ of *Y*. We have to prove that $fA = (X, f\tilde{\mu}_A, f\tilde{\lambda}_A)$ is an i-v intuitionistic fuzzy BRK-ideal of *X*. For any $x', y' \in Y$ there exist $x, y \in X$ such that $f(x) = x'$ and $f(y) = y'$, also we consider $f(0) = 0'$. Then we have
$f\tilde{\mu}_A(0) = \tilde{\mu}_A(f(0)) = \tilde{\mu}_A(0') \ge \tilde{\mu}_A(x') = \tilde{\mu}_A(f(x)) = f\tilde{\mu}_A(x)$
and
$f\tilde{\lambda}_A(0) = \tilde{\lambda}_A(f(0)) = \tilde{\lambda}_A(0') \le \tilde{\lambda}_A(x') = \tilde{\lambda}_A(f(x)) = f\tilde{\lambda}_A(x)$
also
$f\tilde{\mu}_A(0*x) = \tilde{\mu}_A(f(0*x)) = \tilde{\mu}_A(f(0)*'f(x))$
$= \tilde{\mu}_A(0'*'x') \ge \min\{\tilde{\mu}_A(0'*'(x'*'y)), \tilde{\mu}_A(0'*'y')\}$
$= \min\{\tilde{\mu}_A(f(0)*'(f(x)*'f(y))), \tilde{\mu}_A(f(0)*'f(y))\}$
$= \min\{\tilde{\mu}_A(f(0)*'f(x*y)), \tilde{\mu}_A(f(0)*'f(y))\}$
$= \min\{\tilde{\mu}_A(f(0*(x*y))), \tilde{\mu}_A(f(0*y))\}$
$= \min\{f\tilde{\mu}_A(0*(x*y)), f\tilde{\mu}_A(0*y)\}$
and
$f\tilde{\lambda}_A(0*x) = \tilde{\lambda}_A(f(0*x)) = \tilde{\lambda}_A(f(0)*'f(x))$
$= \tilde{\lambda}_A(0'*'x') \le \max\{\tilde{\lambda}_A(0'*'(x'*'y)), \tilde{\lambda}_A(0'*'y')\}$
$= \max\{\tilde{\lambda}_A(f(0)*'(f(x)*'f(y))), \tilde{\lambda}_A(f(0)*'f(y))\}$
$= \{\tilde{\lambda}_A(f(0)*'f(x*y)), \tilde{\lambda}_A(f(0*y))\}$
$= \max\{\tilde{\lambda}_A(f(0*(x*y))), \tilde{\lambda}_A(f(0*y))\}$
$= \max\{f\tilde{\lambda}_A(0*(x*y)), f\tilde{\lambda}_A(0*y)\}$

Hence, $fA = (X, f\widetilde{\mu}_A, f\widetilde{\lambda}_A)$ is an i-v intuitionistic fuzzy BRK-ideal of *X*.

Definition 3.7. An interval-valued intuitionistic subset $A = (X, \widetilde{\mu}, \widetilde{\nu})$ of *X* has "sup" and "inf " properties if for any subset *K* of *X*, there exist $m, n \in K$ such that
$\widetilde{\mu}_A(m) = \sup_{m\in K} \widetilde{\mu}_A(m)$ and $\widetilde{\nu}(n) = \inf_{n\in K} \widetilde{\nu}(n)$.

Definition 3.8. Let f be a mapping from a set *X* to a set *Y*. If $A = (X, \widetilde{\mu}_A, \widetilde{\nu}_A)$ is an interval-valued intuitionistic fuzzy subset X, then the interval-valued intuitionistic fuzzy subset $B = (Y, \widetilde{\mu}_B, \widetilde{\lambda}_B)$ of *Y* is define by

$$\widetilde{\mu}_A \ f^{-1}(y) = \widetilde{\mu}_B(y) = \begin{cases} \sup_{x\in f^{-1}(y)} \widetilde{\mu}_A(x), \ \text{if} \ f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 \qquad\qquad\qquad\qquad \text{otherwise} \end{cases},$$

$$\widetilde{\nu}_A \ f^{-1}(y) = \widetilde{\lambda}_B(y) = \begin{cases} \inf_{x\in f^{-1}(y)} \widetilde{\nu}_A(x), \ \text{if} \ f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 1 \qquad\qquad\qquad\qquad \text{otherwise} \end{cases},$$

for all $y \in Y$ is called the image of $A = (X, \widetilde{\mu}_A, \widetilde{\nu}_A)$ under *f*.

Similarly, if $B = (Y, \widetilde{\mu}_B, \widetilde{\lambda}_B)$ is an interval-valued intuitionistic fuzzy subset of *Y*, then the interval-valued intuitionistic fuzzy subset $A = (X, \widetilde{\mu}_A, \widetilde{\nu}_A)$ of *X* defined by $\widetilde{\mu}_B(f(x)) = \widetilde{\mu}_A(x)$ and $\widetilde{\nu}_A(f(x)) = \widetilde{\lambda}_B(x)$ for all $x \in X$, is said to be the inverse image of $B = (Y, \widetilde{\mu}_B, \widetilde{\lambda}_B)$ under *f*.

In what follows, let *X* denote a BRK-algebra unless otherwise specified, we begin with the following theorem.

Theorem 3.9. An onto homomorphic image of an interval-valued intuitionistic fuzzy BRK-ideal of *X* with sup and inf property is also an interval-valued intuitionistic fuzzy BRK-ideal.

Proof: Let $f : X \to Y$ be an onto homomorphism of BRK-algebras $(X; *, 0)$ and $(Y; *, 0')$, $A = (X, \widetilde{\mu}_A, \widetilde{\nu}_A)$ be an interval-valued intuitionistic fuzzy BRK-ideal of *X* with sup and inf properties and $B = (X', \widetilde{\mu}_B, \widetilde{\lambda}_B)$ is the image of $A = (X, \widetilde{\mu}_A, \widetilde{\nu}_A)$ under *f*. By definition 3.8 we get $\widetilde{\mu}_B(x') = \widetilde{\mu}_A \ f^{-1}(x') = \sup_{x\in f^{-1}(y)} \widetilde{\mu}_A(x) \ (\neq \phi)$ and $\widetilde{\lambda}_B(x') = \widetilde{\nu}_A \ f^{-1}(x') = \inf_{x\in f^{-1}(y)} \widetilde{\nu}_A(x) \ (\neq \phi)$ for all $x' \in Y$, with $\sup \phi = 0$ and $\inf \phi = 1$.

Since $A = (X, \widetilde{\mu}_A, \widetilde{\nu}_A)$ is an interval-valued intuitionistic fuzzy BRK-ideal of *X*, we have $\widetilde{\mu}_A(0) \geq \widetilde{\mu}_A(x)$ and $\widetilde{\nu}_A(0) \leq \widetilde{\nu}_A(x)$ for all $x \in X$. Consider that $0 \in f^{-1}(0')$. Therefore, $\widetilde{\mu}_B(0') = \widetilde{\mu}_A f^{-1}(0') = \sup_{t\in f^{-1}(0')} \widetilde{\mu}_A(t) = \widetilde{\mu}_A(0) \geq \widetilde{\mu}_A(x)$ for all $x \in X$

which implies that $\tilde{\mu}_B(0') \geq \sup_{t\in f^{-1}(y)} \tilde{\mu}_A(t) = \tilde{\mu}_B(x')$ for any $x' \in Y$,

and

$\tilde{\lambda}_B(0') = \tilde{\nu}_A f^{-1}(0') = \inf_{t\in f^{-1}(0')} \tilde{\nu}_A(t) = \tilde{\nu}_A(0) \leq \tilde{\nu}_A(x)$ for all $x \in X$

which implies that $\tilde{\lambda}_B(0') \leq \inf_{t\in f^{-1}(y)} \tilde{\nu}_A(t) = \tilde{\lambda}_B(x')$ for any $x' \in Y$.

For any $x', y', z' \in Y$, let $x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$, and $0_0 \in f^{-1}(0')$ be such that $\tilde{\mu}_A(0_0 * x_0) = \sup_{t\in f^{-1}(0'*x')} \tilde{\mu}_A(t)$, $\tilde{\mu}_A(0_0 * y_0) = \sup_{t\in f^{-1}(0'*y')} \tilde{\mu}_A(t)$ and

$$\tilde{\mu}_A(0_0 * (x_0 * y_0)) = \tilde{\mu}_B\{f(0_0 * (x_0 * y_0))\} = \tilde{\mu}_B(0' * (x' * y'))$$

$$= \sup_{(0_0*(x_0*y_0))\in f^{-1}(0'*(x'*y'))} \tilde{\mu}_A(0_0 * (x_0 * y_0)) = \sup_{t\in f^{-1}(0'*(x'*y'))} \tilde{\mu}_A(t).$$

Then

$$\tilde{\mu}_B(0' * x') = \sup_{t\in f^{-1}(0'*x')} \tilde{\mu}_A(t) = \tilde{\mu}_A(0_0 * x_0) \geq r\min\{\tilde{\mu}_A(0_0 * (x_0 * y_0)), \tilde{\mu}_A(0_0 * y_0)\}$$

$$= r\min\{\sup_{t\in(0'*(x'*y')} \tilde{\mu}_A(t), \sup_{t\in(0'*y')} \tilde{\mu}_A(t)\}$$

$$= r\min\{\tilde{\mu}_B(0' * (x' * y')), \tilde{\mu}_B(0' * y')\}.$$

Also

$\tilde{\nu}_A(0_0 * x_0) = \inf_{t\in f^{-1}(0'*x')} \tilde{\nu}_A(t)$, $\tilde{\nu}_A(0_0 * y_0) = \inf_{t\in f^{-1}(0'*y')} \tilde{\nu}_A(t)$ and

$$\tilde{\nu}_A(0_0 * (x_0 * y_0)) = \tilde{\lambda}_B\{f(0_0 * (x_0 * y_0))\} = \tilde{\lambda}_B(0' * (x' * y'))$$

$$= \inf_{(0_0*(x_0*y_0))\in f^{-1}(0'*(x'*y'))} \tilde{\nu}_A(0_0 * (x_0 * y_0)) = \inf_{t\in f^{-1}(0'*(x'*y'))} \tilde{\nu}_A(t).$$

Then

$$\tilde{\lambda}_B(0' * x') = \inf_{t\in f^{-1}(0'*x')} \tilde{\nu}_A(t) = \tilde{\nu}_A(0_0 * x_0) \leq \max\{\tilde{\nu}_A(0_0 * (x_0 * y_0)), \tilde{\nu}_A(0_0 * y_0)\}$$

$$= \max\{\inf_{t\in(0'*(x'*y')} \tilde{\nu}_A(t), \inf_{t\in(0'*y')} \tilde{\nu}_A(t)\}$$

$$= \max\{\tilde{\lambda}_B(0' * (x' * y')), \tilde{\lambda}_B(0' * y')\}.$$

Hence $B = (X', \tilde{\mu}_B, \tilde{\lambda}_B)$ is an interval-valued intuitionistic fuzzy BRK-ideal of *Y*.

Definition 3.10. Let $A = (X, \tilde{\mu}_A, \tilde{\nu}_A)$ be an interval-valued intuitionistic fuzzy BRK-ideal of *X* and let $(\tilde{\omega}_1, \tilde{\omega}_2) \in D[0,1]$ be such that $\tilde{\omega}_1 + \tilde{\omega}_2 \leq [1,1]$. A non empty set $\tilde{L}(A; \tilde{\omega}_1, \tilde{\omega}_2) := \{x \in X \mid \tilde{\mu}_A(x) \geq \tilde{\omega}_1,\ \tilde{\nu}_A(x) \leq \tilde{\omega}_2\}$ is called an interval-valued intuitionistic level subset of *A*.

Theorem 3.11. Let $A=(X,\tilde{\mu}_A,\tilde{\nu}_A)$ be an interval-valued intuitionistic fuzzy set in *X*. Then *A* is an interval-valued intuitionistic fuzzy BRK-ideal of *X* if and only if the non empty set $\tilde{L}(A;[\tilde{\omega}_1,\tilde{\omega}_2])$ is a BRK-ideal of X, for every $(\tilde{\omega}_1,\tilde{\omega}_2)\in D[0,1]$.

Proof: Assume that $A=(X,\tilde{\mu}_A,\tilde{\nu}_A)$ is an interval-valued intuitionistic fuzzy BRK-ideal of *X* and let $(\tilde{\omega}_1,\tilde{\omega}_2)\in D[0,1]$ be such that $x\in \tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)$. Then

$\tilde{\mu}_A(0)\geq \tilde{\mu}_A(x)\geq \tilde{\omega}_1$, and $\tilde{\nu}_A(0)\leq \tilde{\nu}_A(x)\leq \tilde{\omega}_2$. Therefore $0\in \tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)$.

Let $x,y\in X$ be such that $(0*(x*y)),(0*y)\in \tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)$. Then $\tilde{\mu}_A(0*(x*y))\geq \tilde{\omega}_1$, $\tilde{\mu}_A(0*y)\geq \tilde{\omega}_1, \tilde{\nu}_A(0*(x*y))\leq \tilde{\omega}_2$ and $\tilde{\nu}_A(0*y)\leq \tilde{\omega}_2$ therefore

$$\tilde{\mu}_A(0*x)\geq r\min\{\tilde{\mu}_A(0*(x*y)),\tilde{\mu}_A(0*y)\}\geq r\min\{\tilde{\omega}_1,\tilde{\omega}_1\}=\tilde{\omega}_1$$
$$\tilde{\nu}_A(0*x)\leq \max\{\tilde{\nu}_A(0*(x*y)),\tilde{\nu}_A(0*y)\}\leq \max\{\tilde{\omega}_1,\tilde{\omega}_2\}=\tilde{\omega}_2$$

and so $0*x\in \tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)$.

Thus $\tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)$ is a BRK-ideal of *X*.

Conversely, assume that $\tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)\ (\neq \phi)$ is a BRK-ideal of *X* for every $(\tilde{\omega}_1,\tilde{\omega}_2)\in D[0,1]$.

That is $\tilde{\mu}_A(x)=\tilde{\omega}_1$ and $\tilde{\nu}_A(x)=\tilde{\omega}_2$, for all $x\in X$, since $0\in \tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)$, therefore $\tilde{\mu}_A(0)\geq \tilde{\omega}_1=\tilde{\mu}_A(x)$, and $\tilde{\nu}_A(0)\leq \tilde{\omega}_2=\tilde{\nu}_A(x)$.

Suppose that there exist $x_0,y_0,z_0\in X$ such that

$\tilde{\mu}_A(0*x_0)<r\min\{\tilde{\mu}_A(0*(x_0*y_0)),\tilde{\mu}_A(0*y_0)\}$,

and $\tilde{\nu}_A(0*x_0)>\max\{\tilde{\nu}_A(0*(x_0*y_0)),\tilde{\nu}_A(0*y_0)\}$.

Let $\tilde{\mu}_A(0*(x_0*y_0))=\tilde{\gamma}_1$, $\tilde{\mu}_A(0*y_0)=\tilde{\gamma}_2$ and $\tilde{\mu}_A(0*x_0)=\tilde{\omega}_1$. Then $\tilde{\omega}_1<r\min\{\tilde{\gamma}_1,\tilde{\gamma}_2\}$.

Taking $\pi_1=\frac{1}{2}[\tilde{\mu}_A(0*x_0)+r\min\{\tilde{\mu}_A(0*(x_0*y_0)),\tilde{\mu}_A(0*y_0)\}]$
$=\frac{1}{2}(\tilde{\omega}_1+r\min\{\tilde{\gamma}_1,\tilde{\gamma}_2\})$.

It follows that

$r\min\{\tilde{\gamma}_1,\tilde{\gamma}_2\}>\pi_1=\frac{1}{2}(\tilde{\omega}_1+r\min\{\tilde{\gamma}_1,\tilde{\gamma}_2\})>\tilde{\omega}_1=\tilde{\mu}_A(0*x_0)$.

Also

Let $\tilde{\nu}_A(0*(x_0*y_0))=\tilde{\gamma}_3$, $\tilde{\nu}_A(0*y_0)=\tilde{\gamma}_4$ and $\tilde{\nu}_A(0*x_0)=\tilde{\omega}_2$. Then $\tilde{\omega}_2>\max\{\tilde{\gamma}_3,\tilde{\gamma}_4\}$.

Taking $\pi_2=\frac{1}{2}[\tilde{\nu}_A(0*x_0)+\max\{\tilde{\nu}_A(0*(x_0*y_0)),\tilde{\nu}_A(0*y_0)\}]$
$=\frac{1}{2}(\tilde{\omega}_2+\max\{\tilde{\gamma}_3,\tilde{\gamma}_4\})$.

It follows that

$\max\{\tilde{\gamma}_3,\tilde{\gamma}_4\}<\pi_2=\frac{1}{2}(\tilde{\omega}_2+\max\{\tilde{\gamma}_3,\tilde{\gamma}_4\})<\tilde{\omega}_2=\tilde{\nu}_A(0*x_0)$.

Therefore $0*x_0\notin \tilde{L}(A;[\tilde{\omega}_1,\tilde{\omega}_2])$, on the other hand

$\tilde{\mu}_A(0*(x_0*y_0))=\tilde{\gamma}_1 \geq r\min\{\tilde{\gamma}_1,\tilde{\gamma}_2\}>\tilde{\omega}_1$ and $\tilde{\mu}_A(0*y_0)=\tilde{\gamma}_2 \geq r\min\{\tilde{\gamma}_1,\tilde{\gamma}_2\}>\tilde{\omega}_1$.
Also
$\tilde{\nu}_A(0*(x_0*y_0))=\tilde{\gamma}_3 \leq \max\{\tilde{\gamma}_3,\tilde{\gamma}_4\}<\tilde{\omega}_2$ and $\tilde{\nu}_A(0*y_0)=\tilde{\gamma}_4 \leq \max\{\tilde{\gamma}_3,\tilde{\gamma}_4\}<\tilde{\omega}_2$, and so
$(0*(x_0*y_0)), 0*y_0 \in \tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)$. It contradicts that $\tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)$ is a BRK-ideal of *X*.
Hence
$\tilde{\mu}_A(0*x) \geq r\min\{\tilde{\mu}_A(0*(x*y)),\tilde{\mu}_A(0*y)\}$, and
$\tilde{\nu}_A(0*x) \leq \max\{\tilde{\nu}_A(0*(x*y)),\tilde{\nu}_A(0*y)\}$, for all $x,y\in X$.
Therefore $\tilde{L}(A;\tilde{\omega}_1,\tilde{\omega}_2)$ is an interval-valued intuitionistic fuzzy BRK-ideal of *X*.

Theorem 3.12. Let *Y* be a subset of *X* and let $A=(X,\tilde{\mu}_A,\tilde{\nu}_A)$ be an interval-valued intuitionistic fuzzy set on *X* defined by

$$\tilde{\mu}_A(x)=\begin{cases}\tilde{\alpha}=[\alpha_1,\alpha_2] & \text{if } x\in Y\\ \tilde{\alpha}_\bullet=[0,0] & \text{otherwise}\end{cases},\quad \tilde{\nu}_A(x)=\begin{cases}\tilde{\beta}=[\beta_1,\beta_2] & \text{if } x\in Y\\ \tilde{\beta}_\bullet=[0,0] & \text{otherwise}\end{cases}.$$

Where $\alpha_1,\alpha_2,\beta_1,\beta_2\in(0,1]$ with $\alpha_1<\alpha_2$, $\beta_1<\beta_2$ and $\alpha_i+\beta_i\leq 1$ for $i=1,2$.
If *Y* is a BRK-ideal of *X*, then *A* is an interval-valued intuitionistic fuzzy BRK-ideal of *X*.
Proof: By theorem (3.11) we consider that $Y=\tilde{L}(A;\tilde{\alpha},\tilde{\beta})$. We show that *A* is an interval-valued intuitionistic fuzzy BRK-ideal of *X*.
Since *Y* is a BRK-ideal then $0\in Y$,

$$\tilde{\mu}_A(0)\geq[\alpha_1,\alpha_2]=\tilde{\mu}_A(x), \text{ and } \tilde{\nu}_A(0)\leq[\beta_1,\beta_2]=\tilde{\nu}_A(x).$$

Let $x,y\in X$. If $(0*(x*y)), (0*y)\in Y$, then $(0*x)\in Y$, and so
$\tilde{\mu}_A(0*x)=[\alpha_1,\alpha_2]\geq r\min\{[\alpha_1,\alpha_2],[\alpha_1,\alpha_2]\}=r\min\{\tilde{\mu}_A(0*(x_0*y_0)),\tilde{\mu}_A(0*y)\}$,
$\tilde{\nu}_A(0*x)=[\beta_1,\beta_2]\leq\max\{[\beta_1,\beta_2],[\beta_1,\beta_2]\}=\max\{\tilde{\nu}_A(0*(x_0*y_0)),\tilde{\nu}_A(0*y)\}$.
If $(0*(x*y)), (0*y)\notin Y$, then $(0*x)\notin Y$, and so
$\tilde{\mu}_A(0*(x*y))=\tilde{\alpha}_\bullet=\tilde{\mu}_A(0*y)\}$ and $\tilde{\nu}_A(0*(x*y))=\tilde{\beta}_\bullet=\tilde{\nu}_A(0*y)\}$
$\tilde{\mu}_A(0*x)=\tilde{\alpha}_\bullet\geq r\min\{\tilde{\alpha}_\bullet,\tilde{\alpha}_\bullet\}=r\min\{\tilde{\mu}_A(0*(x*y)),\tilde{\mu}_A(0*y)\}$,
$\tilde{\nu}_A(0*x)=\tilde{\beta}_\bullet\leq\max\{\tilde{\beta}_\bullet,\tilde{\beta}_\bullet\}=\max\{\tilde{\nu}_A(0*(x*y)),\tilde{\nu}_A(0*y)\}$.
If $(0*(x*y))\in Y$ and $(0*y)\notin Y$, then $(0*x)\notin Y$, and so
$\tilde{\mu}_A(0*x)=\tilde{\alpha}_\bullet\geq r\min\{\tilde{\alpha},\tilde{\alpha}_\bullet\}=r\min\{\tilde{\mu}_A(0*(x*y)),\tilde{\mu}_A(0*y)\}$,
$\tilde{\nu}_A(0*x)=\tilde{\beta}_\bullet\leq\max\{\tilde{\beta},\tilde{\beta}_\bullet\}=\max\{\tilde{\nu}_A(0*(x*y)),\tilde{\nu}_A(0*y)\}$
Similarly for the case $(0*(x*y))\notin Y$ and $(0*y)\in Y$ we get
$\tilde{\mu}_A(0*x)\geq r\min\{\tilde{\mu}_A(0*(x*y)),\tilde{\mu}_A(0*y)\}$, and
$\tilde{\nu}_A(0*x)\leq\max\{\tilde{\nu}_A(0*(x*y)),\tilde{\nu}_A(0*y)\}$
Therefore *A* is an interval-valued intuitionistic fuzzy BRK-ideal of *X*.
This completes the proof.

Theorem 3.13. Let *Y* be a subset of *X* and let $A=(X,\tilde{\mu}_A,\tilde{\nu}_A)$ be an interval-valued intuitionistic fuzzy set on *X* defined by

$$\tilde{\mu}_A(x)=\begin{cases}\tilde{\alpha}=[\alpha_1,\alpha_2] & \text{if } x\in Y\\ \tilde{\alpha}_{\bullet}=[0,0] & \text{otherwise}\end{cases}\qquad \tilde{\nu}_A(x)=\begin{cases}\tilde{\beta}=[\beta_1,\beta_2] & \text{if } x\in Y\\ \tilde{\beta}_{\bullet}=[0,0] & \text{otherwise}\end{cases}.$$

where $\alpha_1,\alpha_2,\beta_1,\beta_2\in(0,1]$ with $\alpha_1<\alpha_2$, $\beta_1<\beta_2$ and $\alpha_i+\beta_i\le 1$ for $i=1,2$.

If *A* is an interval-valued intuitionistic fuzzy BRK-ideal of *X*, then *Y* is a BRK-ideal of *X*.

Proof: Assume that *A* is an interval-valued intuitionistic fuzzy BRK-ideal of *X*, let $x\in Y$ $\tilde{\mu}_A(0)\ge\tilde{\mu}_A(x)=[\alpha_1,\alpha_2]$ and $\tilde{\nu}_A(0)\le\tilde{\nu}_A(x)=[\beta_1,\beta_2]$ then $0\in Y$.

If $(0*(x*y)),(0*y)\in Y$, then $\tilde{\mu}_A(0*(x*y))=[\alpha_1,\alpha_2]=\tilde{\mu}_A(0*y)$, and so $\tilde{\mu}_A(0*x)\ge r\min\{\tilde{\mu}_A(0*(x*y)),\tilde{\mu}_A(0*y)\}=r\min\{[\alpha_1,\alpha_2],[\alpha_1,\alpha_2]\}=[\alpha_1,\alpha_2]$, also

$\tilde{\nu}_A(0*x)\le\max\{\tilde{\nu}_A(0*(x*y)),\tilde{\nu}_A(0*y)\}=\max\{[\beta_1,\beta_2],[\beta_1,\beta_2]\}=[\beta_1,\beta_2]$.

This implies that $0*x\in Y$

Hence, *Y* is BRK-ideal of *X*.

4. Conclusions

In this paper, we have introduced the concept of Intuitionistic Fuzzy BRK-ideal of BRK-algebra with Interval-valued Membership and Non Membership Functions and studied their properties. In our future work, we introduce the concept of $(\in,\in\vee q)$ -Fuzzy BRK - ideal of BRK-algebra . I hope this work would serve as a foundation for further studies on the structure of BRK-algebras.

REFERENCES

1. K.T.Atanassov, Intuitionistic fuzzy sets, VIIITKRs Session, Sofia (deposed in Central Science-Technical Library of Bulgarian Academy of Science 1697/84), 1983 (in Bulgarian).
2. K.T.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1) (1986) 87-96.
3. K.T.Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications, Studies in fuzziness and soft computing, vol. 35, Heidelberg, New York, Physica-Verl., 1999.
4. K.T.Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 31(3) (1989) 343 - 349.
5. R.K.Bandaru, On BRK-algebras, *Int. J. Math. Mathematical Sci.*, 2012 (2012), Article ID952654, 12 pp.
6. T.Bej and M.Pal, Doubt Atanassov's intuitionistic fuzzy Sub-implicative ideals in BCI-algebras, *International Journal of Computational Intelligence Systems*, 8 (2) (2015) 240-249.
7. O.R.Elgendy, Fuzzy BRK-ideal of BRK-algebra, *JP Journal of Algebra, Number Theory and Applications*, 36 (3) (2015) 231-240.
8. O.R.Elgendy, Anti Fuzzy BRK-ideal of BRK-algebra, *British J. math. and Comp. Sci.*, 10 (6) (2015) 1-9.
9. O.R.Elgendy, Cubic BRK-ideal of BRK-algebra, *Annals of Fuzzy Mathematics and Informatics*, Submitted, 2016.

10. Q.P.Hu and X.Li, On BCH-algebras, *Math. Sem. Notes Kobe Univ.*, 11 (2) (1983) 313-320.
11. Y.Imai and K.Iséki, On axiom systems of propositional calculi, XIV. Proc. Japan Academy. 42 (1966) 19 -22.
12. K.Iséki, An algebra related with a propositional calculus, *Proc. Japan Acad.,* 42 (1966) 26-29.
13. C.B.Kim and H.S.Kim, On BG-algebras, *Math. J.*, 36 (1992) 15 - 20.
14. J.Neggers and H.S.Kim, On B-algebras, *Math. Vensik* , 54 (2002) 21 - 29.
15. J.Neggers, S.S.Ahn and H.S.Kim, On Q-algebras, *Int. J. Math. Sci.,* 27 (12) (2001) 749 -757.
16. T.Senapati, M.Bhowmik, M.Pal, Interval-valued intuitionistic fuzzy BG-subalgebras, *The Journal of Fuzzy Mathematics*, 20(3) (2012) 707-720.
17. T.Senapati, M.Bhowmik, M.Pal, Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy H-ideals in BCK/BCI-algebras, *Notes on Intuitionistic Fuzzy Sets*, 19 (1) (2013) 32-47.
18. A.Walendziak, On BF- algebras, *Math. Slovaca*., 57 (2) (2007) 119 -128.
19. L.A.Zadeh, The concept of a lingusistic variable and its application to approximate reasoning, *Information Sciences*, 8 (1975) 199- 249.