A New Multiplicative Arithmetic-Geometric Index

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Abstract. In this paper, we propose a new topological index: first multiplicative arithmetic geometric index of a molecular graph. A topological index is a numeric quantity from the structural graph of a molecule. In this paper, we compute multiplicative sum connectivity index, multiplicative product connectivity index, multiplicative atom bond connectivity index, multiplicative geometric-arithmetic index for titania nanotubes. Also we compute the multiplicative arithmetic-geometric index for titania nanotubes.

Keywords: molecular graph, first multiplicative arithmetic-geometric index.

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1. Introduction

Let $G$ be a finite simple connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex $v$ is the number of vertices adjacent to $v$. Any undefined term here may be found in Kulli [1].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of Mathematical chemistry which has an important effect on the development of the chemical sciences. A single number that can be used to characterize some property of the graph of a molecule is called a topological index of that graph. There are numerous molecular descriptors, which are also referred to as topological indices, see [2].

One of the best known and widely used topological index is the product connectivity index or Randić index, introduced by Randić in 1975.

Motivated by the definition of the product connectivity index and its wide applications, Kulli [3] introduced the multiplicative product connectivity index, multiplicative sum connectivity index, multiplicative atom bond connectivity index and multiplicative geometric-arithmetic index of a graph as follows:

The multiplicative sum connectivity index of a graph $G$ is defined as

$$XII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}$$

(1)

The multiplicative product connectivity index of a graph $G$ is defined as
The multiplicative atom bond connectivity index of a graph $G$ is defined as

$$\chi_{II}(G) = \prod_{uv \in E(G)} \frac{1}{d_G(u)d_G(v)}. \quad (2)$$

The multiplicative geometric-arithmetic index of a graph $G$ is defined as

$$ABC_{II}(G) = \prod_{uv \in E(G)} \frac{d_G(u) + d_G(v) - 2}{d_G(u)d_G(v)}. \quad (3)$$

The multiplicative geometric-arithmetic index of a graph $G$ is defined as

$$GA_{II}(G) = \prod_{uv \in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u) + d_G(v)}. \quad (4)$$

These topological indices were also studied, for example, in [4, 5].

Motivated by the definition of the multiplicative geometric-arithmetic index of a molecular graph $G$, we introduce the multiplicative arithmetic-geometric index of a graph as follows:

The multiplicative arithmetic-geometric index of a graph $G$ is defined as

$$AG_{II}(G) = \prod_{uv \in E(G)} \frac{d_G(u) + d_G(v)}{2\sqrt{d_G(u)d_G(v)}}. \quad (5)$$

Many other multiplicative topological indices were studied, for example, in [6, 7, 8, 9, 10, 11, 12].

Numerous technological applications, titania is studied in material science. In this paper, we compute multiplicative connectivity indices of titania nanotubes $TiO_2[m, n]$.

2. Results for titania nanotubes $TiO_2[m, n]$

The titania nanotubes usually symbolized as $TiO_2[m, n]$ for $m, n \in N$, in which $m$ is the number of octagons $C_8$ in a row and $n$ is the number of octagons $C_8$ in a column. The graph of $TiO_2[m, n]$ is shown in Figure 1.

![Figure 1: The graph of TiO$_2$ [m, n] nanotubes with m = 5 and n = 4](image)

By algebraic method, we obtain $|V(TiO_2[m, n])| = 6n(m+1)$ and $|E(TiO_2[m, n])| = 10mn + 8n$. Let $G = TiO_2[m, n]$. 

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Also by algebraic method, we obtain edge partitions of $G$ based on the sum of degrees of the end vertices, as given in Table 1.

<table>
<thead>
<tr>
<th>$d_G(u), d_G(v)/uv \in E(G)$</th>
<th>$E_6 = (2, 4)$</th>
<th>$E_7 = (2, 5)$</th>
<th>$E_8 = (3, 4)$</th>
<th>$E_9 = (3, 5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>$6n$</td>
<td>$4mn + 2n$</td>
<td>$2n$</td>
<td>$6mn - 2n$</td>
</tr>
</tbody>
</table>

**Table 1:** Computing the number of edges for $TiO_2[m, n]$ nanotube

**Theorem 1.** The multiplicative sum connectivity index of $TiO_2$ nanotube is given by

$$XII(TiO_2) = \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{7}\right)^{2mn+2n} \times \left(\frac{1}{8}\right)^{3mn-n}.$$

**Proof:** Let $G = TiO_2[m, n]$. From equation (1) and by cardinalities of the edge partitions of $TiO_2$ nanotube, we have

$$XII(G) = \prod_{uv \in E(G)} \frac{1}{d_G(u) + d_G(v)}$$

$$= \left(\frac{1}{\sqrt{2+4}}\right)^{6n} \times \left(\frac{1}{\sqrt{2+5}}\right)^{4mn+2n} \times \left(\frac{1}{\sqrt{3+4}}\right)^{2n} \times \left(\frac{1}{\sqrt{3+5}}\right)^{6mn-2n}$$

$$= \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{7}\right)^{2mn+2n} \times \left(\frac{1}{8}\right)^{3mn-n}.$$

**Theorem 2.** The multiplicative product connectivity index of $TiO_2$ nanotube is given by

$$\chi II(G) = \left(\frac{1}{8}\right)^{3n} \times \left(\frac{1}{10}\right)^{2mn+n} \times \left(\frac{1}{12}\right)^{n} \times \left(\frac{1}{15}\right)^{3mn-n}.$$

**Proof:** Let $G = TiO_2[m, n]$. From equation (2) and by cardinalities of the edge partitions of $TiO_2$ nanotube, we have

$$\chi II(G) = \prod_{uv \in E(G)} \frac{1}{d_G(u)d_G(v)}$$

$$= \left(\frac{1}{\sqrt{2\times4}}\right)^{6n} \times \left(\frac{1}{\sqrt{2\times5}}\right)^{4mn+2n} \times \left(\frac{1}{\sqrt{3\times4}}\right)^{2n} \times \left(\frac{1}{\sqrt{3\times5}}\right)^{6mn-2n}$$

$$= \left(\frac{1}{8}\right)^{3n} \times \left(\frac{1}{10}\right)^{2mn+n} \times \left(\frac{1}{12}\right)^{n} \times \left(\frac{1}{15}\right)^{3mn-n}.$$

**Theorem 3.** The multiplicative atom bond connectivity index of $TiO_2$ nanotube is given by

$$ABCII(TiO_2) = \left(\frac{1}{2}\right)^{2mn+4n} \times \left(\frac{5}{12}\right)^{n} \times \left(\frac{2}{5}\right)^{3mn-n}.$$

**Proof:** Let $G = TiO_2[m, n]$. From equation (3) and by cardinalities of the edge partitions of $TiO_2$ nanotube, we have
Theorem 4. The multiplicative geometric-arithmetic index of $TiO_2$ nanotube is given by
\[ ABCII(G) = \prod_{u \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u)d_G(v)}} \]
\[ = \left( \frac{2 + 4 - 2}{2 \times 4} \right)^{6n} \times \left( \frac{2 + 5 - 2}{2 \times 5} \right)^{4mn + 2n} \times \left( \frac{3 + 4 - 2}{3 \times 4} \right)^{2n} \times \left( \frac{3 + 5 - 2}{3 \times 5} \right)^{6mn - 2n} \]
\[ = \left( \frac{1}{2} \right)^{2mn + 4n} \times \left( \frac{5}{12} \right)^{n} \times \left( \frac{2}{5} \right)^{3mn - n} . \]

Proof: Let $G = TiO_2 \ [m, n]$. From equation (4) and by cardinalities of the edge partitions of $TiO_2$ nanotube, we have
\[ GAI (TiO_2) = \left( \frac{2\sqrt{2}}{3} \right)^{6n} \times \left( \frac{\sqrt{10}}{7} \right)^{4mn + 2n} \times \left( \frac{\sqrt{3}}{7} \right)^{2n} \times \left( \frac{\sqrt{15}}{4} \right)^{6mn - 2n} . \]

Theorem 5. The multiplicative arithmetic-geometric index of $TiO_2$ nanotube is given by
\[ AGII(TiO_2) = \left( \frac{3}{2\sqrt{2}} \right)^{6n} \times \left( \frac{7}{2\sqrt{10}} \right)^{4mn + 2n} \times \left( \frac{7}{4\sqrt{3}} \right)^{2n} \times \left( \frac{4}{\sqrt{15}} \right)^{6mn - 2n} . \]

Proof: Let $G = TiO_2 \ [m, n]$. From equation (5) and by cardinalities of the edge partitions of $TiO_2$ nanotube, we have
\[ AGII(G) = \prod_{u \in E(G)} \sqrt{\frac{d_G(u) + d_G(v)}{d_G(u)d_G(v)}} \]
\[ = \left( \frac{2 + 4}{2\sqrt{2} \times 4} \right)^{6n} \times \left( \frac{2 + 5}{2\sqrt{2} \times 5} \right)^{4mn + 2n} \times \left( \frac{3 + 4}{2\sqrt{3} \times 4} \right)^{2n} \times \left( \frac{3 + 5}{2\sqrt{3} \times 5} \right)^{6mn - 2n} \]
\[ = \left( \frac{3}{2\sqrt{2}} \right)^{6n} \times \left( \frac{7}{2\sqrt{10}} \right)^{4mn + 2n} \times \left( \frac{7}{4\sqrt{3}} \right)^{2n} \times \left( \frac{4}{\sqrt{15}} \right)^{6mn - 2n} . \]
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