Intern. J. Fuzzy Mathematical Archive
Vol. 12, No. 2, 2017, 49-53
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 15 April 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v12n2a1

International Journal of
Fuzzy Mathematical
Archive

A New Multiplicative Arithmetic-Geometric Index

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli@gmail.com

Received 6 April 2017; accepted 15 April 2017

Abstract. In this paper, we propose a new topological index: first multiplicative arithmetic geometric index of a molecular graph. A topological index is a numeric quantity from the structural graph of a molecule. In this paper, we compute multiplicative sum connectivity index, multiplicative product connectivity index, multiplicative atom bond connectivity index, multiplicative geometric-arithmetic index for titania nanotubes. Also we compute the multiplicative arithmetic-geometric index for titania nanotubes.

Keywords: molecular graph, first multiplicative arithmetic-geometric index.

AMS Mathematics Subject Classification (2010): 05C05, 05C12, 05C35

1. Introduction

Let G be a finite simple connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. Any undefined term here may be found in Kulli [1].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of Mathematical chemistry which has an important effect on the development of the chemical sciences. A single number that can be used to characterize some property of the graph of a molecule is called a topological index of that graph. There are numerous molecular descriptors, which are also referred to as topological indices, see [2].

One of the best known and widely used topological index is the product connectivity index or Randić index, introduced by Randić in 1975.

Motivated by the definition of the product connectivity index and its wide applications, Kulli [3] introduced the multiplicative product connectivity index, multiplicative sum connectivity index, multiplicative atom bond connectivity index and multiplicative geometric-arithmetic index of a graph as follows:

The multiplicative sum connectivity index of a graph G is defined as

$$XII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}} \tag{1}$$

The multiplicative product connectivity index of a graph G is defined as

$$\chi II(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)d_G(v)}}. \tag{2}$$

The multiplicative atom bond connectivity index of a graph G is defined as

$$ABCII(G)=\prod_{uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}. \tag{3}$$

The multiplicative geometric-arithmetic index of a graph G is defined as

$$GAII(G)=\prod_{uv\in E(G)}\frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}. \tag{4}$$

These topological indices were also studied, for example, in [4, 5].

Motivated by the definition of the multiplicative geometric-arithmetic index of a molecular graph G,we introduce the multiplicative arithmetic-geometric index of a graph as follows:

The multiplicative arithmetic-geometric index of a graph G is defined as

$$AGII(G)=\prod_{uv\in E(G)}\frac{d_G(u)+d_G(v)}{2\sqrt{d_G(u)d_G(v)}}. \tag{5}$$

Many other multiplicative topological indices were studied, for example, in [6, 7, 8, 9, 10, 11, 12].

Numerous technological applications, titania is studied in material science. In this paper, we compute multiplicative connectivity indices of titania nanotubes $TiO_2[m, n]$.

2. Results for titania nanotubes TiO_2 [*m*, *n*]

The titania nanotubes usually symbolized as $TiO_2[m, n]$ for $m, n \in N$, in which m is the number of octagons C_8 in a row and n is the number of octagons C_8 in a column. The graph of TiO_2 [*m*, *n*] is shown in Figure 1.

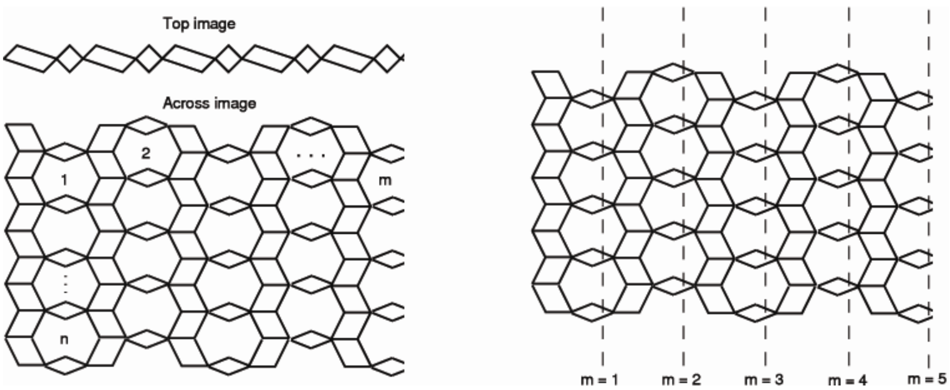

Figure 1: The graph of TiO_2 [*m*, *n*] nanotubes with $m = 5$ and $n= 4$

By algebraic method, we obtain $|V(TiO_2 [m, n])| = 6n(m+1)$ and $|E(TiO_2 [m, n])| = 10mn + 8n$. Let $G = TiO_2 [m, n]$.

Also by algebraic method, we obtain edge partitions of G based on the sum of degrees of the end vertices, as given in Table 1.

$d_G(u), d_G(v)/uv\in E(G)$	$E_6 = (2, 4)$	$E_7=(2, 5)$	$E_7 = (3, 4)$	$E_8 = (3, 5)$
Number of edges	$6n$	$4mn + 2n$	$2n$	$6mn - 2n$

Table 1: Computing the number of edges for $TiO_2[m, n]$ nanotube

Theorem 1. The multiplicative sum connectivity index of TiO_2 nanotube is given by

$$XII(TiO_2)=\left(\frac{1}{6}\right)^{3n}\times\left(\frac{1}{7}\right)^{2mn+2n}\times\left(\frac{1}{8}\right)^{3mn-n}.$$

Proof: Let $G = TiO_2\ [m, n]$. From equation (1) and by cardinalities of the edge partitions of TiO_2 nanotube, we have

$$XII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)+d_G(v)}}$$
$$=\left(\frac{1}{\sqrt{2+4}}\right)^{6n}\times\left(\frac{1}{\sqrt{2+5}}\right)^{4mn+2n}\times\left(\frac{1}{\sqrt{3+4}}\right)^{2n}\times\left(\frac{1}{\sqrt{3+5}}\right)^{6mn-2n}$$
$$=\left(\frac{1}{6}\right)^{3n}\times\left(\frac{1}{7}\right)^{2mn+2n}\times\left(\frac{1}{8}\right)^{3mn-n}.$$

Theorem 2. The multiplicative product connectivity index of TiO_2 nanotube is given by

$$\chi II(G)=\left(\frac{1}{8}\right)^{3n}\times\left(\frac{1}{10}\right)^{2mn+n}\times\left(\frac{1}{12}\right)^{n}\times\left(\frac{1}{15}\right)^{3mn-n}.$$

Proof: Let $G = TiO_2\ [m, n]$. From equation (2) and by cardinalities of the edge partitions of TiO_2 nanotube, we have

$$\chi II(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)d_G(v)}}$$
$$=\left(\frac{1}{\sqrt{2\times4}}\right)^{6n}\times\left(\frac{1}{\sqrt{2\times5}}\right)^{4mn+2n}\times\left(\frac{1}{\sqrt{3\times4}}\right)^{2n}\times\left(\frac{1}{\sqrt{3\times5}}\right)^{6mn-2n}$$
$$=\left(\frac{1}{8}\right)^{3n}\times\left(\frac{1}{10}\right)^{2mn+n}\times\left(\frac{1}{12}\right)^{n}\times\left(\frac{1}{15}\right)^{3mn-n}.$$

Theorem 3. The multiplicative atom bond connectivity index of TiO_2 nanotube is given by

$$ABCII(TiO_2)=\left(\frac{1}{2}\right)^{2mn+4n}\times\left(\frac{5}{12}\right)^{n}\times\left(\frac{2}{5}\right)^{3mn-n}$$

Proof: Let $G = TiO_2\ [m, n]$. From equation (3) and by cardinalities of the edge partitions of TiO_2 nanotube, we have

$$ABCII(G)=\prod_{uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}$$

$$=\left(\sqrt{\frac{2+4-2}{2\times 4}}\right)^{6n}\times\left(\sqrt{\frac{2+5-2}{2\times 5}}\right)^{4mn+2n}\times\left(\sqrt{\frac{3+4-2}{3\times 4}}\right)^{2n}\times\left(\sqrt{\frac{3+5-2}{3\times 5}}\right)^{6mn-2n}$$

$$=\left(\frac{1}{2}\right)^{2mn+4n}\times\left(\frac{5}{12}\right)^{n}\times\left(\frac{2}{5}\right)^{3mn-n}.$$

Theorem 4. The multiplicative geometric-arithmetic index of TiO_2 nanotube is given by

$$GAII(TiO_2)=\left(\frac{2\sqrt{2}}{3}\right)^{6n}\times\left(\frac{2\sqrt{10}}{7}\right)^{4mn+2n}\times\left(\frac{4\sqrt{3}}{7}\right)^{2n}\times\left(\frac{\sqrt{15}}{4}\right)^{6mn-2n}.$$

Proof: Let $G = TiO_2\ [m, n]$. From equation (4) and by cordinalities of the edge partitions of TiO_2 nanotube, we have

$$GAII(G)=\prod_{uv\in E(G)}\frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$$

$$=\left(\frac{2\sqrt{2\times 4}}{2+4}\right)^{6n}\times\left(\frac{2\sqrt{2\times 5}}{2+5}\right)^{4mn+2n}\times\left(\frac{2\sqrt{3\times 4}}{3+4}\right)^{2n}\times\left(\frac{2\sqrt{3\times 5}}{3+5}\right)^{6mn-2n}$$

$$=\left(\frac{2\sqrt{2}}{3}\right)^{6n}\times\left(\frac{2\sqrt{10}}{7}\right)^{4mn+2n}\times\left(\frac{4\sqrt{3}}{7}\right)^{2n}\times\left(\frac{\sqrt{15}}{4}\right)^{6mn-2n}.$$

Theorem 5. The multiplicative arithmetic-geometric index of TiO_2 nanotube is given by

$$AGII(TiO_2)=\left(\frac{3}{2\sqrt{2}}\right)^{6n}\times\left(\frac{7}{2\sqrt{10}}\right)^{4mn+2n}\times\left(\frac{7}{4\sqrt{3}}\right)^{2n}\times\left(\frac{4}{\sqrt{15}}\right)^{6mn-2n}.$$

Proof: Let $G = TiO_2\ [m, n]$. From equation (5) and by cardinalities of the edge partitions of TiO_2 nanotube, we have

$$AGII(G)=\prod_{uv\in E(G)}\frac{d_G(u)+d_G(v)}{2\sqrt{d_G(u)d_G(v)}}$$

$$=\left(\frac{2+4}{2\sqrt{2\times 4}}\right)^{6n}\times\left(\frac{2+5}{2\sqrt{2\times 5}}\right)^{4mn+2n}\times\left(\frac{3+4}{2\sqrt{3\times 4}}\right)^{2n}\times\left(\frac{3+5}{2\sqrt{3\times 5}}\right)^{6mn-2n}$$

$$=\left(\frac{3}{2\sqrt{2}}\right)^{6n}\times\left(\frac{7}{2\sqrt{10}}\right)^{4mn+2n}\times\left(\frac{7}{4\sqrt{3}}\right)^{2n}\times\left(\frac{4}{\sqrt{15}}\right)^{6mn-2n}.$$

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. I.Gutman and N.Trinajstić, Graph theory and molecular orbitals. Total □-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.* 17, (1972) 535-538.
3. V.R.Kulli, Multiplicative connectivity indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 12(2) (2016) 169-176.
4. V.R.Kulli, Multiplicative connectivity indices of $TUC_4C_8[m,n]$ and $TUC_4[m,n]$ nanotubes, *Journal of Computer and Mathematical Sciences*, 7(11) (2016) 599-605.
5. V.R.Kulli, Multiplicative connectivity indices of nanostructures, *Journal of Ultra Scientist of Physical Sciences*, A, 29(1) (2017) 1-10.
6. V.R.Kulli, Some new multiplicative geometric-arithmetic indices, *Journal of Ultra Scientist of Physical Sciences*, A, 29(2) (2017) 52-57.
7. M.Eliasi, A.Iranmanesh and I.Gutman, Multiplicative versions of first Zagreb index, *MATCH Commun. Math. Comput. Chem.*, 68 (2012) 217-230.
8. I.Gutman, Multiplicative Zagreb indices of trees, *Bull. Soc. Math. Banja Luka*, 18 (2011) 17-23.
9. V.R.Kulli, First multiplicative *K* Banhatti index and coindex of graphs, *Annals of Pure and Applied Mathematics*, 11(2) (2016) 79-82.
10. V.R.Kulli, Second multiplicative *K* Banhatti index and coindex of graphs, *Journal of Computer and Mathematical Sciences*, 7(5) (2016) 254-258.
11. V.R.Kulli, On *K* hyper-Banhatti indices and coindices of graphs, *International Journal of Mathematical Archive,* 7(6) (2016) 60-65.
12. V.R.Kulli, On multiplicative *K* Banhatti and multiplicative *K* hyper-Banhatti indices of *V*-Phenylenic nanotubes and nanotorus, *Annals of Pure and Applied Mathematics*, 11(2) (2016) 145-150.